Explaining Rising Wage Inequality: Explorations With a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents

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Large increase in earnings inequality in last 20-30 years due to increases in the “Returns to Skill”

In response to these changes, many polices that promote skill formation have been formulated

We develop a dynamic GE model of schooling and OJT

- Skill-biased technical change yields results that match the recent rise in wage inequality
- Prices do not equal wages - inequality growth largest for young workers
- Cross-section estimates of rates of return are misleading

Outline

“Micro” model of schooling and On-The-Job Training

Extending the model to general equilibrium

Empirical methodology and estimates of the model

The changing wage structure

Conclusions and Extensions

A model of Schooling and On the Job Training

An individual chooses human capital investment through school and on the job to maximize the present value of after-tax earnings

$$V^S(\theta) = \sum_{a=1}^{\bar{a}} \left(\frac{1}{1+r}\right)^a E_a$$

We allow individuals to choose between two schooling levels, High School and College. We follow them at the end of high school.

Once they enter the labor force they spend a fraction of their time, I_a^S, on the job investing in human capital, and the rest working.

The earnings in school group S at age a are

$$E_a^S = R_a^S H_a^S \left(1 - I_a^S\right),$$

where R_a^S is the rental rate on human capital and H_a^S is the stock of human capital.

Human capital on the job is produced according to the production function

$$H_{a+1}^S = A^S(\theta) I_a^{\alpha_S} H_a^{\beta_S} + (1 - \sigma^S) H_a^S .$$

We add heterogeneity to the model by assuming that persons can be indexed by θ. We allow the initial stock of human capital in each schooling group, $H_0^S(\theta)$ and the productivity parameter in human capital $A^S(\theta)$ to depend on θ.

For each θ we can solve for the optimal level of human capital investment over the lifecycle and form $V^S(\theta)$.

People choose schooling to maximize lifetime utility

$$\hat{S} = \text{argmax}[V^S(\theta) - D^S - \varepsilon^S],$$

where D^S is the present value of direct costs of schooling, and ε^S is the nonpecuniary benefits of schooling.

The distribution of ε^S may depend on θ.

Individuals make savings and consumption choices to maximize utility

$$\sum_{a=1}^{\bar{a}} \delta^a \frac{C_a^\gamma - 1}{\gamma}$$

subject to the budget constraint,

$$\sum_{a=1}^{\bar{a}} \left(\frac{1}{1+r} \right)^a C_a = V^S(\theta) - D^S.$$

Embedding the model in a General Equilibrium Framework

- Skill is perfectly substitutable across ages, but not substitutable across schooling groups.
- There are three factors of production High School Human Capital, College Human Capital, and Physical Capital.
- The model is embedded in an Auerbach- Kotlikoff style overlapping generations model.

Each period there are $\overline{a}$ cohorts.

We assume that the distribution of heterogeneity θ is identical within cohorts.

We obtain aggregate stocks according to

$$\begin{aligned}
\bar{H}_t^S &= \sum_{t_c=t-a_R}^{t-1} \int H_{t-t_c}^S(\theta, P_{t_c})(1 - I_{t-t_c}^S(\theta, P_{t_c}))N^S(\theta, t_c)dG(\theta) \\
\bar{K}_t &= \sum_{t_c=t-a_R}^{t-1} \int \sum_{s=1}^{\bar{S}} K_{t-t_c}^S(\theta, P_{t_c})N^S(\theta, P_{t_c})dG(\theta).
\end{aligned}$$

Demand Side of Model

We assume a competitive economy with aggregate production function

$$F_t\left(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t\right).$$

The rental rates in each period thus take the form,

$$
\begin{aligned}
R_t^1 &= \frac{\partial F_t\left(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t\right)}{\partial \bar{H}_t^1} \\
R_t^2 &= \frac{\partial F_t\left(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t\right)}{\partial \bar{H}_t^2} \\
r_t &= \frac{\partial F_t\left(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t\right)}{\partial \bar{K}_t}
\end{aligned}
$$

Estimating the Human Capital Production Function

- We use wage and schooling data on white males from the NLSY
- We assume that there are four observable θ types which we define according to AFQT quartile.
- We assume that the interest rate is fixed at $r = 0.05$ and that rental rates are fixed and normalized to one.

For any given (a, θ, S) and any set of parameters π we can calculate the optimal wage

$$w(a, \theta, S; \pi).$$

We assume that these wages are measured with error and we estimate the parameters, π, using nonlinear least squares, minimizing

$$\sum_{i=1}^{N} \sum_{a} \left(w_{i,a}^{*} - w(a, \theta, S; \pi) \right)^2$$

where $w_{i,a}^{*}$ is the observed wage.

Human Capital Production

$$H_{a+1}^S = A^S(\theta) I_a^{\alpha s} H_a^{\beta s} + (1-\sigma) H_a^S$$

$$S = 1, 2$$

	High School ($S = 1$)	College ($S = 2$)
α	0.945(0.017)	0.939(0.026)
β	0.832(0.253)	0.871(0.343)
$A(1)$	0.081(0.045)	0.081(0.072)
$H_{a_R}(1)$	9.530(0.309)	13.622(0.977)
$A(2)$	0.085(0.053)	0.082(0.074)
$H_{a_R}(2)$	12.074(0.403)	14.759(0.931)
$A(3)$	0.087(0.056)	0.082(0.077)
$H_{a_R}(2)$	13.525(0.477)	15.614(0.909)
$A(4)$	0.086(0.054)	0.084(0.083)
$H_{a_R}(4)$	12.650(0.534)	18.429(1.095)

Figure 1: Predicted vs Actual Hourly Wages (in 1992 dollars) by AFQT Quartile (High School Category)

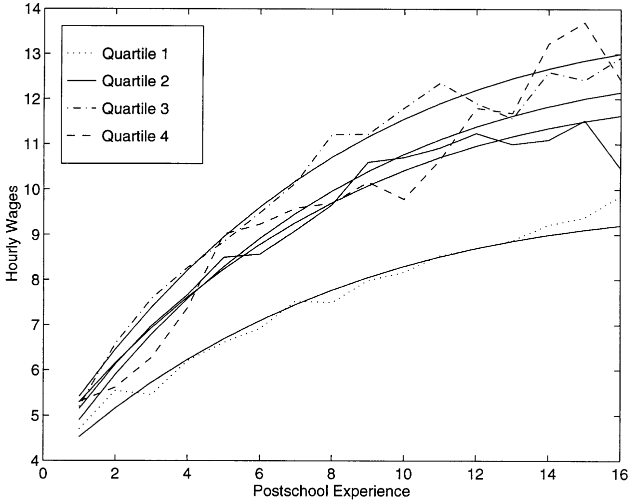


Figure 2: Predicted vs Actual Hourly Wages (in 1992 dollars) by AFQT Quartile (College Category)

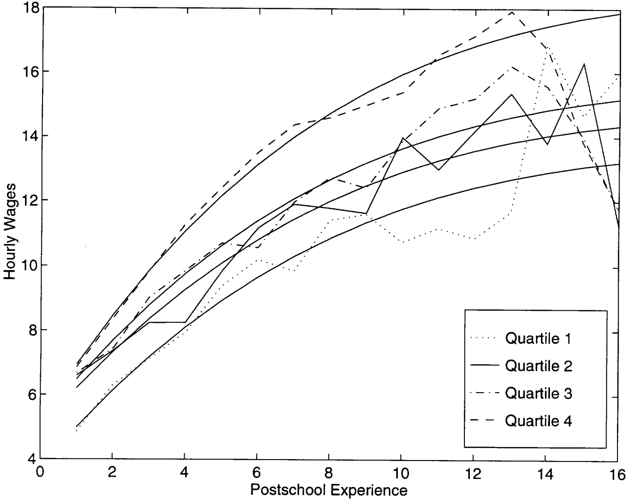


Figure 3A: Comparison of Mincer vs. Estimated Investment Profiles
High School

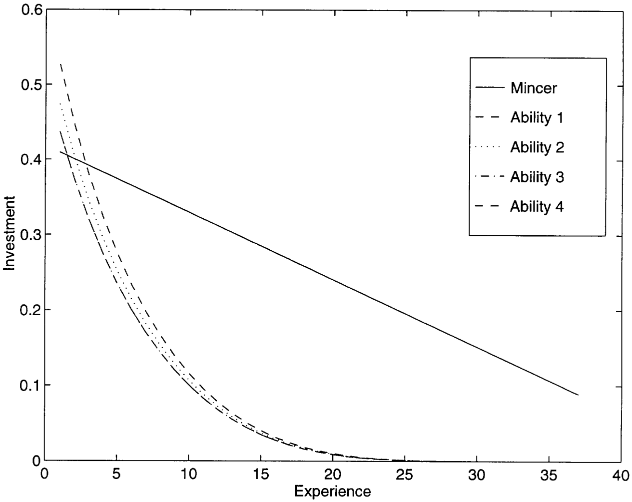
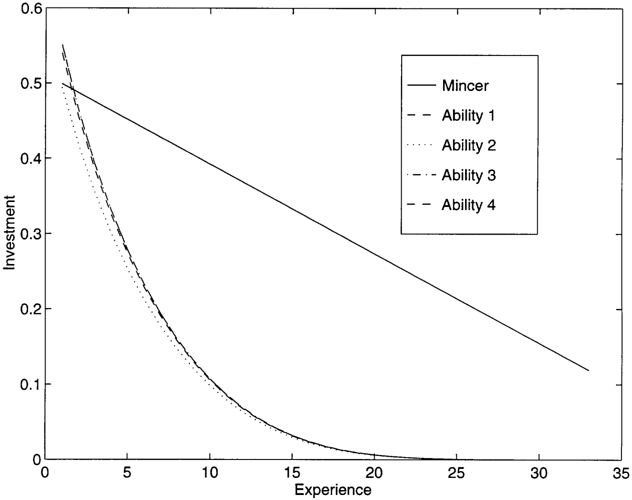


Figure 3B: Comparison of Mincer vs. Estimated Investment Profiles College



Given these estimated parameters, we can obtain the present value of earnings for each type as college graduates or high school graduates, $\widehat{V_\theta^S}$.

for Human Capital Production Function
and Schooling Decision
Units are thousands of Dollars

Human Capital Production		
	High School ($S = 1$)	College ($S = 2$)
$H^S(1)$	8.042(0.0.094)	11.117(0.424)
$H^S(2)$	10.0634(0.118)	12.271(0.325)
$H^S(3)$	11.1273(0.155)	12.960(0.272)
$H^S(4)$	10.361(0.234)	15.095(0.323)
Present Value Earnings 1	260.304(3.939)	289.618(12.539)
Present Value Earnings 2	325.966(5.075)	319.302(10.510)
Present Value Earnings 3	360.717(6.352)	337.260(9.510)
Present Value Earnings 4	335.977(8.453)	393.138(11.442)

College Decision: Attend College if

We assume that the nonpecuniary tastes for college are normally distributed, so

$$\Pr\left(\text{Coll} \mid D^S, \theta\right) = \Phi\left(\frac{(1-\tau)\left(V_\theta^2 - V_\theta^1\right) - D^S + \mu_\theta}{\sigma_\varepsilon}\right)$$

Using data on state tuition we estimate this model as a probit.

College Choice Equation

$P(\delta^2 = 1) = \Lambda(-\lambda D^2 + \alpha(\theta))$

	Probit Parameters	Average Derivatives
λ	0.166(0.062)	-0.0655(0.025)
$\alpha(1)$	-1.058(0.097)	-
$\alpha(2)$	-0.423(0.087)	0.249(0.037)
$\alpha(3)$	0.282(0.089)	0.490(0.029)
$\alpha(4)$	1.272(0.101)	0.715(0.018)
Sample Size:		
Persons	869	1069
Person Years	7996	11626

(1) D^2 is the discounted tuition cost of attending college.

(2) $\alpha(\theta)$ is the nonparametric estimate of $(1-\tau)[V^2(\theta) - V^1(\theta)]$, the monetary

Present Value Earnings 1	[illegible]	[illegible]

College Decision: Attend College if

$$(1-\tau)V^2(\theta) - D^2 + \varepsilon_i \geq (1-\tau)V^1(\theta)$$

$$\varepsilon_\theta \sim N(\mu_\theta, \sigma_\varepsilon)$$

σ_ε (Std. deviation of ε)	22.407(8.425)
Nonpecuniary costs by ability level	
μ_1 (Lowest Ability Quartile)	-53.0190(16.770)
μ_2 (Second Ability Quartile)	-2.8173(12.760)
μ_3 (Third Ability Quartile)	29.7712(11.540)
μ_4 (Highest Ability Quartile)	-28.6494(16.966)

(1) $V^i(\theta)$ is the monetary value of going to schooling level i for a person of AFQT quartile

Estimating/Calibrating Utility and Aggregate Production Parameters

We take

$$\begin{aligned} \delta &= 0.96 \\ \gamma &= 0.10 \end{aligned}$$

We calibrate the model to "look like" the NLSY in the original steady state:

$$\begin{aligned} r &= 0.05 \\ R^1 &= 2.00 \\ R^2 &= 2.00 \end{aligned}$$

In order to match the capital-output ratio, we need a transfer from old cohorts to young. We take an exogenous transfer from a cohort as it retires and give it to a new cohort as it is born. This transfer is approximately $30,000.

We estimate a nested CES production function allowing for a linear time trend in $\log[a_1/(1-a_1)]$

$$a_3\left(a_2\left(a_1(\bar{H}_t^1)^{\rho_1}+(1-a_1)(\bar{H}_t^2)^{\rho_1}\right)^{\rho_2/\rho_1}+(1-a_2)\bar{K}_t^{\rho_2}\right)^{1/\rho_2}$$

Table 3

Estimates of Aggregate Production Function
Estimated from Factor Demand Equations (III-1) and (III-2)
1965-1990
Allowing for Technical Progress Through a Linear Trend
(Standard Errors in Parentheses)

Instruments	ρ_1	Implied Elasticity of Substitution (σ_1)	Time Trend	ρ_2	Implied Elasticity of Substitution (σ_2)	Time Trend
OLS (Base Model)	0.306	1.441	0.036	-0.034	0.967	-0.004
	(0.089)	(0.185)	(0.004)	(0.200)	(0.187)	(0.007)
Percent Working Pop. < 30 & Defense Percent of GNP	0.209	1.264	0.039	-0.036	0.965	-0.004
	(0.134)	(0.215)	(0.005)	(0.200)	(0.187)	(0.007)
Defense Percent of GNP	0.157	1.186	0.041	-0.171	0.854	-0.008
	(0.125)	(0.175)	(0.004)	(0.815)	(0.594)	(0.024)
Percent Working Pop. < 30	0.326	1.484	0.036	0.364	1.572	0.007
	(0.182)	(0.400)	(0.006)	(1.150)	(2.842)	(0.034)

We use $\rho_1 = 1.441$ and $\rho_2 = 0$ based on those estimates.

We calibrate (a_1, a_2, a_3) and the transfer to yield prices (r, R^1, R^2) and a capital-output ratio of 4 in the initial steady state.

Skill-Biased Technical Change

- Economy Starts in Steady State based on NLSY
- Unexepected shock resulting in a constant decline in $\log[a_1/(1-a_1)]$ for 30 years
- Declines by 3.6% as in our estimates
- Perfect foresight
- Transition period of 200 years

Figure 5: Estimated Trend in a_1 for 30 years

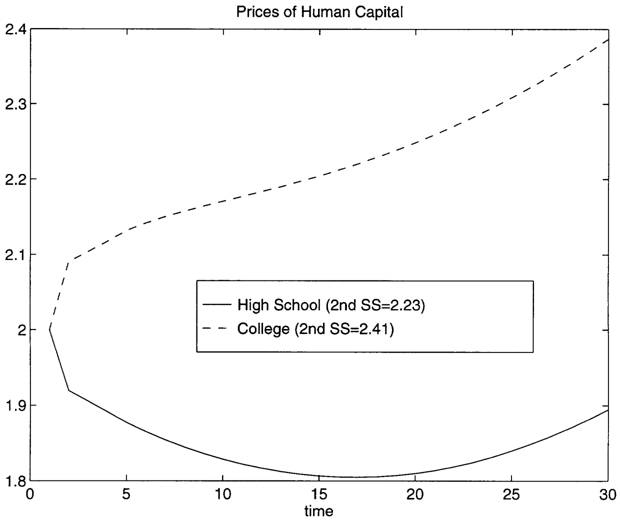
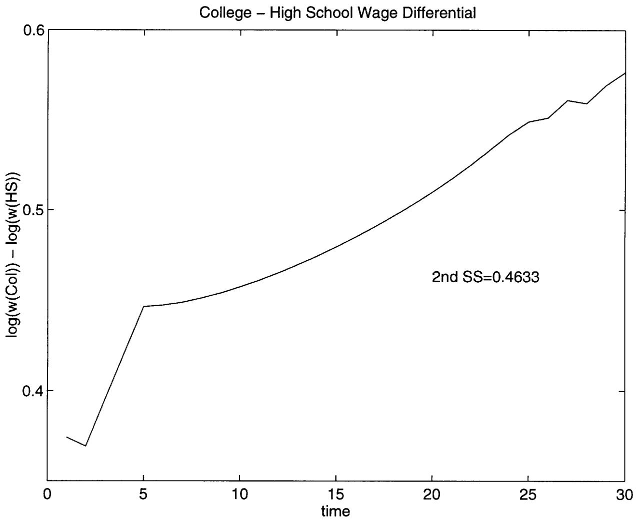


Figure 6: Estimated Trend in a_1 for 30 years



In response to change in relative demand for college graduates

- Increased enrollment in college
- Adjustment occurs only for Young, old do not return to college
- Young “overcompensate” for old
- As old die off, there is an oversupply of college graduates
- Return to College Falls
- This Yields Cycles

Figure 11A: Estimated Trend in a_1 for 30 years

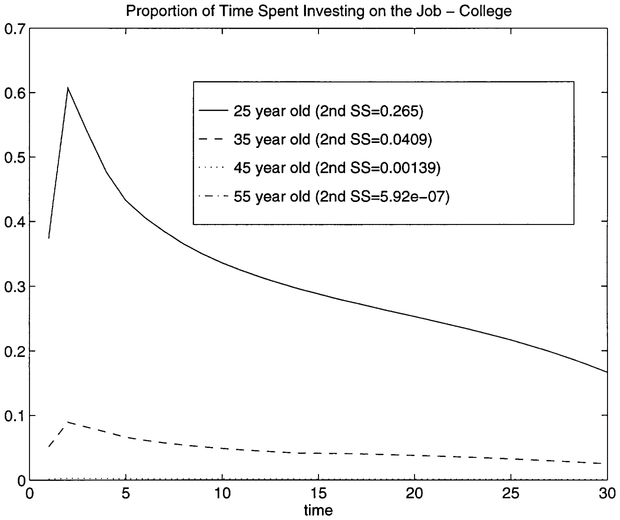


Figure 11B: Estimated Trend in a_1 for 30 years

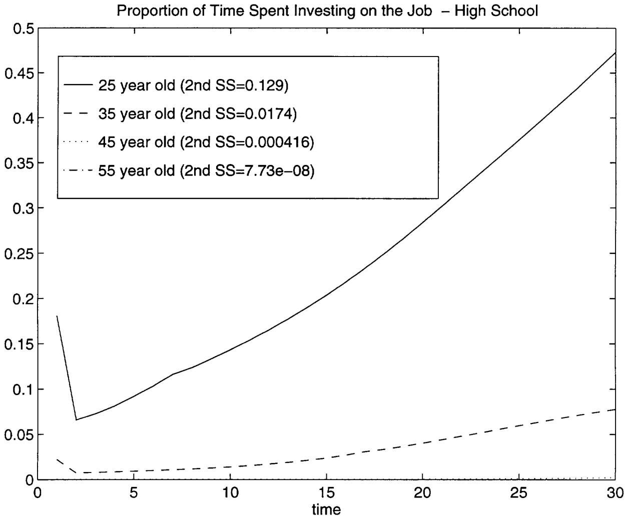


Figure 10: Estimated Trend in a_1 for 30 years

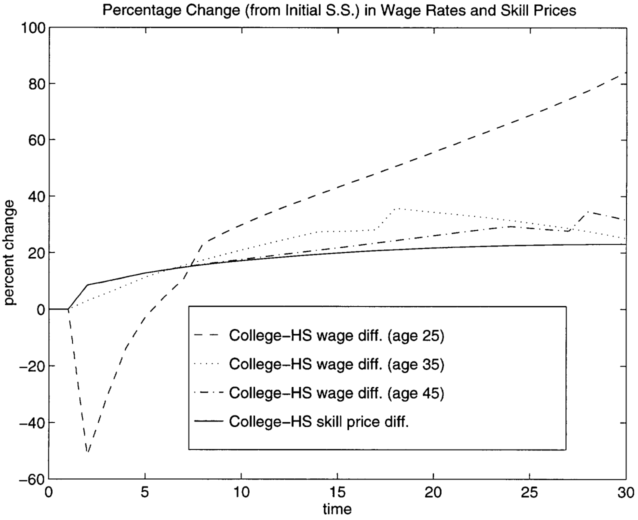


Figure 20: Estimated Trend in a_1 for 30 years
Baby boom (Expansion of Cohort Size by 32%) between years 1965-80

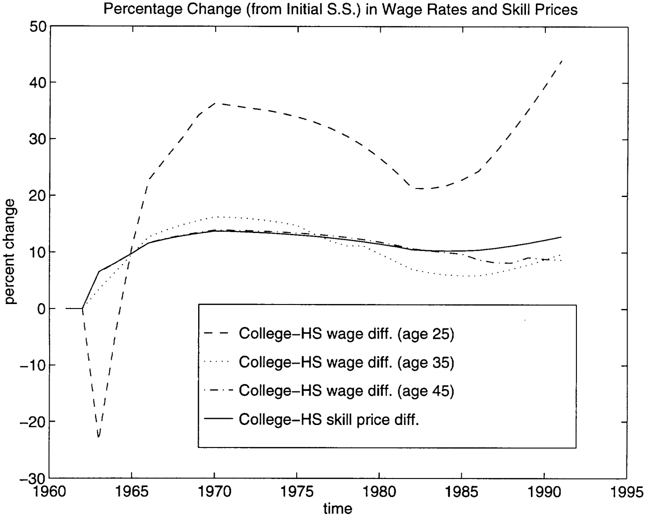


TABLE I

U. S. REAL WEEKLY WAGE CHANGES FOR FULL-TIME WORKERS, 1963–1987[a]

Group	Change in log average real weekly wage (multiplied by 100)			
	1963–1971	1971–1979	1979–1987	1963–1987
All	19.2	−2.8	−0.3	16.1
Gender:				
Men	19.7	−3.4	−2.4	13.9
Women	17.6	−0.8	6.1	22.9
Education (years of schooling):				
8–11	17.1	0.3	−6.6	10.9
12	16.7	1.4	−4.0	14.1
13–15	16.4	−3.4	1.5	14.4
16+	25.5	−10.1	7.7	23.1
Experience (men):				
1–5 years	17.1	−3.5	−6.7	6.8
26–35 years	19.4	−0.6	0.0	18.8
Education and Experience (men):				
Education 8–11				
Experience 1–5	20.5	1.5	−15.8	6.2
Experience 26–35	19.3	−0.4	−1.9	17.0
Education 12				
Experience 1–5	17.4	0.8	−19.8	−1.6
Experience 26–35	14.3	3.2	−2.8	14.7
Education 16+				
Experience 1–5	18.9	−11.3	10.8	18.4
Experience 26–35	28.1	−4.0	1.8	25.9

The Effects of Skill-Biased Technology Change

- Movements in measured wages are different from movements in skill prices, especially for young workers
- Without intervention, economy converges to a new steady state with lower wage inequality than before the technology change
- In the long run, society is richer and all types are better off. In the short run, low ability/low skilled workers caught in the transition are worse off.
- In the new steady state, there are more high skilled workers, but human capital per skilled worker is lower
- During transition periods, cross-section estimates of "returns" to skill are substantially different from the actual returns faced by cohorts making educational decisions

Summary

- We develop an empirically-grounded dynamic overlapping generations general-equilibrium model of skill formation with heterogeneous human capital
- Model roughly consistent with changing wage structure
- We distinguish between effects measured in a cross-section and the effects on different cohorts