Problem Set 2
Labor Economics
Prof. Taber
Due: Wed Oct. 7 in class

We will think of a number of different models of compensating differentials. For some of these you can solve with pencil and paper, but for many you will have to solve numerically using whatever software you prefer. The of a job amenity A which is a good thing and is 1 if the amenity is present at the job and 0 if not (it could be a health insurance plan). We think that people prefer working at jobs with A but A is costly to provide for the firm. (The fact that it is a positive amenity rather than a negative will change things some)

I will add one thing to this that we didn't have in lecture notes. There is free entry in the market of firms with a high cost of providing the goods-so in equilibrium it must be that firms with $A = 0$ make no profit. Assume this throughout these problems.

Think of the following cases.

1. Workers are all equally productive with preferences

$$U_i(C, A) = C + \alpha_i A.$$

Assume that α_i is uniform [0,1].

Profits of firms can be written as

$$f - \beta_j A - W_A$$

where β_j is the cost of providing the amenity and W_A is the wage they pay depending on the amenity. Suppose β_j is uniform $[\underline{\beta}, \overline{\beta}]$ figure out the two wages and the fraction of jobs that have the amenity (as a function of $f, \underline{\beta}$, and $\overline{\beta}$).

2. Same as 1, but rather than assuming β_j is uniform assume that $\log(\beta_j)$ it has a logistic distribution:

$$Pr\left(\log(\beta_j) \leq b\right) = \frac{e^b}{1 + e^b} = \frac{1}{e^{-b} + 1}$$

In this case you don't need to solve it numerically, just write down the formula to solve for the compensating differential $\Delta = W_0 - W_1$.

3. Same as 2 but now let α_i have a distribution of

$$\log(\alpha_i) = a + \varepsilon_i$$

where ε_i is logistic.

4. Go back to the conditions in Problem 1 but assume further that β_j is uniform $[0, 1]$ Wages now depend on both θ_i and and A, call that $W_A(\theta_i)$. Assume that ε_i is independent of θ_i. What do $W_A(\theta_i)$ and the fraction of each θ_i doing the job with amenity A look like? In the cross section what will be the relationship between W_A and W_B on average?

5. Now lets change utility to log utility

$$U_i(C, A) = \log(C) + \alpha_i A$$

and assume that $\theta_i = 0.5$ with probability $\frac{1}{2}$ and $\theta_i = 1$ with probability $\frac{1}{2}$. Now $f = 1$. Continue to assume that α_j and β_j are uniform $[0, 1]$. Now what are $W_0(\theta_i), W_1(\theta_i)$ and the fraction of each θ_i doing the job with amenity $A = 1$? Which ability type is more likely to work with the job amenity. In this case you are going to have to figure out a numerical way to solve for the wages. Also keep in mind that it is one set of firms that is hiring both types of workers so the marginal firm will end up being indifferent between four things which type of worker$\times$where to offer A.

6. Now continue to think of log utility and that β_j it has a uniform distribution, but now get rid of heterogeneity across individual in terms of preferences and ability and assume $\alpha_i = 0.5$ for everyone. In a standard compensating differential model what are the wages and proportion doing the two jobs? Now suppose you are a worker facing those wages. What fair lottery would maximize their expected utility? Calculate the expected value with and without the lottery.