Parental Beliefs, Social Learning, and Intergenerational Mobility

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November 29, 2023

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Abstract

This paper examines the role of information frictions in shaping parental beliefs over child development and intergenerational mobility by incorporating social learning into a heterogeneous-agent model of overlapping generations. Information frictions implicit in my model of social learning influence individual-specific beliefs about the return to parental investments in children's human capital and distort parental investment choices. I calibrate the model using data from the U.S. and show that its predictions are consistent with the evidence from a randomized controlled trial that changed parents' beliefs by providing them with information about child development. Using the calibrated model, I show that, in equilibrium, distortions in beliefs amplify the persistence of earnings across generations by 8.7%. A low-cost, large-scale policy that provides low-income parents with information about the return to parental investments generates a 4.2% increase in intergenerational mobility, not only because low-income parents are more certain about the impact of their investments and so increase investments in their children's human capital, but because these choices spill over into the beliefs held by the next generation.

*University of Wisconsin - Madison. I am grateful for comments from Ananth Seshadri, Ken West, Jeff Smith, J. Carter Braxton, Garrett Anstreicher, J.C. Lazzaro, Matt Wiswall, and Anson Zhou as well as numerous seminar participants at the University of Wisconsin-Madison and the Federal Reserve Bank of Minneapolis. I thank the Baby's First Years Study team and the Institute for Research on Poverty for their support. This material is based upon work supported by the National Science Foundation Graduate Research Fellowship Program under Grant No. DGE-1747503. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.

Early childhood investments have long-lasting implications for children's outcomes in adulthood (e.g. Cunha et al. (2010); Cunha and Heckman (2007); Ermisch et al. (2005); Elango et al. (2015); Almond and Currie (2010)).[1] However, it has also been shown that parents' early childhood investments depend on the returns they expect such investments to yield.[2] For example, List et al. (2021) document large variation in beliefs across socioeconomic status and show that informational interventions that increase subjective perceptions about the importance of parental inputs, increase parental investments. The long-run macroeconomic effects of providing information to low-income parents are not clear because small-scale, short-run empirical studies are not able to capture the general equilibrium effects arising from dynamic subjective beliefs and human capital.

This paper studies the long-run effects of such policies by incorporating heterogeneous subjective beliefs and information frictions — stemming from a failure to account for selection bias — in a social learning environment into a macroeconomic model of overlapping generations.[3] Since information frictions impact intergenerational mobility and low-cost informational interventions are effective at achieving a better allocation of investments in human capital, it is important to understand the long-run impact such policies would have at a large scale. Addressing these questions necessitates a quantitative model of intergenerational mobility that is equipped to study the macroeconomic implications of influencing individual-specific subjective beliefs about the return to investments in children's human capital. I find that the presence of this social learning environment amplifies the intergenerational earnings elasticity by 8.7% and that eliminating the distortions in subjective beliefs generated by information frictions would increase mobility.[4] I use the quantitative model to study the implications of a large-scale, low-cost informational intervention similar to that of List et al. (2021). I find that permanently implementing this policy for low-income parents would increase earnings mobility by 4.2%.

In this paper, I make two contributions. First, I embed individual-specific subjective beliefs about the productivity of parental investments in children's human capital into a model of intergenerational mobility (e.g. Becker and Tomes (1986)) with social learning (e.g. Frick et al. (2022)). Second, I quantify the model and study the long-run macroeconomic implications of scaling-up a policy intervention that targets parental expectations. In the model, I depart from the common assumption of perfect information by introducing subjective beliefs obtained via social learning. Individuals initial subjective beliefs about the return to parental investments are determined by

[1] Early childhood is often referred to as the first five years of life in among developmental psychologists.

[2] See for example, Cunha et al. (2013), Caucutt et al. (2017), Attanasio et al. (2019), and Cunha et al. (2023).

[3] This mechanism has recently been documented by Frick et al. (2022) who study the implications of misperceptions in social interactions with assortativity neglect.

[4] The international elasticity of earnings (IGE) measures the persistence of earnings across generations and is a commonly used measure of mobility.

their own experiences— the inputs they received as a child and adulthood outcomes. Individuals then learn by observing the experiences of their social connections as well. The composition of social connections that an individual has will contribute to their individual-specific subjective beliefs about the impact of parental investments on children's human capital formation. Both the fact that parents are *uncertain* about the impact of parental investments and the fact that parents receive information about the impact of parental investments from *social interactions* are important for evaluating policy interventions that aim to influence parental beliefs. We gain two key insights from introducing social learning. The first is that parents face a trade-off between investing early in childhood when human capital investments make future investments more productive, but the returns are more uncertain, and investing later in childhood when human capital investments are less valuable if early investment were low, but parents are more certain about the impact of their investments. The second insight is that when an individual learns from social interactions but does not know the extent to which her social connections are representative of the aggregate distribution, subjective beliefs about the importance of parental inputs have the potential to be biased. The notion that individuals believe their social interactions to be representative is known in the literature as assortativity neglect (Frick et al., 2022).

I estimate the model using the simulated method of moments to match data from the U.S. on the dispersion of parental beliefs across the distribution of income. I use data on the distribution of economic connectedness derived from Meta (formerly Facebook) in Chetty et al. (2022) to discipline the distribution of social interactions across the income distribution.[5] The model predicts that social segregation generates information frictions that shape subjective beliefs about the returns to parental investment. Parents with a disproportionate number of high earnings social connections overestimate the return to investment, on average, increasing their investments when young and crowding in investments in late childhood as a result of complementary dynamics between investments made over time (Cunha and Heckman, 2007). Parents with disproportionately low earnings social connections underestimate the return to investment on average, reducing investment when young. These dynamics amplify the persistence of earnings across generations.

The randomized controlled trial (RCT) data from List et al. (2021)'s experiment is used to test the validity of the model's predictions about the effect of subjective beliefs on parental investments. The empirical findings suggest that the informational intervention has statistically significant and persistent effects on subjective beliefs. A one standard deviation increase in subjective beliefs about the importance of childhood investments is associated with a $13-18\%$ increase in parental investment rates (List et al., 2021). In the quantitative model, I simulate individual-specific beliefs

[5]Economic connectedness measures the degree to which an individual or group of individuals have friends who are above-median income (Chetty et al., 2022). In this framework, income is thought of as synonymous with labor earnings though this is a substantial abstraction.

and parental investment behavior across the life-cycle. In the model simulated data, increasing mean subjective beliefs by one standard deviation increases parental investment rates by 11.7% in the first period of parenting.

Considering the quantitative model's ability to replicate these empirical estimates, I perform two quantitative exercises to examine the aggregate implications of individual-specific subjective beliefs in a context with social learning. First, I eliminate economic segregation in the distribution of social connections by having individuals learn from a representative distribution of social interactions. Doing so increases average investments among the households with the lowest individual-specific subjective beliefs about the return to parental investment and decreases average investments among those with the highest subjective beliefs. I find that eliminating social segregation would increase earnings mobility by 8.7% as measured by the intergenerational earnings elasticity.[6]

Second, I use the quantitative model to study a large-scale, permanent version of List et al. (2021)'s Newborn program which targets low-income parents' subjective expectations and knowledge in Chicago.[7] The treatment in this randomized controlled trial consisted of providing parents of young children with four ten-minute informational videos about the importance of parental investments prior to routine pediatric appointments, known as well-child visits.[8] The intervention's cost is about \$143-\$150 per participant. I scale-up this intervention by providing parents in the bottom two deciles of the income distribution with an experimenter's signal that conveys correct, but imprecise information about the return to parental investments before they make investment choices. I find that the policy increases low-income parents' subjective beliefs about the importance of childhood investments for human capital development by about 0.07 standard deviations, increasing earnings mobility by 4.2%, and reduces cross-sectional variation in earnings and human capital by 10.2%. The first key mechanism is that providing information to low-income parents increases their subjective beliefs and investment choices. This leads their children to have higher human capital and earnings mobility and increases the subjective beliefs about the return to parental investments in future generations, which are informed by parent's investment choices. Secondly, the policy reduces uncertainty, which increases investments made in early childhood by low-income parents and thereby crowds in later-in-life investments as well.

The remainder of this section discusses the related literature. In section 1, I present the quan-

[6]The intergenerational earnings elasticity is a commonly used measure of mobility. It measures the expected percent increase in children's future earnings from a one percent increase in parent's earnings.

[7]The other study, called the Home Visiting program, was a more comprehensive intervention that included both information and home visits that taught parents about how to create a rich language environment for children in practice. I do not use these data since the interventions included additional resources apart from information.

[8]Well-child visits are routinely recommended appointments in the first 18 months of life where children typically receive common vaccines for things like Polio, MMR, Chickenpox, DTap etc.

titative model of intergenerational mobility with social learning and human capital investment. Section 2 discusses estimation of the model using data on parental beliefs and social interactions from the U.S., and compares the simulated results to the empirical data. In section 3, I examine the implications of social learning on intergenerational earnings mobility in a counterfactual exercise that eliminates the biases introduced by social learning. Section 4 discusses the results of scaling up an informational policy intervention that provides low-income parents with information about the return to parental investment. Finally, I conclude in section 5.

Related literature. Recent empirical literature studies subjective beliefs about the returns to human capital investments. These studies analyze the effects of informational interventions on parental subjective beliefs and subsequent investment behavior (e.g. Cunha et al. (2013); Boneva and Rauh (2018); List et al. (2021)) using randomized controlled trials, and highlight two features of subjective beliefs over the production of children's skills.[9] The first is the large variation in the subjective beliefs of parents and how they differ across socioeconomic status (List et al., 2021). The second is that parents are responsive to information about the importance of parental inputs for child development, suggesting that information frictions are relevant for investment choices, especially for low-income parents (Jensen, 2010; Cunha et al., 2013). In this paper, these properties are explicitly modeled in parental decision making which allows us to study long-run, macroeconomic implications of informational policies. The model is estimated (and validated) using data collected by List et al. (2021) to align with parents' investment response to changes in their subjective beliefs.

Prior theoretical work on social learning has illustrated that connections to more educated or affluent people can be valuable for transferring information and shaping beliefs in many contexts (e.g. Montgomery (1991); Calvo-Armengol and Jackson (2004); Putnam (2016); Jackson (2021)). Furthermore, it has been shown that subjective beliefs about the aggregate environment, including the return to schooling, adjust systematically in response to the experiences of social connections (Alt et al., 2022; Maddock and Glanz, 2005; Boneva and Rauh, 2018). Social learning in this context is similar to Frick et al. (2022) who theoretically study the implications of assortativity in local social interactions where individuals suffer from assortativity neglect in the sense that they believe the people they interact with to be a representative sample of society as a whole.[10] In this paper, I argue that social connections are valuable for transferring information among parents

[9] A related literature study students' subjective beliefs about the return to schooling in developing economies (e.g. Nguyen (2008); Attanasio and Kaufmann (2009); Attanasio and Kaufmann (2014); Jensen (2010)).

[10] Other work from the social learning literature include Manski (1993), Fogli and Veldkamp (2011), and Amador and Weill (2012). Relative to Fogli and Veldkamp (2011)'s social learning technology this paper makes two key deviations: (i) the precision of beliefs is not inherited across generations, but each generation is equally as uncertain before they learn socially (ii) assortativity neglect in social interactions introduces systematic biases.

about the importance of childhood environment thereby shaping parental investment decisions.[11]

This paper contributes to the macroeconomics literature that studies intergenerational mobility and inequality in models of the life-cycle. This literature emphasizes the role of market failures within and across generations that may lead to inefficiently low parental investments (e.g. Becker and Tomes (1986); Solon (2014); Lochner (2007); Del Boca et al. (2014); Daruich (2018); Lee and Seshadri (2019); Caucutt and Lochner (2020)). Policies that insure risky investments in children's human capital, borrow against future generations' earnings to finance investments today, or alleviate intertemporal borrowing constraints over the life-cycle have been studied in Lee and Seshadri (2019), Caucutt and Lochner (2020), Heckman and Mosso (2014) and Daruich (2018), among others. Most of these models of intergenerational mobility, however, assume that parents have full information about their children's human capital and the production function of that human capital with the exception of Bellue (2023), who studies a setting with residential segregation that generates information frictions across neighborhoods in a similar fashion to the information frictions that arise in this paper. Departing from full information is a natural next step in the quantitative intergenerational mobility literature given the wave of empirical evidence that documents subjective parental beliefs about children's human capital formation technology (e.g. Cunha et al. (2013); Cunha (2021); List et al. (2021)). Relative to Bellue (2023) who models local learning with common average beliefs within neighborhood in a static model, social learning in my framework differs in three main aspects: (i) learning is dynamic which gives rise to a trade-off in the timing of parental investments that interacts with uncertainty about the relevance of investments, (ii) social learning is not confined spatially and social interactions allow for the diffusion of perceptions across type, and (iii) this is the first paper to introduce *individual-specific* subjective beliefs that can rationalize the RCT evidence. I make both theoretical and quantitative contributions to this literature by introducing social learning and individual-specific subjective beliefs into a model that allows us to study policy interventions that target parental expectations.

[11]The literature in developmental psychology that studies the sources of parental knowledge has motivated the way social learning is modeled in what follows. Survey evidence shows that the vast majority (75% - 87%) of parents of young children seek information from nonprofessional sources such as their own family, friends, other parents, or from their own experiences (Koepke and Williams, 1989). These nonprofessional sources were the ones parents felt they learned the most from and the most frequent source of parenting information (Koepke and Williams, 1989; Schultz and Vaughn, 1999).

1 Quantitative Model

1.1 Model Overview

To illustrate how subjective parental beliefs influence earnings mobility I build a theory of intergenerational mobility with social learning. There are overlapping generations of parents and children; parents learn about the return on parental investments from those with whom they have social interactions. Parents use subjective beliefs to inform parental investments in their children's human capital. The model incorporates social learning (e.g. Fogli and Veldkamp (2011); Frick et al. (2022); Bellue (2023)) into a model of overlapping generations (e.g. Becker and Tomes (1986)). Parents have uncertain subjective beliefs about the productivity of investments in children's human capital and face inter-temporal credit constraints. Parent-specific subjective beliefs reflect the information about the return on parental investments from childhood experiences and the experiences of social interactions. Therefore, parents' subjective beliefs are distorted when their distribution of social connections is not representative, and they neglect the fact that the signals they observe suffer from selection bias. Furthermore, with finite social connections and risk-aversion, parents face a trade-off between investing early in childhood when human capital investments make later investments more productive but optimal investment is subject to more uncertainty, and investing later in childhood when parents are more certain about the efficacy of their investments. In the remainder of this section, I describe the model economy in detail and define an equilibrium.

Demographics and timing Each life-cycle is $T = 4$ periods with two life-cycle stages: childhood and parenting. Let model age be denoted by $j \in \{1, .., T\}$. Each model period corresponds to eleven years. Individuals are heterogeneous in their age j, human capital h, and beliefs, normally distributed with mean $\mu_{j,i,\theta}$ and standard deviation $\sigma_{j,i,\theta}$, over the return to parental investments during parenting. I will denote variables pertaining to the current generation of parents without superscripts, the prior generation's variables with a superscript p (grandparents), and next generation's variables with a superscript c (children).

In childhood ($j = 1, 2$), individuals do not make any decisions. When an individual becomes a parent in $j = 3$, she realizes her level of human capital, which is determined by the investment decisions of her parents, and has a child of her own. In the parenting stage ($j = 3, 4$), parents receive signals, and update their beliefs about the productivity of parental investments each period. Then they decide how much to borrow or save and how much to invest in the human capital of their child. The model ends when children leave the household. Figure 1 illustrates the timeline of life-cycle stages that individuals go through in the model.

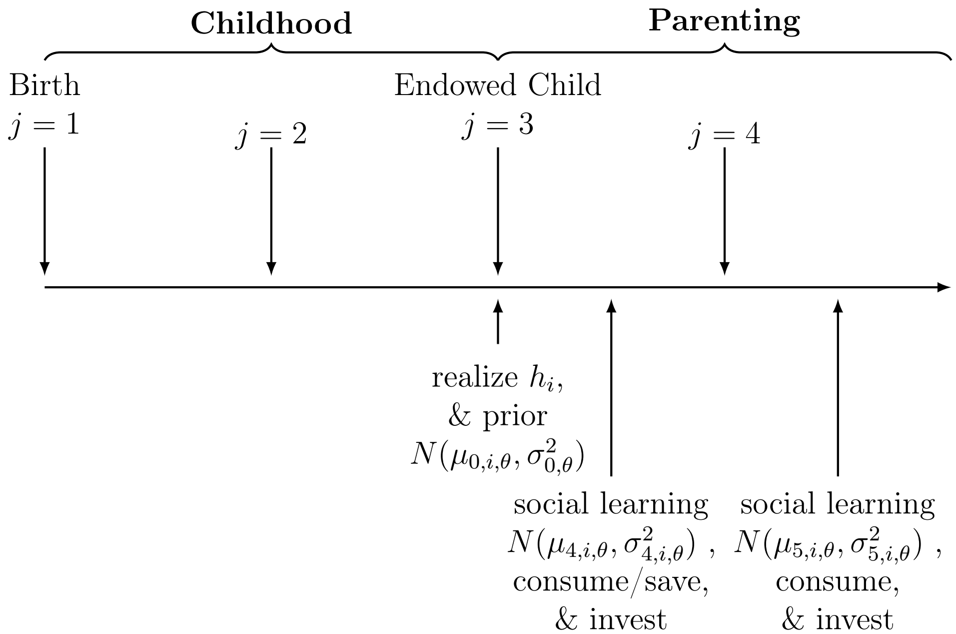


Figure 1: Life-cycle Stages

Wages and human capital The earnings technology and human capital production function are kept simple in order to focus on the role of social learning. Wages are a deterministic function of human capital h, which is constant in adulthood,

$$w(h) = \kappa_1 + \kappa_2 h, \tag{1}$$

where κ_1 is the intercept and κ_2 is the earnings slope.

Children's initial human capital h_0^c is correlated with parental human capital h,

$$ln(h_0^c) = \rho_c ln(h) + ln(\nu), \tag{2}$$

where ρ_c is the persistence of parental human capital to children's initial human capital at birth and $ln(\nu) \sim N(\mu_{ln(\nu)}, \sigma^2_{ln(\nu)})$ is a normally distributed exogenous shock to children's initial human capital.

And finally, children's human capital h^c, evolves based on parental investment x,

$$ln(h^{c\prime}) = \lambda ln(h^c) + \theta ln(x) + ln(\eta), \tag{3}$$

where λ is the self-productivity, θ is the productivity of parental investments, and $ln(\eta) \sim$

$N(0, \sigma^2_{ln(\eta)})$ is a normally distributed shock to human capital.[12] I further simplify the setup by letting the production function of children's human capital be equal in both periods of investment and assuming that the elasticity of parental investments be homogeneous for all parents.[13]

When $\lambda > 0$ and $\theta > 0$ the production function is concave in current the child's human capital level and investments. Consequently, the technology of child human capital formation features dynamic complementarity where prior investments in children's human capital make current investments more productive (e.g. Cunha and Heckman (2007)).[14][15]

Learning In this model economy, parents learn about the investment elasticity, θ, before making investment choices. Departing from full information by introducing learning to the quantitative intergenerational mobility literature is motivated by the wave of recent empirical evidence that documents subjective parental beliefs about children's human capital formation technology (e.g. Cunha et al. (2013); Cunha (2021); List et al. (2021)). In this environment learning has two features: (i) learning is social in nature and (ii) parents are unable to account for selection bias in their social interactions. Three facts from the empirical literature motivate the way in which the social learning environment is modelled in this paper. As discussed in Frick et al. (2022), systematic discrepancies in individual subjective beliefs about the aggregate environment, whether it be the return to schooling or political attitudes, are widely documented.[16]. And moreover, systematic discrepancies in individuals' perceived and actual social interactions have been documented by Breza et al. (2018) and Gallo et al. (2023), among others. Second, empirical evidence shows that there are gaps in average subjective beliefs across socioeconomic status about the importance of parental investments (List et al., 2021). If individuals learn socially but were able to account for the

[12] Self-productivity is the notion that a higher current stock of human capital is associated with future higher skills.

[13] While it has been documented that the elasticity of investment is decreasing in children's age, relaxing this assumption does not change the main channel induced by assortative social connections. However, relaxing the assumption that this productivity of parental investments is equal across parental types could diminish the role of the social learning channel. I argue that this assumption is not so unreasonable in the next subsection. In particular, the experimental results suggest that parents are responsive to information which implies that there is at least some misallocation as result of information frictions, especially among poor parents.

[14] This human capital production function is similar to that in Lee and Seshadri (2019) who have an age-specific, Cobb-Douglas production function. Allowing for age-specific technologies such that θ_j varies between early and late childhood and defining $\lambda = 1 - \theta_j$ would result in the production function that is estimated in Lee and Seshadri (2019).

[15] With complementarity between children's human capital and investments the introduction of public investments into such a framework would amplify the parental investment behavior. If children's human capital and investments are substitutes crowding out private investment relatively more for low human capital parents, parental investment behavior would decline more for children of low human capital parents than for children of high human capital parents.

[16] See also Alt et al. (2022); Gallo et al. (2023); Breza et al. (2018); Maddock and Glanz (2005); Boneva and Rauh (2018)

selection bias in their social networks, then, on average, parents' subjective beliefs should be equal. The fact that the gap in average beliefs is not zero implies that either parents are not accounting for selection bias or the returns to investment are different across socioeconomic status. The third fact is that providing parents with information about the return to parental investments changes their subjective beliefs and investment behavior (List et al., 2021; Cunha et al., 2013; Jensen, 2010). If the true returns to parental investment were different across groups then providing accurate information to parents should have no impact on their subsequent investment behavior. Parents would have been optimizing before receiving information and so their investment behavior should be unchanged after the intervention. Since this is not the case and parents are responsive to the information intervention in the sense that both their subjective beliefs and investment behavior are revised, the difference in mean beliefs must (at least partially) be due to bias. In this social learning environment, parents perceive their set of social interactions to be representative, and are thus unable to account for the selection bias when infering the return to children's human capital using their social connections' experiences.

I incorporate social learning over the return to parental investments, θ, that gives rise to individual-specific subjective beliefs. In particular, assume that parents do not observe the true value of the return to parental investment (θ) or their child's human capital level (h^c) when they make their investment decisions. In each period of parenting, parents draw an independent set of signals from a distribution of social connections. Parents use their sets of signals to update beliefs about the return to parental investments in a Bayesian fashion. Therefore, parental investment choices in each period will be informed by their draw of social interactions.

Priors At the beginning of adulthood in $j = 3$, an individual's human capital is realized which is a function of her parents' human capital and past investments according to 4.

$$ln(h_i) = \underbrace{\lambda^2 \rho_c ln(h_i^p) + \lambda^2 ln(\mu_{ln(\nu)})}_{\text{Contribution of initial human capital}} + \underbrace{\theta(\lambda ln(x_{i,3}^p) + ln(x_{i,4}^p))}_{\text{Contribution of parental investments}} + \underbrace{\varepsilon_i}_{\text{Contribution of shocks}}, \quad (4)$$

where h_i^p is individual i's parents' human capital, $\mu_{ln(\nu)}$ is the mean of the shock to initial human capital, $x_{i,j}^p$ are parental investments when the parent is age j, and ε_i is a combined error term.[17] All values in (4) with the exception of θ and ε_i are observed by the parent and lay the foundation

[17] The combined error term is define as

$$\varepsilon = \lambda^2(ln(\nu) - ln(\mu_{ln(\nu)})) + \lambda ln(\eta_1) + ln(\eta_2)$$

It combines all the exogenous shocks into a mean zero error term.

for her prior beliefs.

Parent i's prior beliefs about the value of θ are normally distributed with mean $\mu_{i,\theta}$ and common variance σ_θ^2. I assume prior beliefs are a function of one's own experience. The parameters of their priors are the mean and variance. Hyper-priors specify the functional form of a parameter.[18] Let the hyper-prior of $\mu_{i,\theta}$ be given by inverting 4

$$\mu_{i,\theta} \sim \mathcal{F}(h_i, h_i^p, x_{i,3}^p, x_{i,4}^p) \tag{5}$$

where

$$\mathcal{F}(h_i, h_i^p, x_{i,3}^p, x_{i,4}^p) = \frac{ln(h_i) - \lambda^2 \rho_c ln(h_i^p) + \lambda^2 ln(\mu_{ln(\nu)})}{\lambda ln(x_{i,3}^p) + ln(x_{i,4}^p))}$$

And assume the prior of the variance is homogeneous such that σ_θ^2 is common across individuals. Together an individual i's prior beliefs are as follows

$$\hat{\theta}_i \sim N(\mu_{\theta;i,0}, \sigma_{\theta;0}^2) \tag{6}$$

Notice that priors are informed by past experiences of investment and the human capital of one's parent. Moreover, prior mean beliefs differ across individual realizations of shocks to human capital.

Social learning Once human capital shocks and thus priors are realized, parents will update their beliefs via social interactions with their friends. Parents receive signals: past parental investments $x_{j=1}^p, x_{j=2}^p$, past parental human capital h^p and human capital h of K social connections.[19] Social connections matter because they determine the composition of observed signals. In the model, an individual i draws the number of above-median social interactions they have from a binomial distribution, $\mathbb{K}_{i,j} \sim B(K, p_i(h))$, where $p_i(h)$ denotes the probability of observing a signal of an above-median income parent, and randomly observe friends within the potential set of above- and below-median social connections in the population. Note that if $p_i(h) = 0.5$, signals are drawn randomly across the distribution of potential social connections. A key assumption is that parents cannot account for the selection in their distribution of social connections— they have assortativity neglect. This comes as a result of not knowing the actual probability $p_i(h)$ that governs the distribution of social interactions for parent i with human capital h, and instead

[18] Also referred to as the second stage prior at times in the Bayesian learning literature. See Berger (1985) for a review of prior information and subjective beliefs in Bayesian analysis.

[19] The number of social connections is assumed to be homogeneous for simplicity. A non-trivial extension to this framework would be to introduce heterogeneous K_i across the distribution of income which could be done using the data in Chetty et al. (2022). Since high-income adults are more likely to have larger social networks this would increase the precision of their subjective beliefs relative to low-income parents.

perceiving social connections to be representative i.e. $p_i(h) = 0.5$.

In period $j = 4$, individuals receive a new set of signals that is independent from those received in period $j = 3$. Again, parent i draws the number of above-median social interactions she has from a binomial distribution, $\mathbb{K}_{i,j} \sim B(K, p_i(h))$ and randomly draws signals within the potential set of above- and below-median social connections in the population. Let $\mathbb{S}_{i,j}$ denote individual i's set of period j signals.

In each period, parents use the signals $\mathbb{S}_{i,j}$ to update their beliefs in a Bayesian fashion before making consumption, savings, and investment decisions. At the beginning of each period parents receive a set of K signals, and use the evolution of children's human capital to infer the return on past parental investment. Bayesian learning with multiple signals is equivalent to the following two-step procedure: (1) infer the productivity of investments in children's human capital using multiple signals and (2) update posterior beliefs using both individual priors and estimates from step 1 (Fogli and Veldkamp, 2011).[20]

The technology of children's human capital formation (3) implies that the log of children's adulthood level of human capital $ln(h)$ is given by

$$ln(h) = \lambda^2 \rho_c ln(h^p) + \theta(\lambda ln(x_3^p) + ln(x_4^p)) + \lambda^2 ln(\mu_\nu) + \varepsilon,$$

With this in mind parents use their set of observed signals $\{x_{3,s}^p, x_{4,s}^p, h_s^p, h_s\}_{s \in \mathbb{S}_{i,j}}$ to estimate the productivity of parental investments θ.[21] We can re-cast the problem with multiple signals in matrix notation as the following regression,

$$H = H^p \rho_c \lambda^2 + \mathcal{V}\lambda^2 + [X_3^p \lambda + X_4^p]\theta + N_1 \lambda + N_2 \tag{7}$$

where H, X_3^p, and X_4^p are $K \times 1$ vectors of $\{ln(h_s), ln(x_{s,3}^p), ln(x_{s,4}^p)\}_{s \in \mathbb{S}_{i,j}}$ respectively, H^p is $\{ln(h_s^p)\}_{s \in \mathbb{S}_{i,j}}$ a vector of parental human capital, $\mathcal{V}$ is a $K \times 1$ vector of the expected initial human capital shock ($ln(\mu_\nu)$), and N_1, N_2 are vectors of $\{ln(\eta_{s,1}), ln(\eta_{s,2})\}_{s \in \mathbb{S}_{i,j}}$ residuals of the same size ($K \times 1$).

Rearranging (7) gives,

$$H - H^p \rho_c \lambda^2 - \mathcal{V}\lambda^2 = X^p \theta + N \tag{8}$$

where we define $X^p = [X_3^p \lambda + X_4^p]$ and $N = [N_1 \lambda + N_2]$.

The estimated coefficient $\hat{\theta}$ for individual i at age j is distributed normally $N(\hat{\mu}_{\theta;i,j}, \hat{\sigma}^2_{\theta;i,j})$ where

[20] Since learning is passive in this framework there is no incentive to strategically make investment choices in order to generate information.

[21] Bayesian social learning with multiple signals is similar in spirit to the social learning technology in Fogli and Veldkamp (2011).

the mean and variance are as follows,

$$\hat{\mu}_{\theta;i,j} = (X^{p\prime}X^p)^{-1}(X^{p\prime}(H - H^p\rho_c\lambda^2 - \mathcal{V}\lambda^2)) \tag{9}$$

$$\hat{\sigma}^2_{\theta;i,j} = \frac{1}{K}(\sigma^2_\varepsilon)(X^{p\prime}X^p)^{-1} \tag{10}$$

where $\sigma^2_\varepsilon = \lambda^4\sigma^2_{ln(\nu)} + \lambda^2\sigma^2_{ln(\eta)} + \sigma^2_{ln(\eta)}$.[22]

Finally, form the posterior belief denoted by $\mu_{\theta;i,j}$ as a linear combination of the estimated value $\hat{\mu}_{\theta;i,j}$ and prior beliefs $\mu_{\theta;i,j-1}$. The posterior mean is

$$\mu_{\theta;i,j} = \frac{\hat{\sigma}^2_{\theta;i,j}}{\hat{\sigma}^2_{\theta;i,j} + \sigma^2_{\theta;i,j-1}}\mu_{\theta;i,j-1} + \frac{\sigma^2_{\theta;i,j-1}}{\hat{\sigma}^2_{\theta;i,j} + \sigma^2_{\theta;i,j-1}}\hat{\mu}_{\theta;i,j} \tag{11}$$

and the posterior variance is

$$\sigma^2_{\theta;i,j} = (\sigma^{-2}_{\theta;i,j-1} + \hat{\sigma}^{-2}_{\theta;i,j})^{-1} \tag{12}$$

Assortativity in social connections generates systematic misinferences across the distribution of human capital and earnings that deviate from the benchmark of unbiased beliefs. These misinferences or biases in beliefs persist due to two effects as highlighted by Frick et al. (2022). First is the "false consensus effect," in which parents' beliefs about the impact of parental inputs on children's future human capital is increasing in the parents' own human capital. The false consensus arises because higher human capital individuals are more likely to have social connections whose distribution of human capital shocks were above average (positive assortativity). Since these parents are more likely to have friends that were "lucky" — (mis)attributing the contribution of human capital shocks to the contribution of parental investments — on average, they overestimate the impact of parental investments on children's human capital. I further discuss the presence of selection bias in parental subjective beliefs in a simple, static framework to illustrate the false

[22] To find the variance of the OLS estimate we need to find the variance of the combined error term. Define the combined error as

$$\varepsilon = \lambda^2(ln(\nu) - ln(\mu_{ln(\nu)})) + \lambda ln(\eta_1) + ln(\eta_2)$$

$$\begin{aligned} Var(\varepsilon) &= Var(\lambda^2 ln(\nu)) + \lambda ln(\eta_1) + ln(\eta_2)) \\ &= Var(\lambda^2 ln(\nu)) + Var(\lambda ln(\eta_1)) + Var(ln(\eta_2)) \\ &= \lambda^4\sigma^2_{ln(\nu)} + \lambda^2\sigma^2_{ln(\eta)} + \sigma^2_{ln(\eta)} \end{aligned}$$

The variance of the coefficient is

$$Var(\hat{\mu}_{\theta;i,j}) = \frac{1}{K}\sigma^2_\varepsilon(X^{p\prime}X^p)^{-1}$$

Plugging in $Var(\varepsilon)$ for σ^2_ε and simplifying gives (10).

consensus effect in Appendix A.1.3. Second is the more "dispersed" parental investment behaviors relative to the benchmark economy of unbiased beliefs. In the model, subjective beliefs are formed using information about the observed distribution of parental investment behaviors, so biases in beliefs are implied by the investment decisions made by parents. That is, the optimal choices and resulting distribution of outcomes imply misinferences in social learning. This ensures that misperceptions are persistent in the sense that, on average, misinferences are not contradicted by the signals received from social interactions (Frick et al., 2022).

Preferences Individuals are risk-averse, paternalistic, and discount the future at rate $\beta \in [0, 1]$.[23] Parents value consumption c according to the utility function $u(c)$, and value their children's expected human capital in adulthood according to parameter α.

1.2 Value Functions

In this section I present the value functions over the life-cycle of an individual beginning when she enters the parenting stage, and makes her own decisions.

Parenting Stage ($j = 3, 4$) Let $V_j(h, \mu_\theta, \sigma_\theta^2, b, x_{j-1})$ denote the value function of an age j parent with human capital h, prior beliefs $N(\mu_\theta, \sigma_\theta^2)$, and assets b and last period investment x_{j-1}. Assume that in the first stage of parenting assets are equal to zero. Since parental preferences are such that parents value their children's human capital directly (as opposed to the stream of consumption for all future generations in the dynasty) there is no incentive to leave children with bequests.[24] Thus, the implications are the same if instead I allow for initial assets to be equal to bequests.

In the current period, each parent updates her beliefs with the signals received from her social connections in combination with her priors using Bayes law, then makes a consumption/savings decision, and decides how much to invest in their child's human capital. Investments, x, increase the child's expected adulthood human capital and thereby parent's utility. Investments are modeled as a goods investment in children's human capital. Extending this to a framework in which parents invest in various inputs (monetary, time, housing, etc.) does not change the inefficiencies in this model as long as parents have subjective beliefs about the average return to these investments. [25]

[23]Paternalistic parents value child outcomes directly, where as altruistic parents care about the utility of their child. The literature is largely divided between these two specifications of parental preferences. See Del Boca et al. (2014) for an example of paternalistic parental preferences.

[24]Paternalistic preferences are key to shutting down the bequest motive. This assumption is useful because it simplifies the parent's problem and reduces the dimensionality of the steady state distribution.

[25]When parents are risk averse, what is key is that parents trade-off investing more early in childhood (making

The decision problem of an age $j = 3$ parent with human capital h have initial prior beliefs $N(\mu_0, \sigma_0^2)$, and assets $b = 0$ is to choose assets for the next period, investment and consumption. Thus the value function is given by,

$$V_3(h, \mu_{\theta;0}, \sigma^2_{\theta;0}, 0, 0) = \max_{b',x} u(c) + \beta \mathbb{E}_{\theta,\nu,\eta} \left[V_4(h, \mu_{\theta;j}, \sigma^2_{\theta;j}, b') \right], \tag{13}$$

subject to the budget constraint,

$$c + x + b' \leq w(h) \text{ for } j = 3,$$

and the borrowing constraint,

$$\underline{b'} \geq f(w(h))$$

where $f(\cdot)$ is a flexible function of income and the wage equation (1). Posterior beliefs after social learning are given by $N(\mu_{\theta;j}, \sigma^2_{\theta;j})$ with $j = 3$ and appear as the prior in period four. Notice that the prior beliefs in the continuation value function indicate a different subjective belief than those that parents entered the period with.[26] Parents form expectations over their children's human capital using their posterior and the fact that initial human capital at birth is assumed to be correlated to parental human capital according to equation (2). Parents then make their consumption/savings and investment decisions using their posterior beliefs and the evolution of children's human capital from equation (3).

The decision problem of an age $j = 4$ parent with human capital h, prior beliefs $N(\mu_\theta, \sigma^2_\theta)$, assets b, and past investment x_3 is given by,

$$V_4(h, \mu_{\theta;j-1}, \sigma^2_{\theta;j-1}, b, x_3) = \max_{x} u(c) + \beta\alpha \mathbb{E}_{\theta,\nu,\eta}[\ln(h^{c'})], \tag{14}$$

$$V_j(h, \mu_\theta, \sigma^2_\theta, b, x^{j-1}) = 0 \ \forall j > 4$$

subject to the budget constraint,

$$c + x \leq w(h) + b(1 + r_f) \text{ for } j = 4,$$

where r_f is the risk-free rate, the wage equation (1), and the child's human capital production function (3).

later investments more productive) when optimal choices are more uncertain, or saving more (borrowing less) to finance investing later in childhood. One could introduce more flexible parental investment by defining x as an aggregate investment with various inputs using a CES aggregator and estimating the elasticities of substitution as is done in some of the quantitative family macroeconomics literature.

[26] The typical conventions of using a $'$ to indicate the state in the next period of a Bellman equation is not used here to be consistent with the notation of prior and posterior beliefs presented above.

In the terminal period, each parent once again updates her beliefs with the signals received from her social connections in combination with her priors (which are the posterior beliefs in period three) using Bayes law, then makes decides how much to invest in their child's human capital and consume under their posterior beliefs which differ from those in the previous period.

Even though parents may have more accurate beliefs now that they have updated their beliefs for a second time, the investments made in period three are irreversible. Parent's use their posterior and the decisions they made in period three with states $(h, \mu_{\theta;j-1}, \sigma^2_{\theta;j-1})$ along with the production function of children's human capital in equation (3) to form expectations about their children's human capital. At the beginning of the next period the child leaves the household and becomes a parent themselves. The parenting stage ends at $j = 4$ as is indicated by the terminal condition in (14). Next, I define an equilibrium of the model.

1.3 Equilibrium

A recursive competitive equilibrium is (1) a sequence of prices, (2) policy functions for consumption, savings/borrowing, and investment, and (3) a stationary distribution of individuals over states $\Omega : j \times h \times \mu_\theta \times \sigma^2_\theta \to [0, 1]$ such that:

1. Given prices $\{r^*_{f,t}, w^*(h)_t\}_{t=0}^{\infty}$ where t denotes time, household policy functions $\{c^*_t, b^*_t, x^*_t\}_{t=0}^{\infty}$ maximize utility,

2. Prices, $\{r^*_{f,t}, w^*(h)_t\}_{t=0}^{\infty}$, clear the asset and labor market,

3. The distribution of agents $\{\Omega(j_t, h_t, \mu_{\theta,t}, \sigma^2_{\theta,t})\}_{t=0}^{\infty}$ is consistent with household policy functions and exogenous shocks to children's human capital ν and η.[27]

2 Model Calibration

In this section, I discuss the calibration of the model. I calibrate the model using data from several U.S. datasets and randomized control trials. The calibration proceeds in three steps. First, some parameter values are chosen externally, from the literature or as a normalization. Second, some parameters are directly estimated from the data. Third, the remaining parameters are estimated to match a series of moments in the data on parental expectations, parental investment, and children's human capital evolution using the simulated method of moments. Table 1 reports the calibrated parameters.

[27]For details on the specific solution algorithm I employ see A.3.

Demographics and Preferences Each period in the model is 11 years. Let preferences over consumption be given by,[28]

$$u(c) = ln(c)$$

The discount factor β is calibrated to an implied annual interest rate of four percent and r_f is set such that $\beta(1 + r_f) = 1$. The altruism parameter α is calibrated to match the average (log) investment to (log) earnings ratio as estimated in early childhood from Baby's First Years which is 0.205.

Social learning The binomial distribution that determines the strength of the social learning mechanism is governed by two parameters: K the number of signals observed each period and p_i the probability of drawing an above-median income social interaction.

I use List et al. (2021)'s RCT data on parental beliefs to discipline the number of signals that parents use to form beliefs. The number of signals that parents receive affects belief updating by determining the relative weights that parents assign to priors and new signals, which we can see in the equations for posteriors (i.e. 12 and 11). In particular, I use data from List et al. (2021)'s Newborn study which elicits subjective parental beliefs using the Subjective Parental Expectations and Knowledge (SPEAK) survey instrument. This instrument contains 57 questions from which each parent is assigned a score. SPEAK is designed to capture parental expectations about the malleability of intelligence, importance of early childhood environments and experiences, and perceptions about media exposure in cognitive and language learning among children (List et al., 2021). To calibrate the number of signals that parents receive, I target the ratio of average beliefs among above-median to below-median socioeconomic status parents which is 1.10. The difference between average beliefs across the income distribution is informative about the number of observations since as K goes to infinity, the extent to which parent's weight their own experiences (which is unbiased) relative to the experiences of their social connections (which suffer from assortativity neglect) goes to zero.

The distribution of probabilities of having an above-median income social interaction p_i, which determines the distribution of signals for parents across the distribution of income, is set to match the distribution of economic connectedness for deciles of the distribution of income in the U.S. documented by Chetty et al. (2022).[29]

[28]Parental preferences are paternalistic. They derive utility from the log of children's human capital which does not display complementarity between the log of parental investments across periods. Since the complementarity of investments across periods is central to individual responses to borrowing constraints, these parental preferences will affect behavior. This is standard in traditional models of human capital development such as Ben-Porath. See A.2 for a discussion of how these parental preference affect behavior.

[29]See Extended Data Table 1 of Chetty et al. (2022).

The degree of economic connectedness measures the share of social connections that an individual i with income rank r_i has who are above median income. This measure of economic connectedness maps directly into the the probability that an individual with income rank r_i observes a signals from a social connection who is above median-income.

Borrowing constraints As in Braxton et al. (2023), I estimate the relationship between earnings and credit constraints directly from the 2001-2004 Survey of Consumer Finances (SCF). [30] In particular, I estimate the following specification,

$$\underline{b}'_i = \psi + \delta w(h_i) + e_i \tag{15}$$

where ψ is the intercept of the borrowing constraint and δ is the slope of the borrowing limit. I estimate this regression for individuals aged 22-43 to align with the age structure of parents in the model's life-cycle, and estimate δ to be 0.21.[31] I calibrate ψ, which determines the extent to which households save or borrow, to match the average ratio of investment among below-median households to above-median households in the BFY data.[32]

Children's human capital Children's initial human capital is determined by equation (2). I calibrate the parameter ρ_c, which captures the persistence in human capital across generations, to match the intergenerational elasticity of earnings (IGE) which is given by γ_1 in the following regression,

$$log(child's\ earnings_i) = \gamma_0 + \underbrace{\gamma_1}_{IGE} log(parent's\ earnings_i) + \epsilon_i$$

The IGE γ_1 captures the expected percent increase in children's permanent earnings associated with a one percent increase in parent's permanent earnings. Davis and Mazumder (2022) estimate the IGE in the U.S. to be 0.41.

Children's initial human capital process also includes shocks to human capital which are calibrated to match data on children's distribution of human capital in early and late childhood from the Panel Study of Income Dynamics Child Development Supplement (PSID CDS). I measure children's skill in the PSID CDS using age-adjusted letter word scores as in Lee and Seshadri (2019).

[30] Braxton et al. (2023) illustrate that the relationship between observed borrowing limits and income is linear.

[31] As in Braxton et al. (2023), I winsorize limits and earnings for the top 5% of individuals and include individuals with zero limits.

[32] Investment is measured in logs plus one.

Table 1: Model Parameters

External

Variable	Value	Description	Source
β	0.65	Discount factor	4% annual rate
r_f	0.54	Risk free return	$\beta(1 + r_f) = 1$
$\mu_{ln(\eta)}$	0	Mean shocks to child human capital	Normalized
σ_0^2	1	Var. initial prior	Normalized
$\bar{EC}_{10}$	0.251	Prob. high-inc social inter. $r_i \leq 10$	Chetty et al. (2022)
$\bar{EC}_{20}$	0.321	Pr high-inc social inter $r_i \in (10, 20]$	Chetty et al. (2022)
$\bar{EC}_{30}$	0.393	Pr high-inc social inter $r_i \in (20, 30]$	Chetty et al. (2022)
$\bar{EC}_{40}$	0.457	Pr high-inc social inter $r_i \in (30, 40]$	Chetty et al. (2022)
$\bar{EC}_{50}$	0.518	Pr high-inc social inter $r_i \in (40, 50]$	Chetty et al. (2022)
$\bar{EC}_{60}$	0.582	Pr high-inc social inter $r_i \in (50, 60]$	Chetty et al. (2022)
$\bar{EC}_{70}$	0.644	Pr high-inc social inter $r_i \in (60, 70]$	Chetty et al. (2022)
$\bar{EC}_{80}$	0.701	Pr high-inc social inter $r_i \in (70, 80]$	Chetty et al. (2022)
$\bar{EC}_{90}$	0.762	Pr high-inc social inter $r_i \in (80, 90]$	Chetty et al. (2022)
$\bar{EC}_{100}$	0.842	Pr high-inc social inter $r_i \in (90, 100]$	Chetty et al. (2022)

Estimated Directly

Variable	Value	Description	Source
θ	0.849	Return to investment	BFY
δ	0.21	Borrowing slope	SCF

Jointly-Calibrated

Variable	Value	Description	Target
ρ_c	0.71	IG corr. initial human capital	IGE
$\mu_{ln(\nu)}$	-1.94	Mean shocks to initial human capital	Avg. skill in early childhood
$\sigma^2_{ln(\nu)}$	4.63	Var. shocks to initial human capital	Variance of investment rate in early childhood
$\sigma^2_{ln(\eta)}$	4.13	Var. shocks to child human capital	Std. dev. skill in late childhood
κ_1	92.0	Earnings intercept	Avg. log earnings
κ_2	0.53	Earnings slope	Avg. investment to earnings
K	35	Number of signals	Belief ratio
λ	0.006	Self productivity of human capital	Avg. skill in late childhood
ψ	-6.96	Borrowing intercept	Avg. log investment to borrowing in early childhood
α	0.52	Altruism	Avg. log investment to earnings in early childhood

I estimate the mean value of the shock to initial children's human capital to match the average human capital in early childhood, as measured by the standardized letter-word (LW) score to have standard deviation one, which is 3.14.[33] I target the variance of investment rates in early childhood in the PSID which is 0.01.

Children's human capital evolves according to the technology of children's human capital formation in equation (3). The productivity of parental investments is the deep parameter over which parents are learning. Given its importance, I use the BFY data to directly estimate θ using an instrumental variables approach. The instrument exploits BFYs' experimental design in which parents were randomized to receive either a high or low cash transfer gifts. The instrument $Z_{i,s}$ is an indicator variable that takes a value of one when a parent is randomly assigned to the high cash transfer treatment group and zero otherwise.[34]

Gennetian et al. (2022) estimate a causal intent-to-treat (ITT) effect of the high cash transfer treatment on parental time and monetary investments. Their findings show that $Z_{i,s}$ has a strong first stage and is a relevant instrument for generating variation in parental investment. For the purposes of this calibration, I measure parental investments in the BFY as the maternal time in child-enriching activities. These activities include frequency of time spent reading, telling stories, playing and building things, and participating in playgroups. I translate these categorical responses to a continuous measure of time in minutes per week spent investing in these activities with estimates of maternal time use (by mother's education) from Kalil et al. (2012) which draws on the American Time Use Survey, as in Gennetian et al. (2022). See Appendix (A.4.1) for more details.

I assume that other types of parental investments are proportional to parental time investments.[35] Let $ln(h^c_{i,s+1})$ denote the log of children's skills as measured by the ASQ of parent i in survey wave s. To measure children's human capital I use the ASQ age-specific scores standardized to have standard deviation one. To estimate the effect of parental investment on children's human capital I estimate an IV specification of the form:

$$ln(h^c_{i,s+1}) = \alpha_0 + \lambda_0 ln(h^c_{i,s}) + \theta\widehat{t_{i,s}} + \mathit{\Gamma}_0\chi_{i,s} + \epsilon_{i,s+1}, \tag{16}$$

$$t_{i,s} = \alpha_1 + \lambda_1 ln(h^c_{i,s}) + \delta_1 Z_{i,s} + \mathit{\Gamma}_1\chi_{i,s} + u_{i,s}, \tag{17}$$

[33] To align with the age structure of the model, this is measured among PSID CDS children aged 11 and under.

[34] The high cash transfer treatment group received a $333 per month unconditional cash transfer and the low cash transfer control group received a $20 per month unconditional cash transfer.

[35] The BFY data does have some transaction level information on child expenditures which would allow one to construct an aggregate measure of both time and monetary investment. However, Gennetian et al. (2022) note that the most common transaction among high-cash-gift recipients is withdrawal of cash from an ATM, amounting to approximately one-third of the total transfer. This limits our ability to interpret expenditures from the debit card transaction data. In the future, I would like to make progress on this by estimating a measurement model of parental investment using the somewhat limited expenditures data.

where $\widehat{t_{i,s}}$ in the second stage regression (equation 16) is the predicted value from the first stage regression (equation 17) and $\chi_{i,s}$ is a vector of controls.[36] For the instrument to be valid, it must be relevant ($cov(Z_{i,s}, \widehat{t_{i,s}}|\chi_{i,s}) \neq 0$) and conditionally exogenous ($cov(Z_{i,s}, \epsilon_{i,s+1}) = 0$).

I confirm that the instrument is relevant, that the high cash transfer treatment (statistically) significantly increases parental time investments positively (Gennetian et al., 2022).[37] For the instrument to be conditionally exogenous, it should be the case that conditional on the controls, the high cash transfer treatment does not operate through other channels that impact children's human capital such as monetary investments in children's education. While this is a strong assumption, the sample characteristics and covariates available in the BFY data are well suited to assuage exogeneity concerns.

In particular, the sample of children in this data are four years old and under. For children of this age, the largest proportion of parental investments by far are parental time investments. It is unlikely that factors like schooling are at play for these young children. One potential factor that may be a threat to identification is children's health. I attempt to address this concern by controlling for maternal characteristics like mother's age and household characteristics like categories of household income and net worth. Given the set of controls and context, assuming that the proportion of total parental investments that are time investments for these children is close to one may not be unreasonable.[38] The coefficient θ, which is estimated to be 0.849, reports how parental investments influence children's subsequent human capital.[39]

The self-productivity, λ, is calibrated to generate average human capital in late childhood that matches the empirical average LW score standardized to have standard deviation one in late childhood which is 4.525 in the PSID CDS.[40] The dispersion of human capital shocks in childhood ($\sigma^2_{ln(\eta)}$) is calibrated to match the standard deviation of human capital as measured by the LW scores in late childhood which is estimated to be 9.56. And finally, the mean human capital shock during childhood ($\mu_{ln(\eta)}$) is normalized to zero.

[36] Covariates include: Mother's age, household income, net worth, number of adults in the household, and experimental site dummies. Results can be found in Appendix A.4.1.

[37] The F-statistic of the first stage regression is 7.13. Due to disclosure restrictions in the data agreement I cannot show the first stage results in full. This is potentially a weak instrument under the set of covariates included in this specification and I hope to improve this estimation in following iterations. Results from other specifications of (16) are pending disclosure.

[38] I discuss the implications of these assumptions in greater detail in Appendix A.4.1.

[39] I plan to conduct a sensitivity analysis by perturbating the estimated value of θ. The literature has varying estimates of the investment productivity; for example, Braxton et al. (2023) estimate this to be 0.11 while Caucutt and Lochner (2012) calibrate a value of 0.47. Lee and Seshadri (2019) calibrate an age-specific productivity of investment which is estimated to be decreasing in age (0.56 in primary school and 0.30 in secondary school).

[40] To coincide with the age structure of the model the average is taken over PSID CDS children aged 11 and older.

Earnings The earnings equation (1) has two parameters to be calibrated: κ_1 and κ_2. The intercept κ_1 is calibrated to match the average log annual earnings of parents in the PSID CDS which is 8.02, and the slope of the earnings function, κ_2, is calibrated to match the rate of investment in the last period of parenting. Using estimates from Lee and Seshadri (2019) I target an average ratio of investment to earnings in late childhood of 0.104.

Table 1 provides the full list of parameter values, and Table 2 displays the calibrated parameters and their calibration targets. The estimated model matches the targeted moments on investment behavior and intergenerational mobility well. The model is able to match the intergenerational earnings elasticity which is measured as the coefficient on a regression of the log of children's earnings on the log of parent's earnings. Importantly, the model is also able to match the ratio of average beliefs of high- and low-income mothers of young children. I discuss non-targeted moments in the next section.

Table 2: Model Calibration

Variable	Value	Target	Model	Data	Source
ρ_c	0.71	IGE	0.33	0.41	Davis and Mazumder (2022)
$\mu_{ln(\nu)}$	-1.94	Avg. skill in early childhood	3.8	3.14	PSID CDS
$\sigma^2_{ln(\nu)}$	4.63	Variance of investment rate in early childhood	0.01	0.01	PSID CDS
$\sigma^2_{ln(\eta)}$	1.96	Std. dev. skill in late childhood	12.4	9.6	PSID CDS
κ_1	92.0	Avg. log earnings	8.4	8.0	PSID CDS
κ_2	0.53	Avg. investment to earnings in late childhood	0.10	0.10	Lee and Seshadri (2019)
K	35	Belief ratio	1.19	1.10	List et al. (2021)
λ	0.006	Avg. skill in late childhood	3.7	4.5	PSID CDS
ψ	-6.96	Avg. log investment to borrowing in early childhood	0.21	0.38	BFY
α	0.52	Avg. log investment to earnings in early childhood	0.21	0.21	BFY

2.1 Model Validation

Under this calibration, I simulate the model for 10,000 parent-child dyads. Since this model examines the role of social learning on intergenerational mobility it is important that it captures the role of information frictions in household investment decisions. I show that the model is able to match the empirical relationship between subjective beliefs and investment choices. After simulating a data set, I show that the model is able to match untargeted evidence from List et al. (2021)'s randomized control trial as validation. Using the empirical data from List et al. (2021)'s Newborn study, I estimate the impact of increased subjective beliefs on parental investment rates as an untargeted moment and verify that the model is able to match the RCT evidence.

The first three columns of Table 3 present the regression results from the following specification at various survey waves

$$rate_{i,t} = \alpha_t + \beta_t \; speak_{i,t} + e_{i,t}$$

where $rate_{i,t}$ is the investment rate of parent i in survey wave t, $speak_{i,t}$ is parental beliefs, and $e_{i,t}$ is an error term. Parental investment rates are measured by direct assessment of the quality of parent-child interactions using the Nursing Child Assessment Satellite Training (NCAST) scale, and beliefs are measured using the standardized SPEAK score to coincide with the measures used by List et al. (2021).[41] The coefficients β_t shown in Table 3 capture the change in parental investment rates associated with a one standard deviation increase in parental beliefs measured at the 6, 12, and 18 month old survey waves.

Table 3: Investment rates & subjective beliefs

	— Dependent variable: parental investment rate —			
	(1)	(2)	(3)	(4)
	6 mo.	12 mo.	18 mo.	Model
Parental beliefs	0.160***	0.182***	0.131**	0.117***
	(0.0446)	(0.0499)	(0.0525)	(0.0113)
Observations	363	372	321	—
R-squared	0.035	0.035	0.019	—

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: The table shows the correlation between parental beliefs and investment rates for parent-child pairs in the Newborn Study at six, twelve, and eighteen months in columns (1)-(3). Column (4) shows the correlation between parental beliefs and investment rates for parent-child pairs in the simulated data at age $j = 3$.

[41] Parent-child interactions are videotaped and coded by trained assessors using the The Nursing Child Assessment Satellite Training (NCAST) scale which consists of 73 binary items. The sum of the positive responses to these items measures parent-child investment quality and can be used for children as young as six months old. See List et al. (2021) for further details. The SPEAK score is described in Appendix A.4.4.

Column four presents the results of an analogous regression from the model simulation where $\mu_{\theta;i,j=3}$ takes the place of $speak_{i,t}$. I find that a one standard deviation increase in subjective beliefs is associated with an 11.7% increase in parental investment rates. The results suggest that the model's prediction about parental behavior are consistent with the empirical evidence.

I analyze the consequences of social learning for children's earnings mobility in the next section.

3 Social Learning and Intergenerational Mobility

Using the calibrated model, I quantitatively examine the implications of social learning on intergenerational earnings mobility. In this counterfactual experiment, I modify the distributions from which agents draw their social interactions. In particular, I model a world in which there is no selection in social connections and instead parents draw social connections from a representative distribution by setting $p_i = 0.5 \forall i$ across the distribution of income.

Table 4: Role of Social Learning

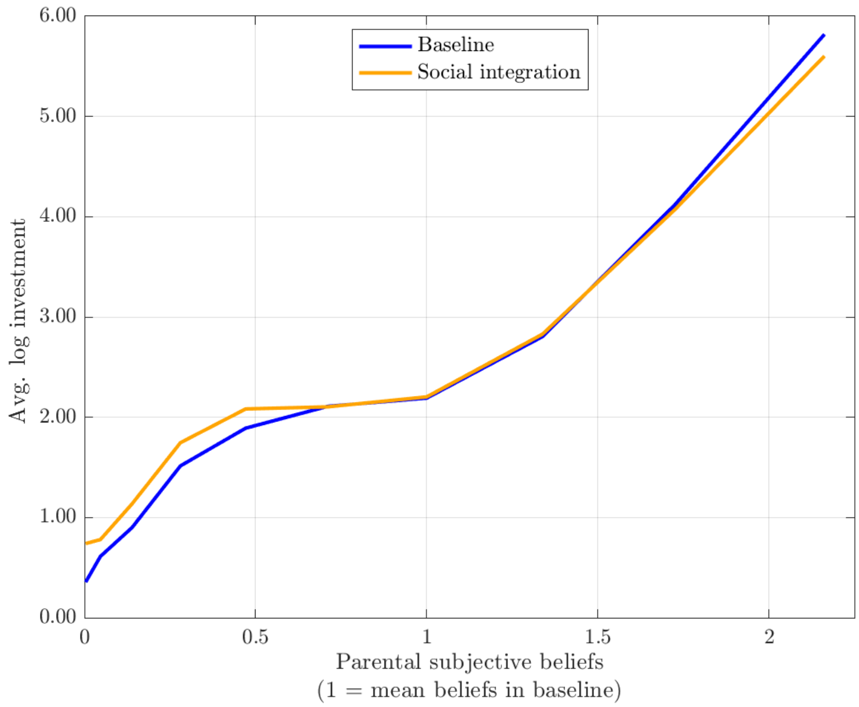
	(1) Baseline	(2) Social integration	(3) $\%\Delta$
IGE	0.333	0.304	8.7%

Notes: The table shows the intergenerational earnings elasticity from regressing the log of children's earnings on the log of parent's earnings from the simulated data. Column (1) shows the IGE for the baseline model, column (2) shows the IGE for the model under social integration (i.e.$p_i = 0.5 \forall i$), and column (3) calculates the percentage change in the IGE between the two.

Columns (1) and (2) of Table 4 present the estimated intergenerational elasticity of earnings (IGE) in the baseline model and the counterfactual of social integration, respectively. In our baseline model, the IGE is 0.333, however under social integration, the IGE declines to 0.304 showing that social learning plays a quantitatively significant role in the amplification of the intergenerational earnings elasticity.

Social integration has two effects on parental subjective beliefs that influence mobility. First, subjective beliefs no longer suffer from selection biases as they do in the baseline model. Since social interactions are representative there is no correlation in the shocks and the level of human capital of social connections in expectation. With less biased beliefs about the return to parental investments, those who had low subjective beliefs in the baseline revise their beliefs and behavior upwards while those who had high beliefs in the baseline revise downwards. This changes their investment behavior so that those who tend to have lower beliefs under social integration are less

Figure 2: Average parental investments over subjective beliefs



Notes: The figure shows the average log of parental investment as a function of parental beliefs (x-axis normalized so that the average posterior beliefs in thhe baseline equal 1). The blue (darker) line corresponds to the baseline economy and the orange line corresponds to the economy under social integration when all distributions of social interactions are representative.

likely to be low income and closer to their borrowing constraint than in the baseline model. When parents are further from their borrowing constraint they increase their investment to a greater extent when their beliefs about the return to parental investments increase, all else equal. Second, parental subjective beliefs are more precise. Since parents are now more likely to have a wider range of social connections they will be more confident about their beliefs over the productivity of parental investments; that is, their subjective beliefs have lower variance. Given that parents are risk averse in this framework their confidence about the return to investments in children's human capital also influences the extent to which they increase their investment when they move closer to or further from their borrowing constraint.

I plot average parental investments (in logs) over average posterior beliefs in $j = 4$ in Figure 2 to illustrate both of these forces. The blue (darker) line depicts investment behavior in the baseline model where a value of one for parental subjective beliefs is equal to the average subjective belief. The orange (lighter) line depicts the average investments given by subjective beliefs under social integration. The increase in investments among parents with low subjective beliefs and the decrease in investments among parents with high subjective beliefs illustrates the revision of beliefs. The second effect, the increase in precision, can be seen in the asymmetrical responses. That is, the increase in log investments among low subjective belief parents is larger than the decrease in investments among the high belief parents.

The results in this section show that earnings mobility is reduced by 8.7% in the model with information frictions generated by social learning relative to the world of social integration in which parents' social connections are representative. This suggests that policies that can reduce the information frictions generated by social learning have the potential to increase children's earnings mobility. In the following section, I will use the quantitative model to investigate how an informational policy intervention would impact aggregate earnings mobility and inequality.

4 Reducing Information Frictions

In this section I use the calibrated model as a laboratory to study the extent to which a large-scale, low-cost information intervention that aims to change parental beliefs can reduce the large persistence in earnings across generations. Recent work by List et al. (2021) analyzes the effects of an information intervention among parents of young children on their beliefs about the impact of parental investments. The study found that the information intervention, which consisted of providing parents with four, ten minute informational videos, increased elicited parental beliefs by 0.38 standard deviations.[42]

I implement this policy in the quantitative model as a permanent policy that is targeted to low-income parents. In particular, I assume that parents who receive this treatment do so at age $j = 3$ after they have observed the signals from their social connections, but before they make investment choices. This reflects the average effect size of the informational intervention on parental beliefs measured one year after treatment (18 months). I implement the information intervention exclusively among parents in the bottom two deciles of the income distribution to target low-income parents as many information interventions in the empirical literature do.[43] In the first period of parenting ($j = 3$), parents receive K signals from their social connections and form posterior beliefs over θ which are denoted $N(\mu_{\theta;i,3}, \sigma^2_{\theta;i,3})$. For parents who receive the information intervention, they also receive an experimenter's signal from the policy intervention that comes from a distribution with a mean equal to the true value of θ and variance 0.122, which is the variance of the intent to treat effect as measured in List et al. (2021). Parents use these $K + 1$ signals to update their beliefs according to Bayes rule.[44]

I study the effects of this policy intervention on average parental behavior, inequality, and intergenerational mobility. Average beliefs among the treated parents increase by 0.066 standard deviations post-intervention. Subsequently, treated parents increase average log investment by 15.5%.[45] Figure 3 presents parental investment behavior over parent's earning percentile in period $j = 3$. The baseline economy (in blue) shows that average log investments are increasing in parental income percentile slowly at first and then rapidly in the top 25%. Under the informational intervention (in orange), parental investments are nearly identical for the top half of the distribution regardless of parental income. Investments in children's human capital for below-median parents are higher than investments made by those in the baseline economy. These parents borrow until they become borrowing constrained to finance early parental investments in their children's human capital. For the top four deciles of the income distribution, investments are similar to investments made in the baseline economy.

Notice that even though only the bottom two deciles receive the experimenter's signal, the increase in mobility for children born to poor parents as a result of the policy spills over into higher beliefs for those children when they become parents. Since their prior beliefs are informed by their own experience, knowing that their parents' investments paid off and moved them up in the income distribution to the third, fourth or fifth deciles, increases their subjective beliefs about the importance of childhood investments and their subsequent investment choices.

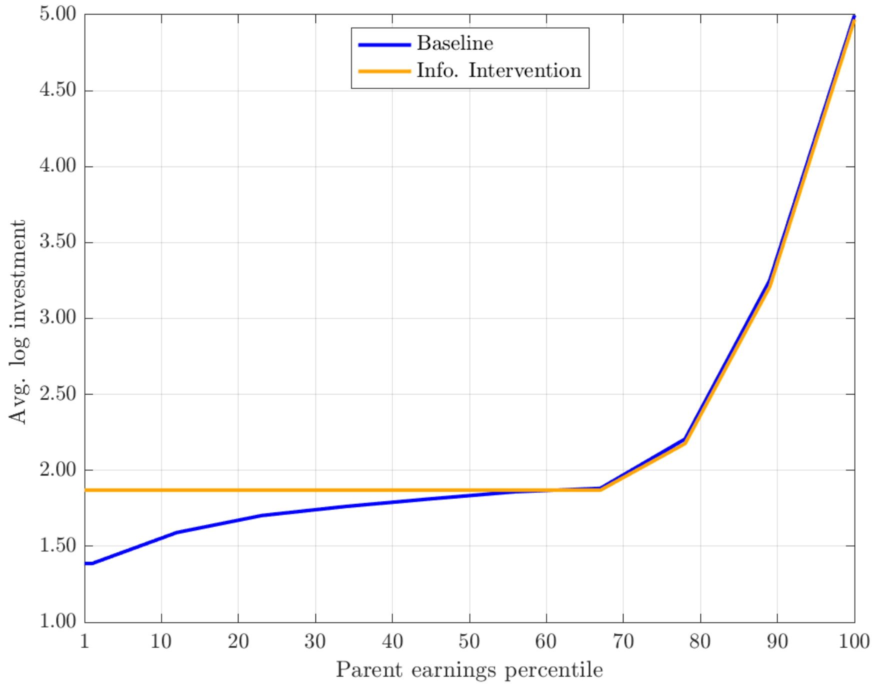
[42]The intent to treat effect size of 0.38 reported in Table 1 of List et al. (2021) is the standardized mean difference in standard deviation units one year after treatment (at 18 mo. survey).

[43]See for example Cunha (2021),Cunha et al. (2023), List et al. (2021).

[44]In particular, parents update beliefs using (11) and (12) with $K + 1$ signals.

[45]The average log investment among treated parents increases from 2.77 to 3.27.

Figure 3: Average parental investments over parental earnings



Notes: The figure shows the average log of parental investment as a function of parental earnings percentile. The blue (darker) line corresponds to the baseline economy and the orange (lighter) line corresponds to the economy under a permanent information policy intervention.

Since aggregate investments in children's human capital are larger, we expect that aggregate human capital is also greater under the policy that provides information to low-income parents. Indeed, average human capital is 2.7% higher in aggregate and 22% higher for children of treated parents (from the bottom 20%).

Table 5: Scaling-up an Information Policy Intervention

	(1) Baseline	(2) Info. Intervention	(3) %Δ
IGE	0.333	0.319	4.2%
Var. human capital	4.4	3.95	-10.2%

Notes: The table shows the intergenerational earnings elasticity and the variance in human capital from the model simulated data. Column (1) shows the results for the baseline model, column (2) shows the results under the permanent information policy intervention, and column (3) calculates the percentage change in the results between the two.

In the equilibrium with the permanent informational intervention the IGE falls to 0.319, which translates to a 4.2% *increase* in intergenerational mobility as shown in the first row of Table 5. This is about half of the size of the increase in mobility one would expect from eliminating information frictions generated by social learning as in the economy with full social integration discussed in the previous section. The second row of Table 5 shows that the variance of human capital is reduced from 4.4 in the baseline to 3.95 under the policy of informational interventions. Hence this economy is *less* unequal in human capital (and thus earnings) relative to the baseline economy.[46]

In this section I show that implementing a large-scale permanent informational policy intervention that provides poor parents with information about the productivity of parental investments for children's human capital formation would increase earnings mobility by 4.2% and reduce cross-sectional inequality in human capital by 10.2%. While other policies that aim to increase equality of opportunity such as the early childhood education program studied in García et al. (2020) have been estimated to increase intergenerational mobility as well, they cost about $80,600 per participant (Daruich, 2018).[47] On the other hand, the informational intervention studied in this counterfactual is estimated to cost only $143-$150 per participant.[48] With such low costs, infor-

[46]Recall that earnings are a deterministic function of human capital.

[47]The North Carolina early childhood education experiment is estimated to cost $54,000 in 2000 ($13,500 per child-year for four years). I use the CPI to estimate the cost in 2019 dollars which is approximately $80,600 for comparison to the estimated cost of the informational intervention.

[48]In 2019 USD.

mational invention policies likely have large scope to increase equal opportunity in a desirable way.

5 Concluding Remarks

Childhood environments are critically important for children's long term outcomes. Yet, parents' beliefs about how important parental inputs are for children's human capital development differ widely. This paper is the first to incorporate individual-specific subjective beliefs which, in part, determine parental investment and later-in-life outcomes into a heterogeneous-agent model of intergenerational mobility. Given that parents tend to seek information about parenting from their family, friends, and other parents (e.g. Koepke and Williams (1989)), I focus on social learning among parents. Parents learn about the importance of investment for children's human capital formation through social interactions before making decisions. However, economic segregation in social connections generates systematic distortions in subjective beliefs and parental investments, and thereby amplifies the persistence of earnings across generations. The model is consistent with experimental evidence from a randomized control trial that documents how changing parental subjective beliefs affects subsequent parental investment behavior.

By introducing social learning to a standard macroeconomic model of overlapping generations, I show that subjective beliefs are relevant for studying aggregate inequality and intergenerational mobility. I find that the intergenerational earnings elasticity declines once social learning distortions are removed. Intergenerational earnings mobility increases by 8.7% as a result of eliminating the information frictions generated by social learning. Moreover, I study the implications of a large-scale, low-cost informational intervention similar to that of List et al. (2021). The policy increases low-income, young parents' subjective beliefs about the importance of childhood investments for human capital development. Permanently implementing this policy would increase earnings mobility by 4.2% and reduce cross-sectional inequality in earnings and human capital by 10.2%. Given that the economic connectedness of social interactions varies by geography, studying the effectiveness of mitigating information frictions among parents across different regions is a fruitful area for future work.

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Appendices

A Appendix

Contents

A.1 Illustrative Models

A.1.1 Illustrative Static Model: Full information

Environment There are two periods, $j = \{1, 2\}$, and two generations, parents and children who are heterogeneous in human capital h. In period $j = 1$, individuals are endowed with a child who has human capital h^c and become parents. In period $j = 2$, the child realizes their skill outcome $h^{c\prime}$ and the model ends. Parents have full information about the environment. They make a parental investment x that affects their child's skill next period $h^{c\prime}$ according to,

$$ln(h^{c\prime}) = \lambda ln(h^c) + \theta ln(x) + ln(\epsilon) \tag{18}$$

where $ln(\epsilon) \sim N(0, \sigma_\epsilon^2)$ is a human capital shock. In period 2, the child realizes their skill outcome $h^{c\prime}$ and the model ends.

Parents choose investment x in order to maximize their utility

$$U(h, h^c) = \max_x ln(c) + \beta\alpha\mathbb{E}[ln(h^{c\prime})]$$

subject to the resource constraint,

$$w(h) = c + x$$

the wage equation,

$$w(h) = \kappa h$$

and the technology of skill formation 18.

Analytical Solution Substitute the budget constraint and technology of child skill formation into the parent's problem.

$$\max_x ln(w(h) - x) + \beta\alpha\mathbb{E}[\lambda ln(h^c) + \theta ln(x) + ln(\epsilon)]$$

where the expectation is taken over shocks to human capital, $ln(\epsilon) \sim N(0, \sigma_\epsilon^2)$.

The first order condition is

$$\frac{1}{w(h) - x} = \beta\alpha\theta\frac{1}{x} \tag{19}$$

Thus, optimal investment is

$$x^* = \frac{\beta\alpha\theta}{1 + \beta\alpha\theta}w(h) \tag{20}$$

In this simple model, the only thing that determines children's outcomes is parental earnings $w(h)$ and exogenous shocks. In the next appendix, I extend this model to a setting where parent's do not have full information and learn about the parameter θ.

A.1.2 Illustrative Static Model: Learning

Environment There are two periods, $j = \{1, 2\}$, and two generations, parents and children who are heterogeneous in human capital h. In period $j = 1$, individuals are endowed with a child who has human capital h^c and become parents. In period 2, the child realizes their skill outcome $h^{c\prime}$ and the model ends. I depart from the full information model above and introduce learning over the skill formation technology. In particular, assume that parents do not observe the true value of θ when they make their investment decision x. Instead, they learn about θ from their own experience. Parents have homogeneous priors over $\theta \sim N(\mu_0, \sigma_0^2)$.[49] They use Bayes law and

[49]This assumption is for simplicity, but can be relaxed without changing the solution.

their own experience — past investment endowment x_{j-1}, and realized human capital h — to form posterior beliefs.

Parents choose investment x in order to maximize their utility

$$U(h, h^c) = \max_x \, ln(c) + \beta\alpha\mathbb{E}[ln(h^{c\prime})]$$

subject to the resource constraint,

$$w(h) = c + x$$

the wage equation,

$$w(h) = \kappa h$$

and the technology of children's skill formation according to,

$$ln(h^{c\prime}) = \lambda h^c + \theta ln(x) + ln(\epsilon) \tag{21}$$

where $ln(\epsilon) \sim N(0, \sigma_\epsilon^2)$ is a human capital shock. However parents have subjective beliefs over $\theta \sim N(\mu, \sigma^2)$. Since parents take expectations over their children's future human capital they use their posterior to make the investment decision. Next, I discuss how beliefs are updated.

Bayesian learning In period 1, parents form beliefs about the return to parental investments. Parents run a regression of their human capital $ln(h)$ on their endowed parental investment $ln(x_{j-1})$,

$$ln(w(h)) = \lambda h^c + \hat{\theta} ln(x_{j-1}) + ln(\epsilon) \tag{22}$$

The estimated coefficient $\hat{\theta}$ has expected value $\hat{\mu}$ and variance $\hat{\sigma}^2$ given by

$$\hat{\mu} = \frac{ln(w(h)) ln(x_{j-1})}{(ln(x_{j-1}))^2} \tag{23}$$

$$\hat{\sigma}^2 = (\sigma_0^{-2} + \hat{\sigma}^{-2})^{-1} \tag{24}$$

Note that $\hat{\mu}$ is an unbiased estimator of θ.

Parents use Bayes Law along with their prior, $N(\mu_0, \sigma_0^2)$, to form a posterior belief denoted by μ.

$$\mu = \frac{\hat{\sigma}^2}{\hat{\sigma}^2 + \sigma_0^2}\mu_0 + \frac{\sigma_0^2}{\hat{\sigma}^2 + \sigma_0^2}\hat{\mu} \tag{25}$$

Optimal choice Parents use their posterior belief μ to make their investment choice.

To solve for the optimal choice substitute the budget constraint and child development into the parental problem.

$$\max_x ln(w(h) - x) + \beta\alpha\mathbb{E}[\lambda h^c + \theta ln(x) + ln(\epsilon)] \tag{26}$$

where $ln(\epsilon) \sim N(0, \sigma_\epsilon^2)$. Which can be written as

$$\max_x ln(w(h) - x) + \beta\alpha(\lambda h^c + \mu ln(x))$$

using the linearity of expectations

The first order condition is

$$\frac{1}{w(h) - x} = \beta\alpha\mu\frac{1}{x} \tag{27}$$

Thus, optimal investment is

$$x^* = \frac{\beta\alpha\mu}{1 + \beta\alpha\mu}w(h) \tag{28}$$

Social learning From this we can see that optimal parental behavior varies with parental income $w(h)$ and subjective beliefs μ. To introduce social learning in a simple way, we can think of parents as simply using Bayes law and a social connections' experience — past investment endowment $x_{\tilde{j-1}}$, and realized human capital $\tilde{h}$ — to form posterior beliefs $N(\mu, \sigma^2)$. Social learning generates information frictions because social connections determine the composition of observed signals.

A.1.3 Selection Bias

In this Appendix, I illustrate the presence of selection bias arising from in the static model with information frictions across the distribution of income. This simple example is able to provide a characterization of the selection bias.

Let parental inputs for the first generation be distributed normally $ln(x_{j-1}) \sim N(\mu_x, \sigma_x^2)$. In period 1, parents and draw a human capital shock $ln(\epsilon) \sim N(0, \sigma_\epsilon^2)$. Assume that $ln(x_{j-1})$ and $ln(\epsilon) \sim N(0, \sigma_\epsilon^2)$ are independent. Skills and wages are then determined by the skill formation technology 21 and earnings equation $w(h) = h$ such that

$$ln(w(h^{c\prime})) = \lambda h^c + \theta ln(x_{j-1}) + ln(\epsilon)$$

Consider a sample of K observations of the data $\{W_i, H_i', X_i'\}_{i=1}^K$ from the population where W_i is a $K \times 1$ vector of $\{ln(w(h^{c\prime})/\kappa)\}_{i=1}^K$, H_i' is a $K \times 1$ vector of $\{\lambda h^c\}_{i=1}^K$, and X_i' is a $K \times 1$ vector of $\{ln(x_{i,j-1})\}_{i=1}^K$. Let S_i be an indicator variable which takes a value of 1 if an observation is included in the observed sample. Lastly, suppose the DGP is given by

$$W_i = H_i' + X_i'\theta + V_i \tag{29}$$

where $\mathbb{E}[V_i|X_i] = 0$ and the productivity $\theta > 0$.

The conditional expectation function of the observed sample is

$$\mathbb{E}[W_i'|H_i, X_i, S_i = 1] = H_i' + X_i'\theta + \mathbb{E}[V_i|X_i, , S_i = 1] \tag{30}$$

If the last term is zero then the regression function for the selected subsample is consistent with the population regression. Therefore, OLS can be used to estimate θ without introducing bias.[50] However, in the case of social learning, this term may not be zero.

Next, make two simplifying assumptions

1. Selection is determined by a threshold of the dependent variable: $S_i = \mathbf{1}(W_i > w)$ (w.l.o.g) for some fixed $w \in \mathbb{R}$[51]
2. The errors are normally distributed $V_i \overset{\text{i.i.d.}}{\sim} N(0, \sigma_\epsilon^2)$

Then, we can write the selection term as:

$$\mathbb{E}[V_i|H_i, X_i, S_i = 1] \overset{1}{=} \mathbb{E}[V_i|H_i, X_i', V_i > w - X_i'\theta] \overset{2}{=} \sigma_\nu \cdot \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right) > 0 \tag{31}$$

where $\lambda(u) := \phi(u)/(1 - \Phi(u))$ denotes the inverse Mills' ratio of a normally distributed random variable. Hence the CEF is given by:

$$\mathbb{E}[W_i'|H_i, X_i, S_i = 1] = H_i' + X_i'\theta + \sigma_\epsilon \cdot \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right) \tag{32}$$

Characterizing the Bias The population regression

$$W = H' + X'\theta + V$$

and the conditional expectation function is

$$\mathbb{E}[W'|H, X] = H + X\theta$$

[50]If parents knew about the selection in their distribution of social connections they could use the unbiased two-step Heckman selection correction estimator to correct for this bias (Heckman, 1974, 1979).

[51]The case where $S_i = \mathbf{1}(W_i \leq w)$ is analogous to those given here.

Running a naive OLS on the sample gives us an estimate of

$$\hat{\theta}_{naive} = (X_i' X_i)^{-1} X_i'(W_i') \tag{33}$$

We can characterize the misspecification bias by substituting 32 into the conditional expectation of 33.

$$\begin{aligned}
\hat{\theta}_{naive} &= \mathbb{E}[(X_i X_i')]^{-1}\mathbb{E}[X_i \mathbb{E}[W_i | H_i, X_i, S_i = 1]] \text{ (Law of Iterated Expectations)} \\
&= \mathbb{E}[(X_i X_i')]^{-1}\mathbb{E}\left[X_i \left(H_i' + X_i'\theta + \sigma_\epsilon \cdot \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right)\right)\right] \\
&= \theta + \underbrace{\sigma_\epsilon \cdot \mathbb{E}[(X_i X_i')]^{-1}\mathbb{E}\left[X_i \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right)\right]}_{\text{bias}}
\end{aligned}$$

Since $\sigma_\epsilon > 0$, the sign of the bias depends on $\mathbb{E}\left[X_i \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right)\right]$. Let true productivity be $\theta > 0$. In general, the sign of the linear combination $\mathbb{E}\left[X_i \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right)\right]$ cannot be determined with K arbitrary. However, if agent i observes a sample of all data above the median, then $\mathbb{E}\left[X_i \lambda\left(\frac{w - X_i'\theta}{\sigma_\epsilon}\right)\right] > 0$.

A.2 Discussion of optimal investment over the life-cycle

Let $g(q, x_3, x_4)$ denote the technology of skill formation for the purposes of this section. Then first order conditions for investments are:

$$u'(c_3) = \beta[u'(c_4) + \beta\alpha\mathbb{E}g_2(h, x_3, x_4)]$$

$$u'(c_4) = \beta[\alpha\mathbb{E}g_3(h, x_3, x_4)]$$

Taking the ratio of these equations gives,

$$\underbrace{\frac{u'(c_3)}{\beta u'(c_4)}}_{\substack{\text{MRS for} \\ \text{consumption}}} = \underbrace{\frac{u'(c_4)}{\beta\alpha\mathbb{E}g_3(h, x_3, x_4)}}_{\substack{\text{MRS for pd. 4} \\ \text{consumption and investment} \\ \text{in human capital}}} + \underbrace{\frac{\mathbb{E}g_2(h, x_3, x_4)}{\mathbb{E}g_3(h, x_3, x_4)}}_{\substack{\text{Tech. rate of} \\ \text{substitution in production} \\ \text{of human capital}}}$$

In the absence of borrowing constraints, optimal investments maximize the discounted value of earnings net of the discounted costs of investment. As noted in Caucutt and Lochner (2020), this is the sense in which the timing of income is irrelevant for investment without the presence of borrowing constraints. The complementarity of investments across periods is central to individual responses to borrowing constraints.

A.3 Equilibrium: Solution Algorithm

I use the following algorithm to solve for a recursive competitive equilibrium of the quantitative model in Section 1.

1. Using the discretized state space for individuals, solve the model backwards starting with the terminal period $j = 4$. The grid is set using Tauchen's method where the maximum value of $ln(h)$ is given by
$$z_N = m\left(\frac{\sigma_\varepsilon^2}{1-\rho_c\lambda^2}\right)^{\frac{1}{2}}$$
where I choose multiple m to be 5, N is 20, and the minimum value is $z_1 = -z_N$.[52]

2. For each part of the state space and in each period of learning, draw social connections and estimate beliefs $(\hat{\mu}_{\theta;i,j}, \hat{\sigma}_{\theta;i,j})$, then average beliefs over 500 draws.

3. Average the beliefs to recover mean beliefs across the state space and the optimal policy functions.

4. Iterate on the (discretized) initial distribution of human capital and prior beliefs until convergence.

A.4 Estimation Details

A.4.1 Human capital and earnings

The Panel Study of Income Dynamics (PSID) is a longitudinal study of a representative sample of U.S. individuals and their families and the Child Deveopment Supplement (CDS) provides further information about a subsample of children and their caregivers. The CDS collects information on the cognitive and socio-emotional development of children as well as information on parents, including their parenting practices, employment, and income. This allows researchers to study the impact of parental characteristics on child development outcomes. The most widely used measures of children's cognitive skills are the Letter-Word (LW) and Applied Problem Solving (AP). I measure children's human capital using the LW scores adjusted to be comparable across children's age as in Lee and Seshadri (2019).

[52] See A.4.5 for details on implementing Tauchen (1986).

Children's initial human capital The distribution of age-adjusted LW scores in early childhood are used to discipline children's initial human capital which is an AR(1) process with correlation ρ_c across generations. Given the age-structure of the model early childhood corresponds to children aged 11 and under, and late childhood corresponds to children aged 12 and above.

Children's human capital production function The Baby's First Years (BFY) study collects data from new mothers on an annual basis about parental investments and spending, parental behavior, and child development. The study is a large-scale multi-site longitudinal unconditional cash transfer randomized controlled trial in the U.S. and has been ongoing over the past four years. Participants were recruited from hospitals in four U.S. metro areas: New York City, New Orleans, Omaha, and the Twin Cities (Minneapolis and St. Paul). The sample includes racially and ethnically diverse low-income mothers and children.

I measure parental investments in the BFY using a measure of maternal time spent in child-enriching activities. These activities include frequency of time spent reading, telling stories, playing and building things, and participating in playgroups. I translate these categorical responses to a continuous measure of time in minutes per week spent investing in these activities with estimates of maternal time use (by mother's education) from Kalil et al. (2012) which draws on the American Time Use Survey, as in Gennetian et al. (2022). For categorical responses of "rarely," assign 0 minutes; for a response of "a few times a month," assign the value for one weekend day; for a response of "weekly," assign a value for three weekend days; for a response of "daily," assign seven days a week by mother's education (as measured in the baseline interview). Education levels coincide with those reported in Kalil et al. (2012) and include less than high school, high school, some college, and college or more.[53]

BFY is an unconditional cash transfer randomized controlled trial experiment which allows for clean identification of the productivity of parental investments. Mothers in the treatment group receive monthly gifts of \$333 (\$3,996/year) — a high cash transfer, while mothers in the control receive a \$20 monthly gift (\$220/year) — a low cash transfer. Gennetian et al. (2022) document the initial findings of the RCT for parental investment behavior. I leverage the exogeneous variation from this experimental design to estimate the productivity of parental investments, or the elasticity of investment to children's human capital, as measured by the Ages and Stages Questionnaire (ASQ).[54] In particular, I use an instrumental variables approach to estimate the following specification,

[53] Gennetian et al. (2022) show that this continuous measure of maternal time investment is qualitatively similar to using the Likert scale index of maternal time in child-enriching activities.

[54] The ASQ is a developmental assessment of cognitive skills for children between birth and six years of age.

$$ln(h^c_{i,s+1}) = \alpha_0 + \lambda_0 ln(h^c_{i,s}) + \theta\widehat{t_{i,s}} + \mathit{\Gamma}_0\chi_{i,s} + \epsilon_{i,s+1}, \tag{34}$$

$$t_{i,s} = \alpha_1 + \lambda_1 ln(h^c_{i,s}) + \delta_1 Z_{i,s} + \mathit{\Gamma}_1\chi_{i,s} + u_{i,s}, \tag{35}$$

where $\widehat{t_{i,s}}$ in the second stage regression (equation 34) is the predicted value of the log of parental time investment from the first stage regression (equation 35) and $\chi_{i,s}$ is a vector of controls.[55] For the instrument to be valid, it must be relevant ($cov(Z_{i,s}, \widehat{t_{i,s}}|\chi_{i,s}) \neq 0$) and conditionally exogenous ($cov(Z_{i,s}, \epsilon_{i,s+1}|\chi_{i,s}) = 0$).

The results are presented in Table 6. Column (2) displays the second stage results of estimating equation 34. And column (3) adds an additional control, the number of gestational weeks, as a proxy for children's initial health status as robustness.

Interpretting the IV results While I had hoped to use the transaction level debit card data on child-specific expenditures to measure monetary investments in children, the most common transaction among the treatment group is withdrawal of cash from an ATM, accounting for more than 30% of the cash transfer on average (Gennetian et al., 2022). In what follows I show that the 2SLS estimator used for calibration scales inversely with parental time investments which are potentially endogenous and argue that is likely that the scale is close to one in this context.

First, I show in general that the 2SLS estimator made under the assumption that parental investments in children's human capital are made in proportion to time investments scales (inversely) with the endogenous regressor.

Setup Suppose we have a dependent variable Y, a potentially endogenous variable T, and an instrument Z. The two stage least squares specification for individual i in survey wave s is

$$Y_{i,s+1} = T_{i,s}\beta_{true} + u_{i,s+1} \tag{36}$$

$$T_{i,s} = Z_{i,s}\beta_1 + \epsilon_{i,s} \tag{37}$$

where $\hat{T}_{i,s}$ in the second stage regression 36 is the predicted value from the first stage regression 37.

The two stage least squares estimator of β_{true} is given by

$$\hat{\beta}_{true} = (\hat{T}'_{i,s}\hat{T}_{i,s})^{-1}(\hat{T}'_{i,s}Y_{i,s+1})$$

[55]Covariates include: Mother's age, Household income bins, Net worth bins, Number of adults in the household, and Experimental site dummies.

Table 6: IV estimation of parental investment elasticity

	———Dependent variable: child ASQ ———	
	(1)	(2)
	IV	IV add health proxy
Parental time investment	0.849	0.900
	(0.610)	(0.628)
Maternal age	0.00138	0.00221
	(0.00831)	(0.00866)
HH income cat. 2	0.103	0.0880
	(0.153)	(0.153)
HH income cat. 3	-0.0592	-0.0570
	(0.150)	(0.154)
HH income cat. 4	-0.194	-0.209
	(0.162)	(0.168)
HH income cat. 5	-0.192	-0.193
	(0.153)	(0.157)
HH income cat. 6	-0.110	-0.114
	(0.195)	(0.202)
HH networth cat. 2	-0.0603	-0.0592
	(0.157)	(0.164)
HH networth cat. 3	0.0708	0.0930
	(0.132)	(0.138)
HH networth cat. 4	-0.181	-0.175
	(0.209)	(0.207)
HH networth cat. 5	0.0276	0.0282
	(0.177)	(0.181)
HH networth cat. 6	0.287	0.323
	(0.280)	(0.294)
No. adults	0.0424	0.0478
	(0.0702)	(0.0708)
Site 2	-0.00125	0.00688
	(0.146)	(0.162)
Site 3	-0.358***	-0.365***
	(0.129)	(0.130)
Site 4	-0.213	-0.217*
	(0.133)	(0.130)
Gestational wks		-0.000752
		(0.0487)
Constant	-3.437	-3.638
	(2.752)	(2.243)
Observations	820	816

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Applied to the problem in this paper, $T_{i,s}$ total parental investments in individual i during survey wave s that impact children's human capital. Now suppose that instead of observing T, we only observe some variable $\mathcal{T}$. Assume $\mathcal{T} \propto T$. Here $\mathcal{T}$ represents parental time investments which are observed, but may only be a share of total parental investment.

The new problem is given by

$$Y_{i,s+1} = \mathcal{T}_{i,s}\beta_{prop} + u_{i,s+1} \tag{38}$$

$$\mathcal{T}_{i,s} = Z_{i,s}\beta_2 + \epsilon_{i,s} \tag{39}$$

where $\mathcal{T}_{i,s} \propto T_{i,s}$

Proof Show that the 2SLS estimator for this problem (equations 38 and 39) scales (inversely) with the endogenous regressor.

Use the property that $\mathcal{T} \propto T \implies \mathcal{T} = kT$ for some constant k. Then the first stage equation 39 can be written as,

$$\begin{aligned}
\mathcal{T}_{i,s} &= kT_{i,s} \\
&= k(Z_{i,s}\beta_1 + \epsilon_{i,s}) \\
&= kZ_{i,s}\beta_1 + k\epsilon_{i,s}
\end{aligned}$$

And,

$$\hat{\mathcal{T}}_{i,s} = Z_{i,s}(k\hat{\beta}_1) \tag{40}$$

$$= k\hat{T}_{i,s} \tag{41}$$

since $\hat{T}_{i,s} = Z_{i,s}\hat{\beta}_1$.

From the second stage equation 38 the 2SLS estimate is

$$\hat{\beta}_{prop} = (\hat{\mathcal{T}}'_{i,s}\hat{\mathcal{T}}_{i,s})^{-1}(\hat{\mathcal{T}}'_{i,s}Y_{i,s+1}) \tag{42}$$

Substituting in 40 we have,

$$\hat{\beta}_{prop} = (k\hat{T}'_{i,s}k\hat{T}_{i,s})^{-1}(k\hat{T}'_{i,s}Y_{i,s+1}) \tag{43}$$

$$= k^{-1}\hat{\beta}_{true} \tag{44}$$

Thus the 2SLS estimator scales inversely with the endogenous regressor. If the proportion k is known then it's possible to back out $\hat{\beta}_{true} = k\hat{\beta}_{prop}$. When k is unknown, a $k < 1$ implies an upward bias of the estimator whereas $k > 1$ implies a downward bias of the estimator.

Next, I argue that it may be reasonable to assume that the scale k is close to one in this context conditional on the set of controls. In the estimation of the productivity of parental investments on children's human capital, I use data from the Baby's First Years Study. The sample of children in this data are four years old and under. This restriction on children's age is helpful for the reason that it is unlikely that other types of parental investments a large impact on the human capital development of children during this stage in the lifecycle. For example, factors like schooling or neighborhood choice are not as relevant for young children before age five. While household resources are certainly important in the formation of children's human capital, I do my best to control for these by including categories of household income, net worth, and number of adults in the household as covariates. In estimating the 2SLS specification, I am most concerned about the role of children's health as a violation of the conditional exogeneity assumption. I control for factors that might influence children's including again household income categories and networth categories as well as mother's age and experimental site dummies for each metro area which may help to pick up environmental factors that are known to impact children's health.

Overall, the largest proportion of parental investments made in children four and under are by far time investments and controlling for a number of maternal and household characteristics gives some confidence that the 2SLS estimator has a relatively small bias. Assuming that time investments are a fraction of total parental investments close to one implies that the estimator is biased upward.

In the case where children's human capital as measured by the ASQ suffers from measurement error, as is commonly assumed in the empirical literature on human capital formation, attenuation bias would bias this estimator downward. This is also somewhat comforting in that the potential biases are moving in opposite directions.

A.4.2 Credit constraints

The Survey of Consumer Finances (SCF) is a nationally representative survey that provides information about income, net worth, and borrowing of U.S. households. I use the 2001 and 2004 waves of the SCF to estimate the relationship between income and credit limits with the following specification,

$$\underline{b}_i' = \psi + \delta w(h_i) + e_i$$

The coefficient δ captures how credit limits evolve across the income distribution. The first column of Table 7 shows that the for each additional dollar of income, individuals' credit limits increased

— Dependent variable: credit limit —	(1)	(2))
Income	0.212***	0.180***
	(0.00849)	(0.0102)
Constant	230.4	5,812***
	(458.3)	(725.9)
Observations	3,064	2,153
R-squared	0.250	0.169
Controls	N	N
Zeros	Y	N
Age	22-43	22-43

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 7: Estimating the borrowing limit

by 21 cents on average. As a robustness check, column (2) shows the result of the same specification excluding individuals with zero income and illustrates a similar relationship.

A.4.3 Social connections

Chetty et al. (2022) derive the distribution of social connections among adults in the U.S. using data from Facebook. Extended Data Table 1 of Chetty et al. (2022) documents the probability of having a social connection across the distribution of income by income decile. I use these estimates directly to calibrate the distribution of social connections across income deciles summarized by p_i, the probability of having an above-median income social connection.

A.4.4 Parental Beliefs

The Newborn Study employs the Survey of Parents Expectations And Knowledge (SPEAK) to elicit parental beliefs about how parents investments impact child development (List et al., 2021). The SPEAK survey was developed by Suskind et al. (2018) and consists of a series of 57 questions covering topics about the development of children such as media use, reading, responsiveness, and languages spoken at home. Parents SPEAK score is the sum of correct answers with higher scores indicating a better understanding of the impact of parental investments for child development. I standardize scores to have a standard deviation of one and work with this measure of parental beliefs to inform the differences in parental beliefs between parents by income. To be sure, the values of the SPEAK score are not informative about parental beliefs in the model, but the

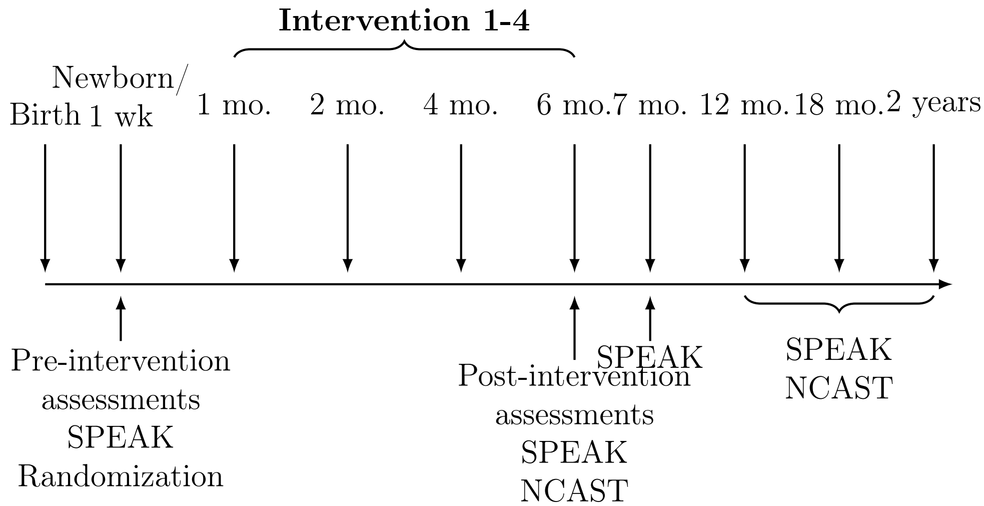

Figure 4: Experimental timeline

dispersion of scores is informative about the beliefs held by heterogeneous parents. I use the SPEAK scores measured at baseline (Newborn/1 week) for estimating the ratio of average parental beliefs.

Figure 4 depicts the timeline of the Newborn Study. The informational intervention was implemented at the child's 1, 2, 4, and 6 month old well-child visits. The measure of parental investments rates presented in 3 uses the NCAST collected at 6 mo., 12 mo., and 2 years.

A.4.5 Discretization

I use Tauchen (1986)'s method to approximate an AR(1) process for human capital (and earnings). This approximation allows us to discretize the Markov chain of transition probabilities in the model using the fact that conditional on some state h, h^c is a normally distributed. I determine the points of the human capital grid in this framework in what follows. The law of motion for human capital is

$$ln(q) = \rho_c \lambda^2 ln(q^p) + \lambda^2 ln(\nu_0) + \theta\lambda ln(x_3^p) + \theta ln(x_4^p) + \lambda ln(\eta_1) + ln(\eta_2)$$

where $ln(\nu_0) \sim N(\mu_{ln(\nu_0)}, \sigma^2_{ln(\nu_0)})$, $ln(\eta_1) \sim N(0, \sigma^2_{ln(\eta)})$, and $ln(\eta_2) \sim N(0, \sigma^2_{ln(\eta)})$ are shocks to human capital.

Let $\tilde{q}$ be a discrete valued process that approximates $ln(q)$, and let $\{q_1, q_2, ..., q_N\}$ be the finite set of possible realizations of $\tilde{q}$. The grid points q_i are evenly distributed across the interval $[q_1, q_N]$ with distance d between grid points.

The transition probability of going from state j yesterday to state k today is given by:

$$
\begin{aligned}
\pi_{jk} &= P(\tilde{q}_t = q_k | \tilde{q}_{t-1} = q_j) \\
&= P(q_k - \frac{d}{2} < \rho_c \lambda^2 q_j + \lambda^2 ln(\nu_0) + \theta \lambda ln(x_3^p) + \theta ln(x_4^p) + \lambda ln(\eta_1) + ln(\eta_2) < q_k + \frac{d}{2}) \\
&= P(q_k - [\rho_c \lambda^2 q_j + \lambda^2 \mu_{ln(\nu_0)} + \theta \lambda ln(x_3^p) + \theta ln(x_4^p)] - \frac{d}{2} \\
&< \lambda^2 (ln(\nu_0) - \mu_{ln(\nu_0)}) + \lambda ln(\eta_1) + ln(\eta_2) \\
&< q_k - [\rho_c \lambda^2 q_j + \lambda^2 \mu_{ln(\nu_0)} + \theta \lambda ln(x_3^p) + \theta ln(x_4^p)] + \frac{d}{2})
\end{aligned}
$$

For an interior point on the grid, the probability is given by

$$
\begin{aligned}
\pi_{jk} = F(&\frac{q_k - [\rho_c \lambda^2 q_j + \lambda^2 \mu_{ln(\nu_0)} + \theta \lambda ln(x_3^p) + \theta ln(x_4^p)] + \frac{d}{2}}{\sigma_\varepsilon^2}) \\
- F(&\frac{q_k - [\rho_c \lambda^2 q_j + \lambda^2 \mu_{ln(\nu_0)} + \theta \lambda ln(x_3^p) + \theta ln(x_4^p)] - \frac{d}{2}}{\sigma_\varepsilon^2})
\end{aligned}
$$

where $\sigma_\varepsilon^2 = \lambda^4 \sigma_{ln(\nu_0)}^2 + \lambda^2 \sigma_{ln(\eta)}^2 + \sigma_{ln(\eta)}^2$ is the combined variance and $F()$ is the standard normal distribution.

For the end points on the grid, the probabilities are

$$
\pi_{j1} = F(\frac{q_1 - [\rho_c \lambda^2 q_j + \lambda^2 \mu_{ln(\nu_0)} + \theta \lambda ln(x_3^p) + \theta ln(x_4^p)] + \frac{d}{2}}{\sigma_\varepsilon^2})
$$

$$
\pi_{jN} = 1 - F(\frac{q_N - [\rho_c \lambda^2 q_j + \lambda^2 \mu_{ln(\nu_0)} + \theta \lambda ln(x_3^p) + \theta ln(x_4^p)] - \frac{d}{2}}{\sigma_\varepsilon^2})
$$

A.4.6 Simulated Method of Moments: Loss Function

Let $\Theta = [\rho_c, \mu_{ln(\nu)}, \sigma^2_{ln(\nu)}, \sigma^2_{ln(\nu)}, \kappa_1, \kappa_2, K, \lambda, \psi, \alpha]$ denote the vector of internally calibrated parameters and M_T denote the set of empirical targets in Table 1. I calibrate the model by minimizing the distance between model moments and the data targets. In particular, I minimize the following loss function.

$$
\hat{\Theta} = \underset{\Theta}{argmin}[M_T - M(\Theta)]' W [M_T - M(\Theta)]
$$

where $M(\Theta)$ is a vector of model simulated moments and the weighting matrix W is the identity matrix.