The Compensating Differentials Model

Christopher Taber

September 20, 2015

Outline

Compensating Differentials

(or equalizing differences)

In the Roy model people only cared about income, but differed in skills

In the simplest version of this model people

- Have identical skills
- Heterogeneity in tastes for jobs

Basic idea is that an employer must pay a premium to get you to do some job you don't want to do

Let D represent a disamenity of work like how dangerous it is

Suppose

- $D = 0$ represents safe jobs that pay W_0
- $D = 1$ represents dangerous jobs that pay W_1

All safe jobs will pay the same because workers are identical and labor market is competitive (and frictionless)

Preferences

$$
\begin{aligned}
& U_i(C, D) \\
C_0 &= I + W_0 \\
C_1 &= I + W_1
\end{aligned}
$$

where I is nonlabor income

Compensating difference is determined by indifferent individual

Lets figure out what the supply curve looks like

Take a really simple case with linear utility so that

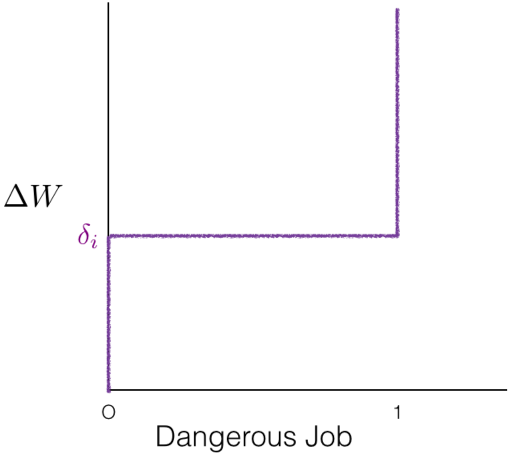
$$U_i(C, D) = C - \delta_i D$$

Then individual i chooses to work in dangerous sector if

$$\begin{aligned} U_i(C_0, 0) &< U_i(C_1, 1) \\ I + W_0 &< I + W_1 - \delta_i \\ \delta_i &< W_1 - W_0 \equiv \Delta W \end{aligned}$$

For person i the supply curve looks like:

ΔW
0
1
Dangerous Job



ΔW
δ_i
0
1
Dangerous Job

Now suppose that δ_i varies over the population with measure G

Let $1(\bullet)$ be the indicator function

Supply of people to dangerous jobs can be written as

$$\begin{aligned} N_1^s(\Delta W) &= \int 1(\delta_i < \Delta W)\, dG(\delta_i) \\ &= G(\Delta W) \end{aligned}$$

Similarly supply to safe jobs is just

$$N_0^s(\Delta W) = 1 - G(\Delta W)$$

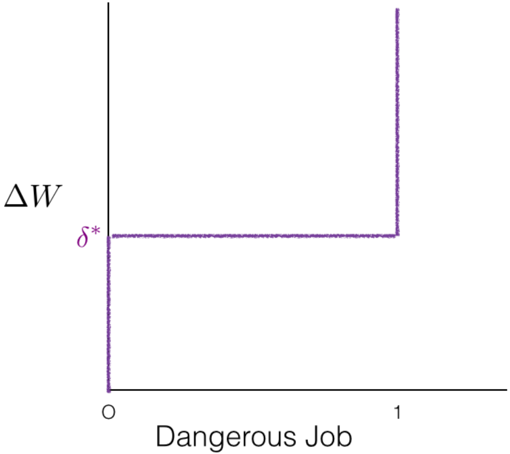
Notice that

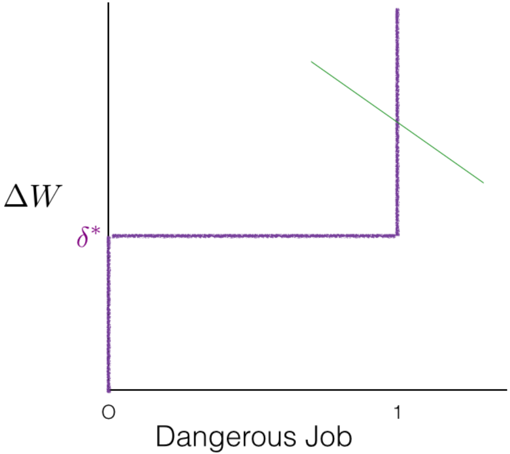
- This is just the CDF of δ_i
- As ΔW increases more people do the dangerous job
- Elasticity of supply

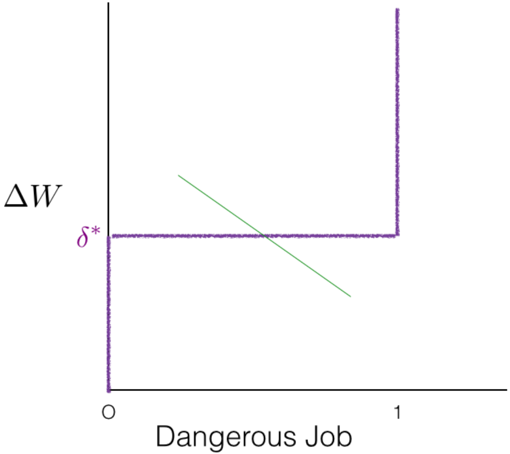
$$\begin{aligned}\frac{\partial \log(N_1^s(\Delta W))}{\partial \log(\Delta W)} &= \frac{\partial \log(G(\Delta W))}{\partial \log(\Delta W)} \\ &= \frac{\Delta W}{G(\Delta W)} g(\Delta W)\end{aligned}$$

so the elasticity depends on the density of people who are indifferent.

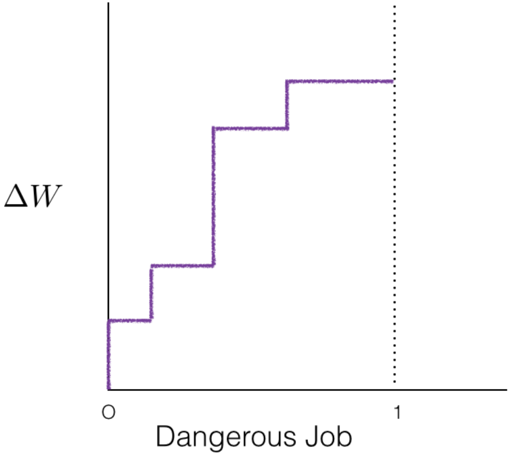
Examples:


ΔW
δ^*
0
1
Dangerous Job

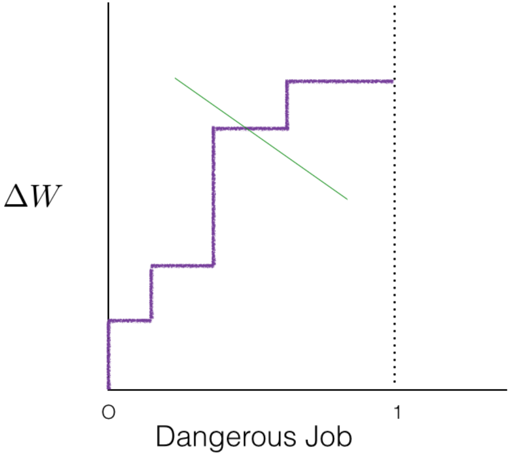

ΔW
δ^*
0
1
Dangerous Job



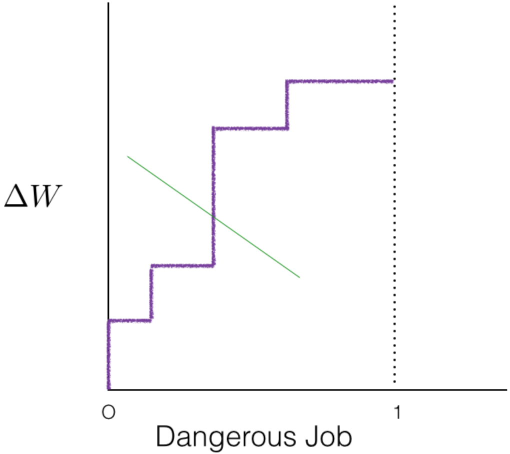
ΔW
δ^*
0
1
Dangerous Job



ΔW
0
1
Dangerous Job



ΔW
0
1
Dangerous Job



ΔW
0
1
Dangerous Job


ΔW
0
1
Dangerous Job

ΔW
0
1
Dangerous Job

ΔW
0
1
Dangerous Job


ΔW
0
1
Dangerous Job

Firm Side

Now lets think about the firm side of the market

It costs money to make the workplace safe

The cost varies across jobs (this is easier for a university than a coal mine)

Each firm (job) hires one worker and there are as many firms as workers

Production for the firm j is F_j

Costs of making the work environment safe is β_j

so profits as a function of working environment is

$$F_j - \beta_j (1 - D) - W_D$$

Thus the workplace is dangerous if

$$\begin{aligned} W_1 &< W_0 + \beta_j \\ \beta_j &> \Delta W \end{aligned}$$

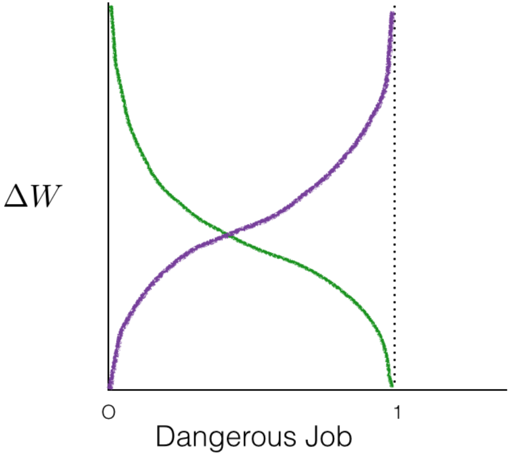
Let F be the distribution of β_j then demand for workers in dangerous jobs is

$$\begin{aligned} N_1^d(\Delta W) &= \int 1(\beta_j > \Delta W) dF(\beta_j) \\ &= 1 - F(\Delta W) \end{aligned}$$

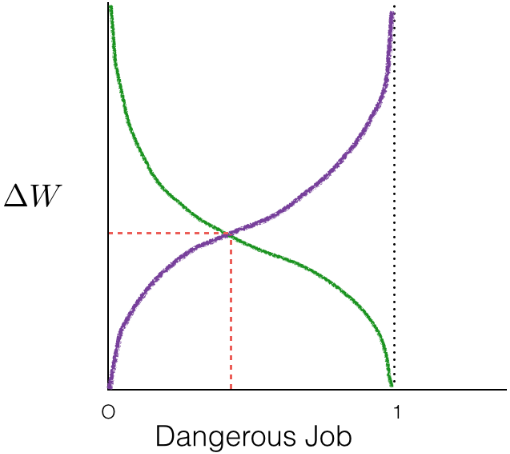
So demand also looks like a cdf.

ΔW
0
1
Dangerous Job

Putting them together



ΔW
0
1
Dangerous Job



ΔW
0
1
Dangerous Job

Hedonic Price Model

More generally suppose that danger is continuous

Let $W(D)$ be the wage paid at mortality rate D

Worker chooses D to maximize

$$U^i(I + W(D), D)$$

so

$$U^i_c W' = -U^i_D$$

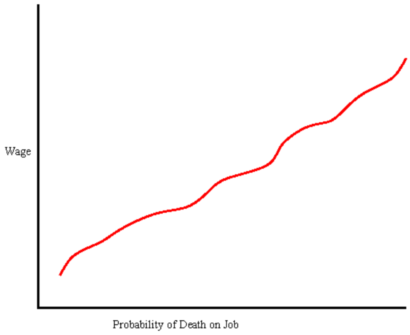
Firm minimizes costs of production

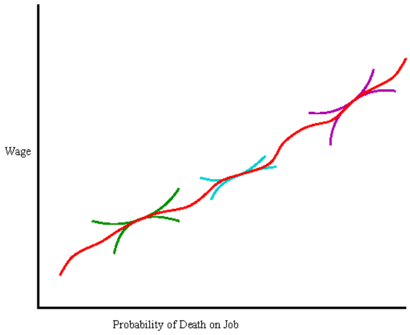
$$W(D) + \beta^j(D)$$

so

$$W' = -\beta^{j\prime}$$

I am not going to get into detail (See Rosen)


Wage
Probability of Death on Job


Wage
Probability of Death on Job

Applications

- Occupational Choice
- Immigration/Migration
- Environment
- Local public finance
- Industry wage differences
- Human capital/Signaling
- Labor Supply

Roback (JPE, 1982)

Jobs located in different cities

Measure value of local amenities

Workers and Firms choose where to locate

Depends on:

- Rent in each place
- Wage rates in each place

To keep things simple I will assume that workers are identical in skills and tastes

Generalizing this so they are just perfect substitutes would be straight forward

Moving costs between places is zero

Workers have utility

$$U(C, \ell_c, A)$$

where

- C is consumption
- ℓ_c is land consumed
- A is the value of amenities-this is determined by where you live

The budget constraint is

$$C + \ell_c r \leq w + I$$

where

- r is rent
- w is labor income
- I is nonlabor income

Let $V(w, r; A)$ be the associated indirect utility function with

$$\begin{aligned} \frac{\partial V(w, r; A)}{\partial w} &\geq 0 \\ \frac{\partial V(w, r; A)}{\partial r} &\leq 0 \\ \frac{\partial V(w, r; A)}{\partial A} &\geq 0 \end{aligned}$$

Since all individuals have to be indifferent between living in different locations, there is a value $\overline{v}$ such that

$$V(w, r; A) = \overline{v}$$

Production

Firms production function depends on land, the number of workers, and potentially the Amenity

$$X = F(\ell_p, N; A)$$

where

- ℓ_p is land used in production
- N is the number of workers
- X is goods produced which have price one
- F is constant returns to scale

Let $C(w, r; A)$ be the unit cost function

We allow free entry so in equilibrium

$$C(w, r; A) = 1$$

where

$$\begin{aligned} \frac{\partial C(w, r; A)}{\partial w} &= \frac{N}{X} > 0 \\ \frac{\partial C(w, r; A)}{\partial r} &= \frac{\ell_p}{X} > 0 \end{aligned}$$

The sign of $\frac{\partial C(w,r;A)}{\partial A}$ is indeterminate depending on the particular amenity

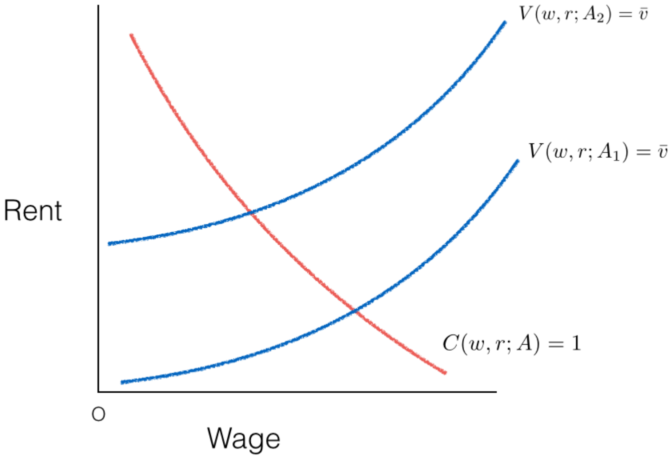
- Ocean, $\frac{\partial C(w,r;A)}{\partial A} < 0$
- Public Schools, $\frac{\partial C(w,r;A)}{\partial A} = 0$
- Regulations on Clean Air, $\frac{\partial C(w,r;A)}{\partial A} > 0$

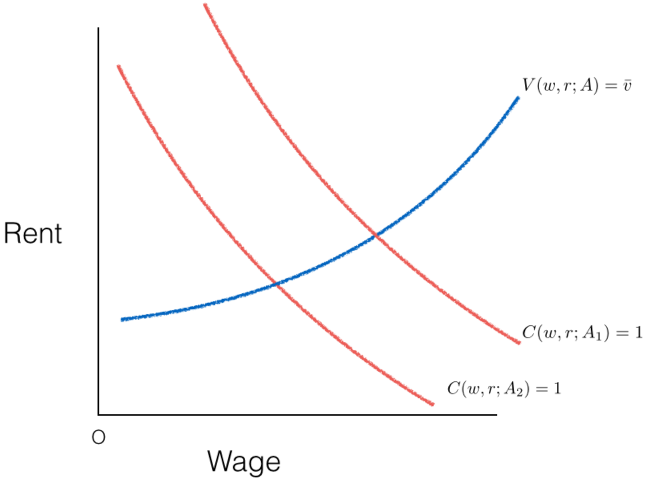
Lets consider two cities with

$$A_2 > A_1$$

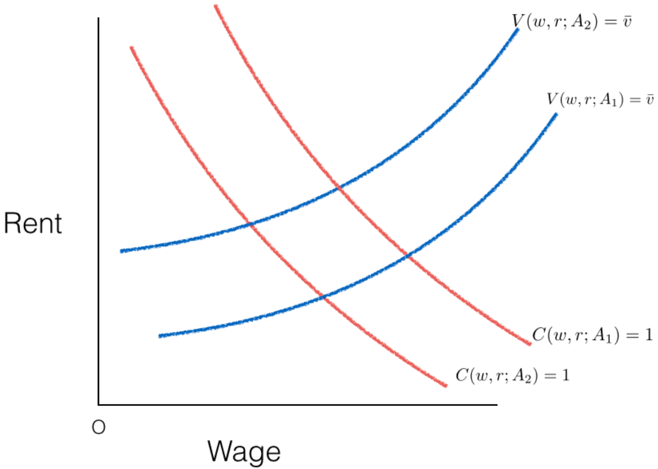
Think about 4 different cases:

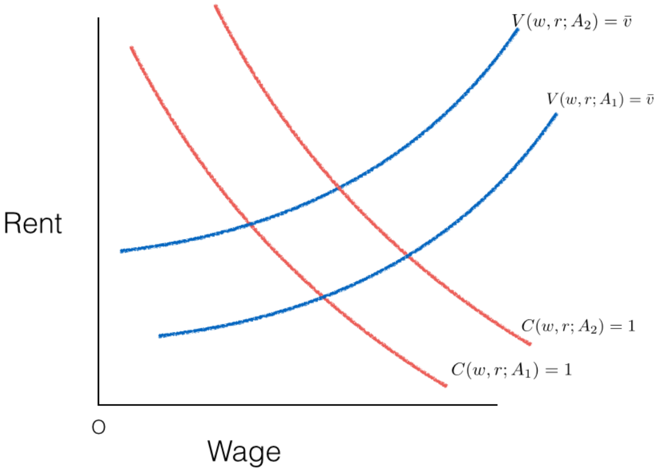
- $\frac{\partial V(w,r;A)}{\partial A} > 0$, $\frac{\partial C(w,r;A)}{\partial A} = 0$
  In this case the trade off between w and r is the same in both cities for the firms
- $\frac{\partial V(w,r;A)}{\partial A} = 0$, $\frac{\partial C(w,r;A)}{\partial A} > 0$
- $\frac{\partial V(w,r;A)}{\partial A} > 0$, $\frac{\partial C(w,r;A)}{\partial A} > 0$
- $\frac{\partial V(w,r;A)}{\partial A} > 0$, $\frac{\partial C(w,r;A)}{\partial A} < 0$


$V(w, r; A_2) = \bar{v}$
$V(w, r; A_1) = \bar{v}$
Rent
$C(w, r; A) = 1$
O
Wage



Rent
Wage
O
$V(w, r; A) = \bar{v}$
$C(w, r; A_1) = 1$
$C(w, r; A_2) = 1$


$V(w, r; A_2) = \bar{v}$
$V(w, r; A_1) = \bar{v}$
Rent
$C(w, r; A_1) = 1$
$C(w, r; A_2) = 1$
O
Wage



$V(w, r; A_2) = \bar{v}$
$V(w, r; A_1) = \bar{v}$
Rent
$C(w, r; A_2) = 1$
$C(w, r; A_1) = 1$
O
Wage

Lets think about equilibrium over a large number of cities

We know that

$$\begin{aligned} C(w, r; A) &= 1 \\ V(W, r; A) &= \overline{v} \end{aligned}$$

So

$$\begin{aligned} \frac{dV}{dA} &= V_w \frac{\partial w}{\partial A} + V_r \frac{\partial r}{\partial A} + V_A = 0 \\ \frac{dC}{dA} &= C_w \frac{\partial w}{\partial A} + C_r \frac{\partial r}{\partial A} + C_A = 0 \end{aligned}$$

Substituting for $\frac{\partial r}{\partial A}$and solving for $\frac{\partial w}{\partial A}$ gives

$$\begin{aligned} C_w V_w \frac{\partial w}{\partial A} + C_w V_r \frac{\partial r}{\partial A} + C_w V_A &= 0 \\ -V_w C_w \frac{\partial w}{\partial A} - V_w C_r \frac{\partial r}{\partial A} - V_w C_A &= 0 \end{aligned}$$

$$\frac{\partial w}{\partial A} = \frac{V_r C_A - C_r V_A}{C_r V_w - V_r C_w}$$
$$\frac{\partial r}{\partial A} = \frac{V_w C_A - C_w V_A}{C_w V_r - V_w C_r}$$

Thus if

$$\begin{array}{lll} C_A = 0 & \frac{\partial w}{\partial A} < 0 & \frac{\partial r}{\partial A} > 0 \\ C_A > 0 & \frac{\partial w}{\partial A} < 0 & \frac{\partial r}{\partial A} ?? \\ C_A < 0 & \frac{\partial w}{\partial A} ?? & \frac{\partial r}{\partial A} > 0 \end{array}$$

Lets think about implementing this model empirically

We want to measure how tastes for cities vary

We can observe how wages and rental rates vary across cities

We can use these to measure “revealed preference” for amenities

We know that

$$
\begin{aligned}
C_w &= \frac{N}{X} \\
C_r &= \frac{\ell_p}{X} \\
-\frac{V_r}{V_w} &= \ell_c
\end{aligned}
$$

The shadow price of the amenity can be defined as

$$
\begin{aligned}
P_A^* &\equiv \frac{V_A}{V_w} \\
&= \frac{-V_w \frac{\partial w}{\partial A} - V_r \frac{\partial r}{\partial A}}{V_w} \\
&= -\frac{\partial w}{\partial A} + \ell_c \frac{\partial r}{\partial A}
\end{aligned}
$$

The worth to the firm

$$
\begin{aligned}
C_A &= -C_w \frac{\partial w}{\partial A} - C_r \frac{\partial r}{\partial A} \\
&= -\frac{N}{X}\frac{\partial w}{\partial A} - \frac{\ell_p}{X}\frac{\partial r}{\partial A}
\end{aligned}
$$

From the point of view of policy, the total value of the amenity can be written as:

$$
\begin{aligned}
NP_A^* - C_A X &= N\left[-\frac{\partial w}{\partial A} + \ell_c \frac{\partial r}{\partial A}\right] + -N\frac{\partial w}{\partial A} + \ell_p \frac{\partial r}{\partial A} \\
&= [N\ell_c + \ell_p]\frac{\partial r}{\partial A}
\end{aligned}
$$

Roback implements this to gauge the value of life in each city

One problem is that we are assuming that worker quality and housing quality is the same across regions

She relaxes this by assuming that workers are perfect substitutes

Worker i in city j earns

$$E_{ij} = w_j L_i$$

and

$$\begin{aligned}
\log(E_{ij}) &= \log(w_j) + \log(L_i) \\
&= Z_j' \gamma + u_j + X_i' \beta + u_i
\end{aligned}$$

TABLE 1

COEFFICIENTS OF CITY CHARACTERISTICS FROM LOG EARNINGS REGRESSIONS IN 98 CITIES

	1	2	3	4
TCRIME 73	$.94 \times 10^{-5}$	$.44 \times 10^{-5}$	$.74 \times 10^{-5}$	$.86 \times 10^{-5}$
	(2.58)	(1.17)	(1.93)	(2.21)
UR 73	$.36 \times 10^{-2}$	$.12 \times 10^{-2}$	$.32 \times 10^{-2}$	$.27 \times 10^{-2}$
	(1.29)	(.43)	(1.14)	(.97)
PART 73	$.24 \times 10^{-3}$	$.13 \times 10^{-3}$	$.37 \times 10^{-3}$	$.34 \times 10^{-3}$
	(1.55)	(.86)	(2.33)	(2.15)
POP 73	$.16 \times 10^{-7}$	$.15 \times 10^{-7}$	$.16 \times 10^{-7}$	$.16 \times 10^{-7}$
	(7.97)	(7.74)	(8.04)	(8.11)
DENSSMSA	$.81 \times 10^{-6}$	$.24 \times 10^{-5}$	$.20 \times 10^{-5}$	$.38 \times 10^{-5}$
	(.29)	(.86)	(.73)	(1.40)
GROW 6070	$.21 \times 10^{-2}$	$.14 \times 10^{-2}$	$.15 \times 10^{-2}$	$.17 \times 10^{-2}$
	(7.84)	(5.66)	(6.06)	(6.47)
HDD	$.20 \times 10^{-4}$			
	(8.48)			
TOTSNOW		$.72 \times 10^{-3}$		
		(3.54)		
CLEAR			$-.64 \times 10^{-2}$	
			(−4.80)	
CLOUDY				$.72 \times 10^{-2}$
				(5.21)
R^2	.4980	.4955	.4960	.4962
F-ratio	424.2	420.0	420.8	421.1
N = 12,001				

NOTE.—Regressions include all personal characteristics. Sample includes 98 cities; t-statistics are in parentheses (see App. for variable definitions).

TABLE 2

COEFFICIENTS OF REGION DUMMIES AND CITY CHARACTERISTICS

NORTHEAST	−.0218	−.0095
	(−2.25)	(−.74)
SOUTH	−.0669	−.0138
	(−6.51)	(−.87)
WEST	−.0354	−.0579
	(−3.46)	(−3.41)
TCRIME 73		$.13 \times 10^{-4}$
		(2.82)
UR 73		$.92 \times 10^{-2}$
		(2.60)
PART 73		$.29 \times 10^{-3}$
		(1.87)
POP 73		$.16 \times 10^{-7}$
		(7.77)
DENSSMSA		$-.13 \times 10^{-5}$
		(−.42)
GROW 6070		$.23 \times 10^{-2}$
		(8.41)
HDD		$.16 \times 10^{-4}$
		(4.86)
R^2	.4900	.4986
F-ratio	479.4	384.0

NOTE.—Regressions include all personal characteristics. Sample includes all 98 cities; t-statistics are in parentheses.

TABLE 3

REGRESSIONS OF THE LOG OF AVERAGE RESIDENTIAL SITE PRICE PER SQUARE FOOT ON CITY CHARACTERISTICS

	1	2	3	4
TCRIME 73	2.5×10^{-5}	1.5×10^{-5}	-4.5×10^{-7}	7.0×10^{-6}
	(.65)	(.38)	(−.01)	(.16)
UR 73	8.9×10^{-2}	8.8×10^{-2}	9.2×10^{-2}	9.1×10^{-2}
	(3.45)	(3.35)	(3.53)	(3.52)
PART 73	2.2×10^{-4}	1.1×10^{-4}	-3.8×10^{-5}	1.4×10^{-4}
	(.15)	(.08)	(−.02)	(.09)
POP 73	6.8×10^{-8}	6.9×10^{-8}	6.8×10^{-8}	6.8×10^{-8}
	(1.80)	(1.78)	(1.76)	(1.76)
DENSSMSA	1.9×10^{-4}	2.0×10^{-4}	2.0×10^{-4}	2.0×10^{-4}
	(3.02)	(3.12)	(3.17)	(3.18)
GROW 6070	1.1×10^{-2}	1.0×10^{-2}	9.9×10^{-3}	1.0×10^{-2}
	(4.34)	(4.11)	(4.03)	(4.00)
HDD	3.5×10^{-5}			
	(1.44)			
TOTSNOW		1.3×10^{-3}		
		(.69)		
CLEAR			1.2×10^{-4}	
			(.09)	
CLOUDY				3.2×10^{-4}
				(.21)
INTERCEPT	−1.73	−1.54	−1.44	−1.53
	(−5.92)	(−5.99)	(−6.51)	(−3.32)
R^2	.5741	.5650	.5623	.5625
F-ratio	14.44	13.92	13.77	13.78

SOURCE.—Data are from U.S. Department of Housing and Urban Development 1973. $N = 83$.

TABLE 4

Implicit Prices of Amenities Computed from Tables 1 and 3

	1	2	3	4
TCRIME 73				
(crimes/100 population)	$−9.25	$ .90	$ −8.05	$ −9.15
UR 73				
(fraction unemployed)	−5.55	20.65	−.70	5.00
PART 73				
(micrograms/cubic meter)	−2.50	−1.40	−4.00	−3.70
POP 73				
(10,000 persons)	−1.50	−1.40	−1.50	−1.50
DENSSMSA				
(100 persons/square mile)	6.30	4.90	5.35	3.35
GROW 6070				
(percentage change in population)	−1.85	−11.95	−13.05	−15.2
HDD				
(1° F colder for one day	−.20			
TOTSNOW				
(inches)		−7.30		
CLEAR				
(days)			69.55	
CLOUDY				
(days)				−78.25

Note.—Measurement units of amenities shown under variable name. Each entry is computed using eq. (5) in the text and evaluated at mean annual earnings. $p_s^* = [k_l(d \log r/ds) - (d \log w/ds)]w$. Average annual earnings = $10,868. Average budget share of land = .035. Negative numbers indicate disamenities, while positive numbers indicate amenities.

TABLE 6

COMPARISON OF QOL 3 RANKINGS OF 20 LARGEST CITIES WITH RANKING OF LIU

Rank	Name	Liu's Rank	QOL 3	Population Rank
1	Los Angeles–Long Beach	10	1.7517	2
2	Anaheim–Santa Ana–Garden Grove	9	1.7363	19
3	San Francisco–Oakland	2	1.5841	6
4	Dallas	5	1.3378	17
5	Baltimore	13	1.0244	12
6	Nassau-Suffolk	. . .	1.0010	9
7	St. Louis	16	.9407	11
8	Milwaukee	8	.9386	20
9	Boston	12	.9296	8
10	Minneapolis	4	.9047	16
11	New York	14	.8962	1
12	Washington, D.C.	3	.8910	7
13	Newark	11	.8853	15
14	Philadelphia	7	.8038	4
15	Houston	6	.7708	14
16	Chicago	18	.7416	3
17	Detroit	17	.6347	5
18	Cleveland	15	.6227	13
19	Seattle-Everett	1	.5871	18
20	Pittsburgh	19	.4961	10

NOTE.—Liu's rank is based on adjusted standardized score of environmental component. Nassau-Suffolk is not included in Liu's (1976) study.

Bergstrom, Soldiers of Fortune

In Essays in Honor of K.J. Arrow, 1986.

Basic idea might not have been original to Bergstrom, but it demonstrates it nicely

Everyone in our country is identical (ex-ante)

We need π fraction of the population to be in the army

The rest $(1 - \pi)$ are farmers

How do we get people to enter the army?

- Volunteer (compensating differential)
- Draft

Let $\overline{w}$ be per capita GDP

Thus it must be the case that

$$\pi C_A + (1-\pi)\, C_F = \pi W_A + (1-\pi)\, W_F = \overline{w}$$

where

- C_j is consumption in job j
- W_j is wage at job j

Let

- $u_A(C_A)$ be utility from consumption if in army
- $u_F\,(C_F)$ be utility from consumption if a farmer
- $$u_A\,(c) < u_F\,(c)$$
- $U_A'' < 0, U_F'' < 0$

Equalizing differentials

People consume what they make so

$$C_j = W_j$$

Need $\widehat{W}_A, \widehat{W}_F$ to satisfy the following two equations:

$$\begin{aligned} u_A\left(\widehat{W}_A\right) &= u_F\left(\widehat{W}_F\right) \\ \pi\widehat{W}_A + (1-\pi)\widehat{W}_F &= \overline{w} \end{aligned}$$

It must be the case that

$$\widehat{W}_F < \widehat{W}_A$$

Draft

π people will be randomly assigned to army; Government chooses C_F, C_A

What is ex-ante optimal?

$$\max_{C_A, C_F} \pi_A u_A(C_A) + (1-\pi) u_F(C_F)$$
$$\text{s.t. } \pi C_A + (1-\pi) C_F = \overline{w}$$

The first order conditions are

$$u'_A(C^*_A) = u'_F(C^*_F)$$

Assume that consumption is valued more on the Farm

$$u'_A(c) < u'_F(c)$$

But this implies that

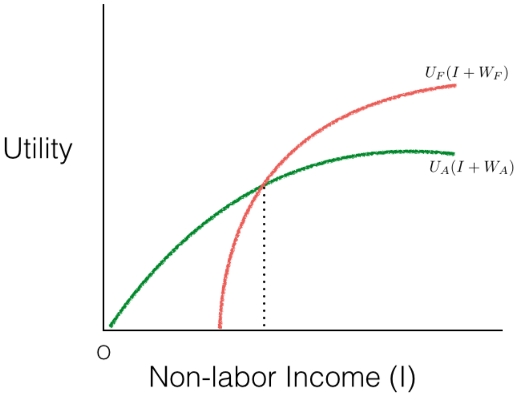
$$C_F > C_A$$

This result is the opposite of compensating differentials

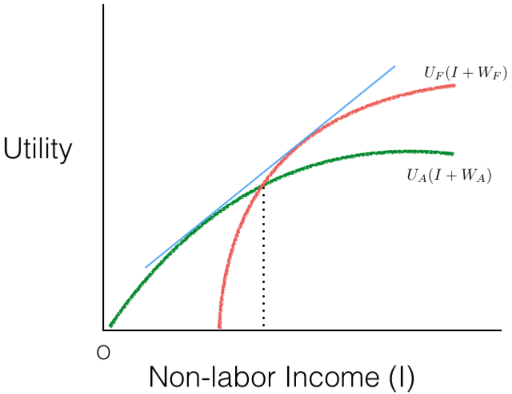
A feasible solution to the problem is to set $C_F = \hat{W}_F$ and $C_A = \hat{W}_A$

Thus ex-ante utility of agents is higher with draft

Compensating differentials seems to be inefficient because levels of utility are equated rather than marginal utility



$U_F(I + W_F)$
Utility
$U_A(I + W_A)$
O
Non-labor Income (I)


$U_F(I + W_F)$
Utility
$U_A(I + W_A)$
O
Non-labor Income (I)

Private Lotteries

Now consider the following lottery

With probability $(1-\pi)$ you win

$$C_F^* - \widehat{W}_F$$

With probability π you lose

$$\widehat{W}_A - C_A^*$$

First notice that this is a fair lottery so it is feasible that it could exist

$$\begin{aligned}
&(1-\pi)\left(C_F^* - \widehat{W}_F\right) - \pi\left(\widehat{W}_A - C_A^*\right) \\
&= (1-\pi)\,C_F^* + \pi C_A^* - (1-\pi)\,\widehat{W}_F - \pi\widehat{W}_A \\
&= \overline{w} - \overline{w} = 0
\end{aligned}$$

What occupation do winners and losers choose?

First recall that

$$u_A\left(\widehat{W}_A\right) = u_F\left(\widehat{W}_F\right)$$

and since

$$u_A'(c) = u_F'(c)$$

everywhere then it must be the case that for $\Delta > 0$

$$u_A\left(\widehat{W}_A + \Delta\right) < u_F\left(\widehat{W}_F + \Delta\right)$$

and

$$u_A\left(\widehat{W}_A - \Delta\right) > u_F\left(\widehat{W}_F - \Delta\right)$$

Now if I win I get

$$\begin{array}{ll} u_A\left(\widehat{W}_A + C_F^* - \widehat{W}_F\right) & \text{Army} \\ u_F\left(\widehat{W}_F + C_F^* - \widehat{W}_F\right) = u_F\left(C_F^*\right) & \text{Farmer} \end{array}$$

Since $C_F^* - \widehat{W}_F$ is positive I will choose to be a farmer.

If I lose I get

$$\begin{aligned} u_A\left(\widehat{W}_A - \left(\widehat{W}_A - C_A^*\right)\right) &= u_A\left(C_A^*\right) \quad && \text{Army} \\ u_F\left(\widehat{W}_F - \left(\widehat{W}_A - C_A^*\right)\right) & && \text{Farmer} \end{aligned}$$

Since $\widehat{W}_A - C_A^*$ is positive I will choose to enter the army.

Thus

- This is a fair gamble
- It is self-regulating:
  - winners choose to be farmers
  - losers choose to enter army
- Gives the optimal ex-ante utility so workers would choose to enter these lotteries