Heterogeneous Human Capital

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Multidimensional skills

Now rather than assume human capital is one dimensional, lets make it multidimensional.

Productivity of a worker

$$\pi' H_t$$

Now if π were homogeneous across firms, the fact that we have multiple skills wouldn't be interesting as we could just define General human capital as $\pi' H$

Thus we want to allow π to vary across firms

Furthermore in a completely frictionless market this wouldn't be that interesting-People could just keep working for identical types of firms and it would be pretty much the same as just general human capital though

- There could be vertical differentiation (management skills grow faster so you don't choose a management job when you are young)
- As in general human capital, the rate of accumulation might be different at different firms

Lets put search frictions in to make this more interesting

Allow for search frictions in a simple way:

- outside offers arrive on the job
- bargaining over wages every period-for simplicity threat point is non-employment
- to keep things simple use two period model-more than two doesn't really change things

Solving model

We will work backwards starting from period 2:

Home production: $\pi_h' H_2$

That means that the second period wage at a type π firm is

$$w_2(H_2, \pi) = \delta \pi' H_2 + (1 - \delta)\, \pi_h' H_2.$$

Let π_1 be the first period firm (given outside the model)

For simplicity allow there to be a different human capital function for each dimension of human capital and each with its own input, so

$$H_2^{(m)} = \mathcal{H}^{(m)}\left(s_1^{(m)}\right)$$

and productivity at the first period firm is

$$\pi_1' H_1 \left(1 - \sum_{m=1}^{M} s_1^{(m)}\right).$$

At the beginning of the second period the worker gets an offer from an outside firm with productivity π

(can think of $\pi = 0$ as no offer)

This gives first period value function

$$V_1(H_1, \pi_1, w_1, s_1)$$
$$= w_1 + \frac{1}{R} E_\pi \max\{\delta\pi_1' \mathcal{H}(s_1) + (1-\delta)\pi_h' \mathcal{H}(s_1),$$
$$\delta\pi' \mathcal{H}(s_1) + (1-\delta)\pi_h' \mathcal{H}(s_1)\}$$

We take the value function at home as $V_1^h(H_1)$ and do not need to worry about exactly how it is determined

The value of the match to the firm is

$$
\begin{aligned}
&\Pi_1(H_1, \pi_1, w_1, s_1) \\
&= \left(1 - \sum_{m=1}^{M} s_1^{(m)}\right) \pi_1' H_1 - w_1 \\
&\quad + \frac{1}{R} Pr\left(\pi_1' \mathcal{H}(s_1) > \pi' \mathcal{H}(s_1)\right)(1-\delta)\left[\pi_1' - \pi_h'\right] \mathcal{H}(s_1)
\end{aligned}
$$

To solve generalized Nash Bargaining problem we pick s_1 and w_1 to maximize

$$
\left[V_1(H_1, \pi_1, w_1, s_1) - V_1^h(H_1)\right]^{\delta} \left[\Pi_1(H_1, \pi_1, w_1, s_1)\right]^{1-\delta}
$$

subject to

$$
0 \leq \sum_{m=1}^{M} s_1^{(m)} \leq 1.
$$

The wage that comes out of this is

$$
\begin{aligned}
w_1 = & \delta \left[\left(1 - \sum_{m=1}^{M} s_1^{(m)} \right) \pi_1' H_1 \right. \\
& + \frac{1}{R} Pr \left(\pi_1' \mathcal{H}(s_1) > \pi' \mathcal{H}(s_1) \right) (1-\delta) \left[\pi_1' - \pi_h' \right] \mathcal{H}(s_1) \Big] \\
& + (1-\delta) \Big[V_1^h(H_1) \\
& \left. - \frac{1}{R} E_\pi \left(\max\{ \delta \pi_1' \mathcal{H}(s_1), \delta \pi' \mathcal{H}(s_1) \} + (1-\delta) \pi_h' \mathcal{H}(s_1) \right) \right]
\end{aligned}
$$

and the first order condition for human capital is

$$
\begin{aligned}
&\pi_1' H_1 \\
=&\frac{1}{R}\left[E_\pi\left(1\left[\pi_1'\mathcal{H}(s_1)\leq \pi'\mathcal{H}(s_1)\right]\left[\pi^{(m)}-(1-\delta)\left(\pi^{(m)}-\pi_h^{(m)}\right)\right]\right)\right. \\
&+\Pr\left(\pi_1'\mathcal{H}(s_1)>\pi'\mathcal{H}(s_1)\right)\pi_1^{(m)}+ \\
&\left.+\ \frac{\partial \Pr\left(\pi_1'\mathcal{H}(s_1)>\pi'\mathcal{H}(s_1)\right)}{\partial \mathcal{H}^{(m)}\left(s_1^{(m)}\right)}(1-\delta)\left(\pi_1'-\pi_h'\right)\mathcal{H}(s_1)\right] \\
&\times \frac{\partial \mathcal{H}^{(m)}\left(s_1^{(m)}\right)}{\partial s_1^{(m)}}
\end{aligned}
$$

- First part of this corresponds to switching jobs
- Second corresponds to staying at the same job
- Third is incentive to invest in skills that are likely to keep the worker at the current job (firm rents)

Generalize or specialize?

It turns out one can get two local optima:

- workers specialize in skills important for the current firm-plan to stay
- workers specialize in general skills-plan to leave

Numerical Example:

- Two Skills
  - First is immutable: $\mathcal{H}^{(1)}(s_1^{(1)}) = H_1$ so that $s_1^{(1)} = 0$.
  - Second is firm specific so has no value outside first period firm: $H_2^{(2)} = A\left(s_1^{(2)}\right)^{\alpha}$
- Simulate two versions of this model
  - Outside value of $\pi^{(1)}$ is standard log normal (with an offer for sure)
  - Outside value of $\pi^{(1)}$ is 1.7 with a probability of an offer of 80%

With $\alpha = 0.4, A = 1.5, 1/R = 0.95, H_1 = (1,1), \pi_1 = (1,1)$

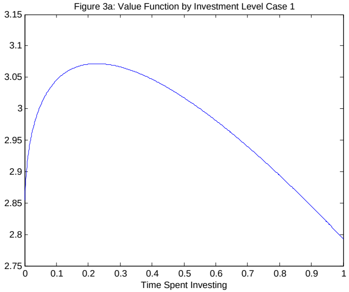
Figure 3a: Value Function by Investment Level Case 1
3.15
3.1
3.05
3
2.95
2.9
2.85
2.8
2.75
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time Spent Investing

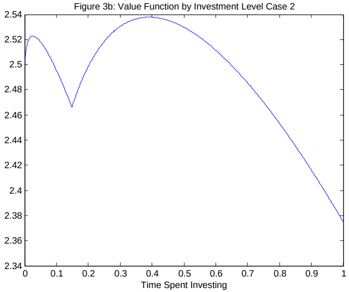

Figure 3b: Value Function by Investment Level Case 2
2.54
2.52
2.5
2.48
2.46
2.44
2.42
2.4
2.38
2.36
2.34
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time Spent Investing

Inneficiencies

Looking at the first order condition one can see that investment is innefficient

$$
\begin{aligned}
&\pi_1' H_1 \\
=&\frac{1}{R}\left[E_\pi\left(1\left[\pi_1'\mathcal{H}\left(s_1\right)\leq\pi'\mathcal{H}\left(s_1\right)\right]\left[\pi^{(m)}-(1-\delta)\left(\pi^{(m)}-\pi_h^{(m)}\right)\right]\right)\right. \\
&+\Pr\left(\pi_1'\mathcal{H}\left(s_1\right)>\pi'\mathcal{H}\left(s_1\right)\right)\pi_1^{(m)}+ \\
&\left.+\frac{\partial\Pr\left(\pi_1'\mathcal{H}\left(s_1\right)>\pi'\mathcal{H}\left(s_1\right)\right)}{\partial\mathcal{H}^{(m)}\left(s_1^{(m)}\right)}(1-\delta)\left(\pi_1'-\pi_h'\right)\mathcal{H}\left(s_1\right)\right] \\
&\times\frac{\partial\mathcal{H}^{(m)}\left(s_1^{(m)}\right)}{\partial s_1^{(m)}}
\end{aligned}
$$

If $\delta = 1$ then the worker would internalize everything and you would get first best.

You can see two problems from first order condition if $\delta < 1$

- Holdup problem: current firm and worker do not internalize rents made by the outside second period firm
- Inneficient invest to keep worker at current firm:
    - Turnover is efficient
    - However, current first loses rents when a marginal worker leaves
    - Thus firm wants to overinvest in specific skills and underinvest in general skills
    - This does not happen if outside offer is threat point-rents on marginal worker are zero

General Human Capital Only

$$\pi_1 H_1 = \frac{1}{R}\left[\Pr\left(\pi_1 \leq \pi\right) E_\pi\left(\pi - (1-\delta)\left(\pi - \pi_h\right) \mid \pi_1 \leq \pi\right)\right.$$
$$\left. + \left[\Pr\left(\pi_1 > \pi\right)\pi_1\right] \frac{\partial \mathcal{H}^{(m)}\left(s_1^{(m)}, H_1\right)}{\partial s_1^{(m)}}\right.$$

You still have holdup problem

Purely firm specific and purely general

Imagine skill 1 is general, skill 2 only has value at current firm

That is in second period if we stay productivity is

$$\pi_1^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)+\pi_1^{(2)}\mathcal{H}^{(2)}\left(s_1^{(2)}\right)$$

but if we leave to another firm it is

$$\pi^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)$$

First order conditions are special cases of the general model above

Consider the term for general human capital

$$\frac{\partial \Pr\left(\pi_1^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)+\pi_1^{(2)}\mathcal{H}^{(2)}\left(s_1^{(2)}\right)>\pi^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)\right)}{\partial \mathcal{H}^{(1)}\left(s_1^{(1)}\right)}$$

this must be negative

Then for specific:

$$\frac{\partial \Pr\left(\pi_1^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)+\pi_1^{(2)}\mathcal{H}^{(2)}\left(s_1^{(2)}\right)>\pi^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)\right)}{\partial \mathcal{H}^{(2)}\left(s_1^{(2)}\right)}$$

this must be positive

In this case we know we overinvest in specific and underinvest in general skills

Industry or Occupation Specific

Imagine there are two sectors.

- Skill 1 is only useful in current (period 1) sector
- Skill 2 is only useful in the other sector
- Let μ_1 be probability that outside offer comes from sector 1

In this case the value function as

$$
\begin{aligned}
&V_1(H_1, \pi_1, w_1, s_1) \\
&= w_1 + \frac{1}{R}\Big[\mu_1 E_\pi \max\left\{\delta\pi_1^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right), \delta\pi^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right)\right\} \\
&\qquad + (1-\mu_1)E_\pi \max\left\{\delta\pi_1^{(1)}\mathcal{H}^{(1)}\left(s_1^{(1)}\right), \delta\pi^{(2)}\mathcal{H}^{(2)}\left(s_1^{(2)}\right)\right\} + (1-\delta)H_h\Big]
\end{aligned}
$$

Again, this is just a special case of our model above.

Note that for investment in skills it only matters when we get an offer from a sector 2 firm, for a sector one firm all that matters is whether $\pi > \pi_1$

We can then show that there will be overinvestment in sector 1 skills and underinvestment in sector 2 skills for the same reasons as above