Structural Estimation of a Model of School Choices: The Boston Mechanism versus Its Alternatives

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We model household choice of schools under the Boston mechanism (BM) and develop a new method, applicable to a broad class of mechanisms, to fully solve the choice problem even if it is infeasible via the traditional method. We estimate the joint distribution of household preferences and sophistication types, using administrative data from Barcelona. Counterfactual policy analyses show that a change from BM in Barcelona to the deferred-acceptance mechanism would decrease average welfare by €1,020, while a change to the top-trading-cycles mechanism would increase average welfare by €460.

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I. Introduction

Designed to broaden households' access to schools beyond their neighborhoods, public school choice systems have been increasingly adopted in many countries.[1] The quality of schools to which students are assigned can have significant long-term effects for individual families as well as important implications on efficiency and equity for a society.[2] How to assign students to schools is of key interest among policy makers and researchers.

One important debate centers around the popular Boston mechanism (BM), which is vulnerable to manipulation (Abdulkadiroğlu and Sönmez 2003). Some cities, including Boston, have replaced BM with less manipulable mechanisms, such as the student-proposing deferred-acceptance mechanism (DA; Gale and Shapley 1962).[3] However, the efficiency and equity comparison between BM and its alternatives remains an open question.

To answer this question, one needs to quantify two essential but unobservable factors underlying households' choices, which is what we do in this paper. The first factor is household preferences, indispensable for comparing welfare across mechanisms even if household choices are observed under each. Moreover, as choices are often not observed under counterfactual scenarios, one needs to predict how households would behave. The knowledge of household preferences alone is not enough for this purpose. Although BM gives incentives for households to act strategically, there may exist nonstrategic households that simply rank schools according to their true preferences.[4] A switch from BM to DA, for example, will induce behavioral changes only among strategic households. The knowledge about the distribution of household types (strategic or nonstrategic) thus becomes a second essential factor.

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[1] Some papers explore changes in families' school choice sets to study how school choice affect students' achievement, e.g., Abdulkadiroğlu, Pathak, and Roth (2009), Hastings, Kane, and Staiger (2009), Lavy (2010), Walters (2013), Deming et al. (2014), and Mehta (2017). Other studies focus on how the competition induced by school choices affects school performance, e.g., Hoxby (2003) and Rothstein (2006).

[2] See Heckman and Mosso (2014) for a comprehensive review of the literature on human development and social mobility.

[3] See Abdulkadiroğlu et al. (2005) for the Boston reform and Pathak and Sönmez (2013) for switches in other cities to less manipulable mechanisms.

[4] There is direct evidence that both strategic and nonstrategic households exist. For example, Abdulkadiroğlu et al. (2006) show that some households in Boston obviously failed to strategize. Calsamiglia and Güell (2018) prove that some households obviously behave strategically.

We develop a model of school choices under BM by households who differ in both their preferences for schools and their strategic types. Nonstrategic households fill out application forms according to their true preferences. Strategic households take admission risks into account to maximize their expected payoffs. A household's expected payoff depends on how it selects and ranks schools on its application list. The standard way to solve this problem selects the best permutation out of the set of schools. This method is applicable only when the choice set is small, because the dimensionality grows exponentially with the number of schools. We utilize two unexploited properties of most allocation mechanisms and show that the full optimization problem can be effectively solved via backward induction even when the household faces a large choice set. This solution method is applicable to a wide range of mechanisms, covering most mechanisms studied in the literature.

We apply our model to a rich administrative data set from Barcelona, where a version of the BM system has been used to allocate students across over 300 public schools. Between 2006 and 2007, there was a drastic change in the official definition of school zones that significantly altered the set of schools a family had priorities for. We estimate our model with the 2006 prereform data via simulated maximum likelihood, and we conduct an out-of-sample validation with the 2007 postreform data. The model matches the data in both years.

In counterfactual policy experiments, we assess the performance of two truth-revealing alternatives to BM: DA and the top-trading-cycles mechanism (TTC).[5] An average household in Barcelona would lose by an amount equivalent to €1,020 in a BM-to-DA change and benefit by €460 in a BM-to-TTC change. There would be both winners and losers in either case, leading to a wide dispersion of welfare changes. The cross-household standard deviation of welfare changes is €7,180 in the former case and €9,630 in the latter. A BM-to-DA change is more likely to benefit those who live in higher-school-quality zones, hence enlarging the cross-zone inequality. In a BM-to-TTC change, the quality of the school zone a household lives in does not affect its chance to win or to lose. While TTC enables 59% of households whose favorite schools are out of their zones to attend such schools, this fraction is only 47% under BM and 42% under DA.

We contribute to the literature on the design of centralized choice systems initiated by Balinski and Sönmez (1999) for college admissions and Abdulkadiroğlu and Sönmez (2003) for public school choices, the latter leading to debates on BM. Some suggest that BM creates an equity problem, as nonstrategic parents may be disadvantaged by strategic ones (e.g., Pathak and Sönmez 2008). Using Boston data under BM, Abdulkadiroğlu

[5] TTC was introduced by Shapley and Scarf (1974) and adapted by Abdulkadiroğlu and Sönmez (2003).

et al. (2006) find that households that obviously failed to strategize were disproportionally unassigned. Calsamiglia and Miralles (2014) show that under certain conditions, the only equilibrium under BM is the one in which families apply for and are assigned to in-zone schools. Chen and Sönmez (2006) and Ergin and Sönmez (2006) show that DA is more efficient than BM in complete-information environments. Miralles (2008), Abdulkadiroğlu et al. (2011), and Featherstone and Niederle (2016) provide examples where BM is more efficient than DA.[6]

Empirical studies that quantify the differences between alternative mechanisms have been sparse.[7] He (2014) estimates an equilibrium model under BM. Under certain assumptions, he estimates household preferences without specifying the distribution of household sophistication types. This approach imposes fewer presumptions on the data but restricts the model's ability to compare across mechanisms. Developed independently at roughly the same time as our paper are those of Hwang (2015) and Agarwal and Somaini (2018).[8] Hwang (2015) set-identifies household preferences, assuming certain simple rules on behavior. Assuming that all households are strategic, Agarwal and Somaini (2018) interpret a household's application as a choice of a probability distribution over assignments, which corresponds to choosing the best permutation of schools.[9] As in our paper, they exploit the observed assignment outcomes and estimate household preferences without having to solve for the equilibrium. They show that a class of mechanisms can be consistently estimated and establish conditions under which preferences are nonparametrically identified. In the application, they estimate a parametric model using data from Cambridge, where each household can rank up to three programs out of 13.

Our paper complements well the three papers mentioned above. We develop a solution method applicable to a wide range of choice mechanisms, which efficiently solves household problems that are unmanageable via the standard method. This new solution method can significantly expand the scope of empirical research on choice mechanisms. We show evidence suggesting the coexistence of strategic and nonstrategic households and estimate both household preferences and the distribution of strategic types in a parametric model. The rich variations in our data allow us to form a more comprehensive view of the alternative mechanisms in terms of not only the overall household welfare but also cross-neighborhood inequality. Moreover, we are able to validate our model with data after a sharp reform, which we view as a positive message for empirical research

[6] See Troyan (2012), Akyol (2014), and Lu (2014) for further discussions on this topic.

[7] With a different focus, Abdulkadiroğlu, Agarwal, and Pathak (2017) show the benefits of centralizing school choice procedures.

[8] De Haan et al. (2015) compare DA with BM, using choice data and survey data on preferences.

[9] In an extension, they allow for the existence of both strategic and nonstrategic households.

that compares different mechanisms via structural models and counterfactual analyses.

Researchers have used out-of-sample fits for model validation, exploiting random social experiments (Wise 1985; Lise, Seitz, and Smith 2004; Todd and Wolpin 2006), lab experiments (Bajari and Hortaçsu 2005), or regime shifts (McFadden and Talvitie 1977; Pathak and Shi 2014).[10] Some studies, including our paper, deliberately hold out data for validation purposes, for example, those of Lumsdaine, Stock, and Wise (1992), Keane and Moffitt (1998), and Keane and Wolpin (2007).

The next section describes the background. Section III describes the model. Section IV explains our estimation and identification strategy. Section V describes the data. Section VI presents the estimation results. Section VII conducts counterfactual experiments, followed by the conclusion. The appendix contains further details and additional tables.

II. Background

A. *The Public School System in Barcelona*

The public school system consists of over 300 public or semipublic schools. Public schools are fully financed by the government and free to attend. The operation of public schools follows government rules. All public schools are largely homogenous in teacher assignment, infrastructure, curricula, and funding per pupil. Semipublic schools are run privately, have more autonomy, and are allowed to charge service fees. On average, of the total funding for semipublic schools, 63% is from the government, 34% from service fees, and 3% from private sources. All public and semipublic schools are subject to the same national limit on class size and have to unconditionally accept—and can accept only—students assigned to them via the centralized procedure. Outside of the system, there are private schools, accounting for only 4% of all schools in Barcelona. Private schools receive no public funding, are subject to few restrictions, and do not participate in the centralized school choice program.[11]

B. *School Choice within the Public School System*

Families get into the public school system via a centralized procedure, in which almost all families participate.[12] Every April, participating families with a child who turns 3 in that calendar year submit a ranked list of up

[10] See Keane, Todd, and Wolpin (2011) for a comprehensive review.

[11] For this reason, information on private schools is very limited. Given the lack of information and the small fraction of schools they account for, we treat private schools as part of the (exogenous) outside option.

[12] In 2007, over 95% of families with a 3-year-old child in Barcelona participated.

to 10 schools.[13] Assignment is via BM. The result is made public between April and May, and enrollment happens in September. If a school is overdemanded, applicants are prioritized according to government rules. Applicants can get priority points for the presence of a sibling in the same school (40 points), in-zone schools (30 points), and some family/child characteristics (e.g., disability [10 points]). Ties in total priority scores are broken through a fair lottery.

Transferring to a different school within the public system is feasible only if the receiving school has a free seat, which is nearly impossible for popular schools. In preschool-to-primary-school transitions, a student has the priority to stay in the same school she enrolled in for preschool. Moreover, students are given priorities to specific secondary schools based on their primary schools.

C. The 2007 Redefinition of Zones

Before 2007, Barcelona was divided into fixed zones. Families had 30 priority points for every in-zone school and 0 for out-of-zone schools, regardless of distance.[14] In 2007, a family's school zone was redefined as the smallest area around its residence that covered the closest three public and three semipublic schools.[15] The reform was announced abruptly on March 27, 2007; families were informed via mail by March 30 and had to submit their lists by April 20.

III. Model

A. Primitives

There are J (public, semipublic) schools distributed across various zones in the city. There is a continuum of households of measure 1 (we use the words "household," "applicant," "student," and "parent" interchangeably). Each household submits an ordered list of schools. Then a centralized procedure assigns students according to their applications, school capacity, and a priority structure.[16] One can choose either the school one is assigned to or the outside option.

[13] Applications after the deadline can be considered only after all on-time applicants have been assigned.

[14] Before 2007, a family had priorities for a set of public schools defined by its public school zone and a set of semipublic schools defined by its semipublic school zone. Throughout the paper, "in-zone" schools refer to the union of these two sets.

[15] There were over 5,300 zones under this new definition (Calsamiglia and Güell 2018).

[16] Since almost all families participate in the application procedure in reality, we assume that the cost of application is zero and that all families participate. This is in contrast with the case of the costly college application, e.g., Fu (2014).

Each school j has a location l_j, a vector w_j of observable characteristics, and a characteristic ζ_j observable to households but not the researcher.[17] No school can accommodate all students, but each student is guaranteed a seat in the system.

Household i has characteristics x_i, a location l_i, idiosyncratic tastes for schools $\epsilon_i = \{\epsilon_{ij}\}_j$, and a type $T \in \{0, 1\}$ (nonstrategic or strategic). Households know their tastes and types, which are unobservable to the researcher. The vector ϵ_i is independent of (x_i, l_i), and ϵ's are mutually independent and identically distributed (i.i.d.) following $F_\epsilon(\epsilon)$.[18] The fraction of strategic households varies with household characteristics and locations, $\lambda(x_i, l_i)$. Conditional on observables, the two types differ only in their behaviors, specified below.

Remark 1. We do not take a stand on why some households are (non)strategic. This would be critical if a policy change could affect the fraction of strategic households. It is less concerning for us because we aim at investigating the impact of replacing BM with some truth-revealing mechanisms, under which all households will rank schools according to their true preferences. Once we recover household preferences and the (current) distribution of types, we can compare the current regime with truth-revealing alternatives without the need to know how household types are determined.

We normalize the ex ante value of the outside option to 0, so that a household's evaluation of each school is relative to its outside option. Let d_{ij} be the distance between household i and school j, and let $d_i = \{d_{ij}\}_j$. Household i's utility from attending school j is given by[19]

$$u_{ij} = U\left(w_j, x_i, d_{ij}, \zeta_j\right) + \epsilon_{ij}.$$

Between application and enrollment (about 6 months), the value of the outside option is subject to a shock $\eta_i \sim$ i.i.d. $N(0, \sigma_\eta^2)$, for example, a wage shock that changes one's ability to pay for the private school. A household knows the distribution of η_i before application and observes η_i afterward. With η_i, applying for schools in the public system provides an option value for households. These shocks also rationalize the data fact that some households opted out despite being assigned to their first choices.

[17] We assume that households have full information about schools. Our data do not allow us to separate preferences from information frictions. Examples of using experiments to study how information affects schooling choices include papers by Hastings and Weinstein (2008) and Jensen (2010).

[18] Each component of ϵ_i follows $N(0, \sigma_\epsilon^2)$.

[19] A likelihood ratio test does not favor a more complex model with zone characteristics added to the utility function. Following the literature on choice mechanisms, we abstract from peer effects and social interactions (see Blume et. al 2011 and Epple and Romano 2011 for reviews). The major complication is the multiple-equilibria problem arising from peer effects and social interactions, even under DA or TTC.

B. *Priority and Assignment*

1. Priority Scores

Let z_l be the zone that contains location l, $I(l_i \in z_{l_j})$ indicate whether i lives in school j's zone, and $\text{sib}_{ij} \in \{0, 1\}$ indicate whether i has a sibling enrolled in j.[20] Household i's priority score for school j (s_{ij}) is given by

$$s_{ij} = x_i^0 a + b_1 I\left(l_i \in z_{l_j}\right) + b_2 \text{sib}_{ij}, \tag{1}$$

where a is the vector of points based on demographics. Equation (1) implies that multiple households may tie in their priority scores. If a school is overdemanded, then households are ranked first by their scores, and tied households are ranked by random lottery numbers drawn after applications are submitted. As a special feature in Barcelona and the rest of Spain, a student's priority score of her first choice carries over for all schools on her application.[21] We take this feature into account in our analyses. Ex ante, it is not clear how much the contrast between BM and other mechanisms would depend on the presence of this special priority feature. However, it is reassuring, as we show below, that our findings are qualitatively in line with studies using data from other countries with standard priority structures.[22]

2. The BM Procedure

Schools are gradually filled up over $R < J$ rounds, where R is the maximum length of an application list.

Round 1.—For each school, consider only the students who have listed it as their first choice and assign seats to them one at a time, following their priority scores from high to low (with random numbers as tiebreakers) until there is either no seat left or no student left who has listed it as her first choice.

Round $r \in \{2, 3, \ldots, R\}$.—Only the rth choices of the students not previously assigned are considered. For each unfilled-up school, assign the remaining seats to these students one at a time, following their priority scores and tiebreaking lottery numbers until there is either no seat left or no student left who has listed it as her rth choice.

The procedure terminates after any step $r \leq R$ when every student is assigned or if the only students who remain unassigned listed no more

[20] Characteristics x_i consist of demographics x_i^0 and the vector $\{\text{sib}_{ij}\}_{j=0}^{J}$.

[21] For example, if a student lists an in-zone sibling school as her first choice, she carries $x_i^0 a + b_1 + b_2$ for all the other schools she listed.

[22] Following a referee's suggestion, we have also simulated an equilibrium under a standard BM and found similar results. Details are in the online appendix.

than r choices. A student who remains unassigned after the procedure ends can propose a leftover school and be assigned to it.

3. Admission Probabilities

The probability of admission to each school j can be characterized by a triplet $(\bar{r}_j, \bar{s}_j, \mathrm{cut}_j)$, where $\bar{r}_j$ is the round at which j is filled up ($\bar{r}_j > R$ if j is a leftover school), $\bar{s}_j$ is the priority score for which lottery numbers are used to break ties for j's slots, and cut_j is the cutoff of the lottery number for admission to j. School j will admit any rth-round applicant before $\bar{r}_j$, any $\bar{r}_j$th-round applicant with $s_{ij} > \bar{s}_j$, and any $\bar{r}_j$th-round applicant with score $\bar{s}_j$ and random lottery number higher than cut_j; and it will reject any other applicant. Once the random lottery numbers are drawn, admissions are fully determined. When making its application decision, a household knows $S_i \equiv \{s_{ij}\}_j$ but not its random number, which makes admissions uncertain in many school-round cases. The assignment procedure implies that the admission probability is (weakly) decreasing in s_{ij} in each round and is (weakly) decreasing over rounds for all scores. In particular, the admission probability to a school in round $r + 1$ for the highest priority score is (weakly) lower than that for the lowest score in round r.

C. Household Problem

We start with the enrollment problem. After seeing the postapplication shock η_i and the assignment result, i chooses between the school it is assigned to and the outside option. The expected value of being assigned to school j is

$$v_{ij} = E_{\eta_i} \max\{u_{ij}, \eta_i\}. \tag{2}$$

If rejected by all schools on its list, i can opt for a school that it prefers the most among the leftover schools (i's backup). The value (v_{i0}) of being assigned to its backup school is given by

$$v_{i0} = \max\{v_{ij}\}_{j \in \text{leftovers}}. \tag{3}$$

1. Application: Nonstrategic Households

A nonstrategic household lists schools according to its true preferences $\{v_{ij}\}_j$. Without further assumptions, any list of length $n (1 \le n \le R)$ that consists of the ordered top n schools according to $\{v_{ij}\}_j$ is consistent with nonstrategic behavior, which makes the prediction of allocation outcomes ambiguous. To avoid such a situation, we impose the following weak requirement: suppose that household i ranks its backup school as its n_i^*th favorite; then the length of i's application list n_i is such that

$$n_i \geq \min\{n_i^*, R\}. \quad (4)$$

That is, when there are still slots left on its application form, a nonstrategic household will list at least up to its backup school.[23]

Let $A_i^0 = \{a_1^0, \ldots, a_{n_i}^0\}$ be an application list for nonstrategic $(T = 0)$ household i, where a_r^0 is the ID of the rth listed school and n_i satisfies condition (4). The elements in A_i^0 are given by

$$\begin{aligned} a_1^0 &= \arg\max_j \{v_{ij}\}_j, \\ a_r^0 &= \arg\max_j \{v_{ij} | j \neq a_{r'<r}\}_j, \quad \text{for } 1 < r \leq \min\{n_i^*, n_i\}. \end{aligned} \quad (5)$$

The rth listed school is one's rth favorite for the entire list if $n_i < n_i^*$ and for the first n_i^* schools if $n_i \geq n_i^*$.[24] Define $\mathbf{A}^0(x_i, \epsilon_i, l_i)$ as the set of lists that satisfy conditions (4) and (5) for a nonstrategic household with (x_i, ϵ_i, l_i). If $n_i^* \geq R$, then the set $\mathbf{A}^0(\cdot)$ is a singleton and the length of the list $n_i = R$. If $n_i^* < R$, then all lists in the set $\mathbf{A}^0(\cdot)$ are identical up to the first n_i^* elements and imply the same outcome.

2. Application: Strategic Households

Taking admission probabilities as given, a strategic household maximizes its expected payoff. This payoff depends on not only which schools are listed but also how they are ordered.[25] Therefore, the standard (direct) solution is to choose the best permutation of schools. Formally, let $\mathbf{P}(J; R)$ be the set of all possible permutations of size 1–R out of elements in J, and let $|\mathbf{P}(J; R)|$ be its size. The standard solution is given by

$$\max_{A \in \mathbf{P}(J;R)} \pi(A, S_i, x_i, l_i, \epsilon_i), \quad (6)$$

where $\pi(A, \cdot)$ is the expected value of list A. Choosing the best permutation has been feasible in previous studies because choice sets (J) were

[23] See the online appendix for further discussions about condition (4).

[24] We do not require that schools listed after one's backup school be ranked, which is a weaker assumption than otherwise.

[25] We assume that strategic households are fully rational, because it is a clear baseline. As a justification, BM has been practiced in Barcelona for over 20 years and is very familiar to households. A more flexible model would allow for partially informed types, which is a straightforward extension to our framework but will impose great challenges for identification. We leave it for future work.

small in those studies. When J is relatively big and R is beyond 1, $\mathbf{P}(J; R)$ soon becomes unmanageably large. In the case of Barcelona, with $J = 317$ and $R = 10$, $|\mathbf{P}(J; R)|$ is over 8.9×10^{24}.

REMARK 2. We have followed the literature in assuming that households take admission probabilities as given and believe that their own choices would not affect the equilibrium. This is a nontrivial assumption. Although not ideal, it allows us to estimate household preferences without having to solve for the equilibrium. Otherwise, the estimation procedure would be very burdensome, if not infeasible, without making some other restrictive assumptions, because of the multiple-equilibria problem embedded in BM. Because of this major advantage, the price-taking assumption has been widely used in the literature, especially when the market is large.[26]

We develop a solution method to break the curse of dimensionality and fully solve the strategic household's problem. Note that although we use data from Barcelona with a particular mechanism and priority structure, our solution method is general and applicable to a broad class of mechanisms, referred to as the class under consideration hereafter.[27] Some examples in this class include BM, constrained and unconstrained DA, first preference first, Chinese parallel, and variants or hybrids of the above.[28] The solution utilizes the following two unexploited properties that are intrinsic of these mechanisms:.

1. Sequentiality.—Although the entire application list is submitted all at once, the ranked schools on the list are considered sequentially in the procedure. During the assignment, the rth listed school (a_r) is relevant only if one is rejected by all previously listed schools. Therefore, a_r should be one's best choice, contingent on reaching that stage, implying that the problem can be solved via backward induction.

2. Reducible history.—Being rejected by previously listed schools may carry information about one's probability of being assigned to a_r, but the information can be fully summarized by objects much simpler than the list $(a_1, \ldots, a_{r-1})$. Therefore, the problem involves a state space with a dimension much lower than $|\mathbf{P}(J; R)|$.[29] In particular, as we show in the online appendix, with some differences in specifics, mechanisms in the class under consideration has the following feature. After being rejected

[26] See, e.g., Abdulkadiroğlu et al. (2011), Azevedo and Hatfield (2015), Kojima (2015), Azevedo and Leshno (2016), Hatfield, Kojima, and Narita (2016), and Agarwal and Somaini (2018).

[27] Agarwal and Somaini (2018) study the same class of mechanisms.

[28] For example, the Cambridge mechanism, serial dictatorship, and the procedure used in London for secondary school assignment since 2005.

[29] The dimensionality of the direct solution is the same as that of a backward induction where all details of the list $(a_1, \ldots, a_{r-1})$ bear information relevant for a_r, which is not the case. That is, the direct method makes the problem unnecessarily complicated.

by $(a_1, \dots, a_{r-1})$, i will be admitted to a_r if a_r still has seats and if i is ranked high enough among those being considered. The latter is fully determined by i's priority and random lottery number for a_r. Among the two factors, i's priority for a_r is determined by predetermined characteristics and, in some instances, the rank position of the school on i's list (i.e., r), but it is independent of the other schools on the list.[30] One's lottery number is drawn after the application, unknown to the applicant when making her decisions. In the case where a household has a single lottery number across all tiebreaking cases, correlation arises between the probabilities of being admitted to the listed schools: being rejected by a_1 as a result of losing the lottery for a_1 reveals that one's lottery number is below cut_{a_1}; being rejected again by a_2 as a result of losing the lottery for a_2 reveals that one's lottery number is below $\min\{\text{cut}_{a_1}, \text{cut}_{a_2}\}$, and so on. However, other than this, $(a_1, \dots, a_{r-1})$ bears no information that is payoff relevant for one's decision on a_r. Therefore, the dimensionality can be reduced considerably. Consider an example where one can list up to three schools out of 12, under a standard BM. Suppose that the numbers of schools filled up in rounds 1–3 are (5, 4, 3). The dimensionality is $|\mathbf{P}(12; 3)| = 1{,}464$ in the direct solution, while it is bounded from the above by 150 in our solution.

Given sequentiality, we explain below how to derive a strategic household's optimal applications list $A_i^1 = \{a_{i1}^1, \dots a_{iR}^1\}$ via backward induction in general. Then, we use BM as an example to show the evolution of the state variables involved in the induction, utilizing the property of reducible history. The online appendix proves that this method fully solves the problem, formally describes the dimensionality involved in the solution, and explains applications of this method to other mechanisms in the class under consideration. Readers not interested in the details can skip to the next section.

Solution via backward induction.—Let $F_i^r \equiv F_i(a_1, \dots a_{r-1})$ be the information relevant for round r that is contained in the history of i being rejected by $(a_1, \dots, a_{r-1})$. Let $p_j^r(S_i|F_i^r)$ be the probability of being admitted to j for a household with scores S_i and j as its rth choice. The contents of F_i^r and the determination of $p_j^r(\cdot)$ vary across mechanisms and depend on the detailed specification of priorities and the usage of lottery numbers in ranking applicants. However, in the class of mechanisms under consideration, given $(S_i, x_i, l_i, \epsilon_i, F_i^R)$, a_R will solve

$$V^R(S_i, x_i, l_i, \epsilon_i, F_i^R) = \max_j \left\{ p_j^R(S_i|F_i^R) v_{ij} + \left(1 - p_j^R(S_i|F_i^R)\right) v_{i0} \right\}.$$

[30] One exception is the BM in Barcelona and Spain in general, where priorities for all listed schools are determined by the priority for the school ranked first. This makes the case in Spain more complicated than regular cases, which can nevertheless be solved efficiently with our method.

In general, given $V^{r+1}(S_i, x_i, l_i, \epsilon_i, \cdot)$ and the state variables $(S_i, x_i, l_i, \epsilon_i, F_i^r)$, with $V^{R+1}(\cdot) = v_{i0}$, the continuation value for i at round $r \leq R$ is given by

$$\begin{aligned} V^r(S_i, x_i, l_i, \epsilon_i, F_i^r) = \max_{j \in J}\{&p_j^r(S_i|F_i^r)v_{ij} \\ &+ \left(1 - p_j^r(S_i|F_i^r)\right)V^{r+1}\left(S_i, x_i, l_i, \epsilon_i, F_i^{r+1}\right)\}. \end{aligned} \tag{7}$$

The process continues until $r = 1$, where a_1 solves

$$V^1\left(S_i, x_i, l_i, \epsilon_i, F_i^1\right) = \max_j\left\{p_j^1(S_i|F_i^1)v_{ij} + \left(1 - p_j^1\left(S_i|F_i^1\right)\right)V^2(S_i, x_i, l_i, \epsilon_i, F_i^2)\right\}.$$

The backward-induction process above constructs an optimal list $(a_1, \ldots, a_R)$.

We show the contents of F_i^r and the determination of $p_j^r(\cdot)$, using as examples the standard BM and the BM used in Barcelona, the latter being a special and more complicated case of the former.[31]

Case 1: school-specific priorities and a single lottery number (standard BM).—When a household has a single lottery number across all tiebreaking cases, correlation arises between admission probabilities across rounds. Losing the lottery for a_r reveals that one's lottery number is below cut_{a_r}. Therefore, the probability of being allocated in round $r + 1$ conditional on being rejected by a_r is (weakly) lower than the unconditional probability. Let $\bar{\xi}_i^r \in [0, 1]$ be the upper bound of one's random number conditional on one's rejection history ($\bar{\xi}_i^1 = 1$). All relevant information reduces to $\bar{\xi}_i^r$, that is, $\mathrm{F}_i^r = \bar{\xi}_i^r$. Constraints for equation (7) are

$$\bar{\xi}_i^{r+1} = \begin{cases} \min\left\{\text{cut}_j, \bar{\xi}_i^r\right\} & \text{if } s_{ij} = \bar{s}_j \text{ and } r = \bar{r}_j, \\ \bar{\xi}_i^r & \text{otherwise.} \end{cases} \tag{8}$$

$$p_j^r(S_i|\bar{\xi}_i^r) = \begin{cases} 1 & \text{if } r < \bar{r}_j \text{ or } \left(r = \bar{r}_j \text{ and } s_{ij} > \bar{s}_j\right), \\ \max\left\{0, \dfrac{\bar{\xi}_i^r - \text{cut}_j}{\bar{\xi}_i^r}\right\} & \text{if } r = \bar{r}_j \text{ and } s_{ij} = \bar{s}_j, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Condition (8) is the updating rule: $\bar{\xi}_i^{r+1}$ will decrease to $\min\{\text{cut}_j, \bar{\xi}_i^r\}$ if i is in the tied priority group and loses the lottery.[32] The second equality in

[31] The easiest case happens when applicants are given i.i.d. school-specific lottery numbers, under which $V^r(\cdot, F_i^r) = V^r(\cdot)$ and the constraint for eq. (7) is

$$p_j^r(S_i|F_i^r) = p_j^r(S_i) = \begin{cases} 1 & \text{if } r < \bar{r}_j \text{ or } (r = \bar{r}_j \text{ and } s_{ij} > \bar{s}_j), \\ 1 - \text{cut}_j & \text{if } r = \bar{r}_j \text{ and } s_{ij} = \bar{s}_j, \\ 0 & \text{otherwise.} \end{cases}$$

[32] Going to round $r + 1$ means that one must have been rejected in round r.

condition (9) follows the uniform distribution with truncated support $[0, \bar{\xi}_i^r]$.

Case 2: constant priority and a single lottery number (Barcelona).—The priority score of one's top-listed school carries over to future rounds. As a result, the continuation values for $r > 1$ depend on the top-listed school (a_1), and S_i in equation (7) now becomes a vector of identical elements, $s_{ia_1}\mathbf{1} \equiv [s_{ia_1}, \ldots, s_{ia_1}]$. With $s_{ia_1}\mathbf{1}$ being the priority score vector, the problem for $r > 1$ remains the same as in case 1. For round 1, one solves the following problem:

$$V^1\left(S_i, x_i, l_i, \epsilon_i, \bar{\xi}_i^1 = 1\right) = \max_{j\in J}\{p_j^1\left(s_{ij}\mathbf{1}|1\right)v_{ij} + \left(1 - p_j^1\left(s_{ij}\mathbf{1}|1\right)\right)V^2\left(s_{ij}\mathbf{1}, x_i, l_i, \epsilon_i, \bar{\xi}_i^2\right)\}, \tag{10}$$

subject to conditions (8) and (9). That is, the choice in round 1 governs the vector of priority scores.

Remark 3. Multiple lists may yield the same value. Let $\mathbf{A}^1(x_i, l_i, \epsilon_i)$ be the set of optimal lists for a strategic household. All lists in the optimal set, including the one derived by backward induction, are identical up to the payoff-relevant part of the lists and imply the same allocation outcome.[33]

IV. Estimation

A. *Further Empirical Specification*

As described in detail in appendix A1, the utility function takes the following form:

$$U\left(w_j, x_i, d_{ij}, \zeta_j\right) = \tau_1 I(\text{single parent}) + \tau_2\left(\text{sib}_{ij} - \text{sib}_{i0}\right) - C\left(d_{ij}\right) + \sum_{e=1}^{3}\left(\delta_{0e} + \delta_{1e}\zeta_j + w_j\alpha_e\right)I(\text{edu}_i = e). \tag{11}$$

In particular, τ_2 is added to i's evaluation of j if a sibling is enrolled in j and subtracted from i's evaluation of all schools if a sibling is in the outside option ($\text{sib}_{i0} = 1$). The term $C(d_{ij})$ is a distance cost function. The second line of equation (11) specifies the part of the utility that varies across households with different education levels.

With potential correlation between school characteristics that are unobserved (ζ_j) and observed (w_j), estimates of α in equation (11) may be

[33] For example, consider a list $A^1 = \{a_1^1, \ldots, a_r^1, \ldots a_R^1\}$; by the specification of $\{u_{ij}\}$, each a_r^1 is generically unique if no school listed before it has a 100% admission rate for the household. However, if for some $r < R$, the admission rate for the rth listed school is 1, then any list that shares the same first r ordered elements is also optimal. See the online appendix for other cases.

inconsistent. Yet one can combine the effects of (w_j, ζ_j) and rewrite the second line of equation (11) as

$$\sum_e \left(\delta_{0e} + \delta_{1e}\kappa_j + w_j\rho_e\right) I(\mathrm{edu}_i = e). \tag{12}$$

The reduced-form parameters ρ and $\{\kappa_j\}_j$ can be consistently estimated, and each of them is some combination of structural parameters α, δ, and ζ. For the goal of this paper, it is sufficient to estimate ρ and $\{\kappa_j\}_j$ instead of α and ζ.[34]

B. The Likelihood

Let parameter vector $\Theta \equiv [\Theta_u, \Theta_T]$, where Θ_u governs household preferences, and Θ_T governs type distribution. Let $O_i \equiv [\tilde{A}_i, \tilde{e}_i | \tilde{j}_i]$ be the observed outcomes for household i, where $\tilde{A}_i$ is the application list, $\tilde{j}_i$ is the assigned school, and $\tilde{e}_i$ is enrollment. Conditional on being type T, the probability of observing O_i is given by

$$\begin{aligned} L_i^T(\Theta_u) = \int \Bigg(& I\left(\tilde{A}_i \in \mathbf{A}^T(x_i, l_i, \epsilon_i; \Theta_u)\right) \\ & \times \left\{ \tilde{e}_i \Phi\left(\frac{\bar{u}_{i\tilde{j}}(\Theta_u) + \epsilon_{i\tilde{j}_i}}{\sigma_\eta}\right) \right. \\ & \left. + (1 - \tilde{e}_i)\left[1 - \Phi\left(\frac{\bar{u}_{i\tilde{j}}(\Theta_u) + \epsilon_{i\tilde{j}_i}}{\sigma_\eta}\right)\right]\right\}\Bigg) dF_\epsilon(\epsilon; \sigma_\epsilon), \end{aligned}$$

where $\mathbf{A}^T(\cdot)$ is the set of model-predicted optimal application lists for a type-T household and $\Phi[(\bar{u}_{i\tilde{j}}(\Theta_u) + \epsilon_{i\tilde{j}_i})/\sigma_\eta]$ is the probability that this household will enroll in $\tilde{j}_i$. Integrating over the type distribution, i's contribution to the likelihood is

$$L_i(\Theta) = \lambda(x_i, l_i; \Theta_T) L_i^1(\Theta_u) + (1 - \lambda(x_i, l_i; \Theta_T)) L_i^0(\Theta_u).$$

The log likelihood of the whole sample is given by $\mathbf{L}(\Theta) = \Sigma_i \ln(L_i(\Theta))$.

C. Identification

We give an overview of the identification in this subsection and leave the formal proof to the online appendix. The identification relies on the following assumptions.

ASSUMPTION A1. There does not exist a vector of household observables x and a school j such that all households with x have probability zero of being admitted to school j.

[34] Equations (11) and (12) are invariant to our counterfactual policy changes.

ASSUMPTION A2. Household tastes ϵ are drawn from an i.i.d. unimodal distribution, with mean normalized to zero, and they are independent of school characteristics, household observables (x, l), and household type (T).

ASSUMPTION A3. At least one continuous variable in the utility function is excluded from the type distribution. Conditional on variables that enter the type distribution function, the excluded variable is independent of household type T.

To illustrate the identification challenge, consider a situation where each household applies to only one school, which is a less favorable situation for identification because we would have less information, and suppose that there is no postapplication shock.[35] If all households are nonstrategic, then the model boils down to a multinomial discrete choice model with a household choosing the highest $\bar{u}_{ij}(\Theta_u) + \epsilon_{ij}$. The identification of such models is well established under very general conditions (e.g., Matzkin 1993). If all households are strategic, then a household considers the admission probabilities $\{p_{ij}\}_j$ and chooses the option with the highest expected value.[36] With the admission probabilities observed from the data, this model is identified with assumption A1.[37] The challenge exists because we allow for a mixture of the two types of households. In the following, we first explain assumptions A2 and A3 and then give the intuition underlying the identification proof.

1. Assumptions A2 and A3 in Our Framework

We observe application lists with different distance-quality-risk combinations with different frequencies in the data. The model predicts that households of the same type tend to make similar application lists. Given assumption A2, the distributions of type-related variables will differ around the modes of the observed choices, which informs us of the correlation between type T and these variables. Assumption A3 guarantees that different behaviors can arise from exogenous variations within a type. To satisfy assumption A3, we need to make some restrictions on how household observables (x_i, l_i) enter type distribution and utility. Conditional on distance, a nonstrategic household may not care too much about living to the left or the right of a school, but a strategic household may be more likely to have chosen a particular side so as to take advantage of the priority-zone

[35] The postapplication shock is identified from the observed allocation and enrollment outcomes.

[36] Agarwal and Somaini (2018) show conditions under which one can nonparametrically identify household preferences when all of them are strategic.

[37] If for all households with x the admission probabilities to j are zero, then the utility for school j for these households is unidentifiable, because the expected value of applying to j is zero regardless of the level of utility.

structure.[38] However, given that households, strategic or not, share the same preferences about school characteristics and distances, there is no particular reason to believe that, everything else being equal, the strategic type will live closer to a particular school than the nonstrategic type would just for pure distance concerns. In other words, because the only difference between a strategic type and a nonstrategic type is whether or not one considers the admission probabilities, which are affected by one's home location only via the zone to which it belongs, we assume that home location l_i enters the type distribution only via the school zone z_{l_i}, that is, $\lambda(x_i, l_i) = \lambda(x_i, z_{l_i})$. In contrast, household utility depends directly on the home-school distance vector d_i. Conditional on being in the same school zone, households with similar characteristics x but different home addresses still face different home-school distance vectors d, as required in assumption A3.

Our exclusion restriction assumption is nontrivial and deserves some further discussion. It assumes away any systematic difference in home-school distance between two types of households living in the same school zone, which may arise from factors unobservable to the researcher. Readers should be aware of this limitation, although the online appendix shows suggestive data evidence that this assumption may not be too unreasonable in our case.[39]

2. The Intuition for Identification

The data contain rich information for identification. First, one can compare a household's listed schools with other schools. As a result of unobserved school characteristics, some seemingly good schools may in fact be unattractive, making them not as popular as they "should have been" among most households. Controlling for such common factors, as we do in the model, a household may still leave out some seemingly good schools because of unobserved tastes. Assumption A2 implies that tastes are independent of household-school-specific admission probabilities, which should not lead to a systematic relationship between a school being listed and a household's chances of getting into this school. However, as is shown in section V, for a large fraction of households, when they left

[38] Without directly modeling households' location choices, we allow household types to be correlated with the characteristics of the school zones they live in. We leave the incorporation of household location choices for future extensions.

[39] We utilize a data fact to test this assumption: although types are not observed for all households, some households are obviously nonstrategic (as defined in sec. IV.C.3). We find no correlation between obvious naivete and various measures of school-home distance, suggesting that this exclusion restriction may not be unreasonable in our case. However, we cannot provide sufficient conditions to validate our assumption, since types are not observed for all households.

out schools better than their listed ones in terms of quality, fees, and distance, in most cases these better schools were ones for which they had lower chances. Such behavior is highly consistent with strategizing instead of truth telling.

Second, one can explore the fact that admission probabilities increase discontinuously with in-zone status. Most illustratively, consider households along the border of two zones. Were all households nonstrategic, applications should be very similar among households along both sides of the border. In contrast, were most households strategic, applications would be very different across the border.

Finally, conditional on (x, z_l), the variation in d induces different behaviors within the same type, and, conditional on (x, z_l, d), different types behave differently. In particular, although households share the same preference parameters, different types of households will behave as if they have different sensitivities to distance. For example, consider households with the same (x, z_l) and a good school j out of their zone z_l. As the distance to j decreases along household addresses, more and more nonstrategic households will apply to j because of the decreasing distance cost. However, the reactions will be much less obvious among the strategic households, because they take into account the risk of being rejected, which remains unchanged no matter how close j is, as long as it is out of z_l. The different distance elasticities among households therefore inform us of the type distribution within (x, z_l). This identification argument does not depend on specific parametric assumptions. For example, Lewbel (2000) shows that similar models are semiparametrically identified when an assumption A3–like excluded variable with a large support exists. However, to make the exercise feasible, we have made parametric assumptions.[40]

3. Obviously Nonstrategic Households

The identification of our model is further facilitated by the fact that we can partly observe household type directly from the data: there is one particular type of "mistake" that a strategic household will never make and is a sufficient (but not necessary) condition to spot a nonstrategic household.[41] Intuitively, if a household's admission status is still uncertain for

[40] We have to use parametric assumptions because semiparametric estimation is empirically infeasible and because the support of d is bounded by the size of the city, which is not large enough relative to the (unbounded) support of household tastes, as required in Lewbel (2000).

[41] If the support of household characteristics is fully conditional on being obviously nonstrategic, then household preferences can be identified by using this subset of households without assumption A1, since ϵ is independent of (x, l). However, our identification does not rely on the existence of obviously nonstrategic households.

all schools listed so far and there is another school j it desires, one should never waste the current slot listing a zero-probability school instead of j, because the admission probabilities decrease over rounds.[42] The idea is formalized in the following claim and proved in the online appendix.[43]

Claim 1. An application list with the following features is sufficient but not necessary evidence that the household must be nonstrategic: (1) for some rth element a_r on the list, the household faces zero admission probability at the rth round; (2) it faces admission probabilities lower than 1 for all schools listed in previous rounds; and (3) it faces a positive but lower-than-100% admission probability for the school listed in a later slot $r' \geq r + 1$, and no school listed between a_r and $a_{r'}$ admits the household with probability 1.

V. Data

We focus on applications among families with children who turned 3 years old in 2006 or 2007 and lived in Barcelona. For each applicant, we observe the application list, the assignment and enrollment outcomes, home address, family background, and the ID of the school(s) her siblings were enrolled in the year of her application. For each school, we observe its type (public or semipublic) and a measure of quality, capacity, and service fees. The online appendix describes our data sources.

A. Admission Thresholds

Assuming that each household is a small player that takes the admission thresholds as given, we can recover all parameters by estimating an individual decision model. Given the observed applications and the priority rules, we make 1,000 copies for each observed application list, involving all participating households (11,871 in 2006), and assign each copy a random lottery number. We simulate the assignment results in this enlarged market to obtain $\{(\bar{r}_j, \bar{s}_j, \mathrm{cut}_j)\}_j$, which are treated as the ones households expected when they applied.[44]

[42] For example, consider an application list (1, 2, 3, …) by household i with S_i, where school 1 is filled up in round 1 with $p_1^1(S_i, \cdot) < 1$, school 2 is filled up in round 1 (hence $p_2^2(\cdot) = 0$), school 3 is filled up in round 3, and $0 < p_3^3(S_i, \cdot) < 1$. This list is irrational, because one has probability 1 of getting into school 3 in round 2 and hence is strictly better off with any list starting with (1, 3) instead of (1, 2).

[43] Abdulkadiroğlu et al. (2006) use a mistake similar to feature 1 in claim 1 to spot nonstrategic households: listing a school overdemanded in the first round as one's second choice.

[44] Agarwal and Somaini (2018) prove consistency of the obtained admission probabilities for the class of mechanisms under consideration.

B. Summary Statistics

For estimation, we drop 3,152 observations whose locations cannot be matched in the GIS (geographic information system),[45] 31 whose outcomes were inconsistent with the official rule, 191 with special-needs children or postdeadline (and hence ineligible) applications, and those with missing information.[46] The final estimation sample has 6,836 observations. Table 1 summarizes school characteristics.[47] Compared to public schools, semipublic schools have higher quality and larger capacities. Table 2 reports household characteristics in the estimation sample.[48] Households had priority for 22 schools on average but with considerable dispersion, depending on their zones.

Panel A of table 3 shows that most of households listed no more than three schools, with 47% listing only one school. High school (HS)-educated parents and single parents tended to list more schools than others. Panel B of table 3 shows the round at which households were assigned. In equilibrium, most households (93%) were assigned to their first choices. However, this does not imply low risk.[49] Among those assigned to their first choices, the average admission probability was 94.6%, with the lowest being 20%. Of all schools, 44% were filled up in round 1, while 40% were leftovers. That is, households face very high stakes: a large number of schools were overdemanded, and once rejected in round 1, most schools one could get into were leftovers. The fact that most households were assigned to their first choice suggests both the prevalence of strategic play and a large amount of coordination in equilibrium. Table 4 summarizes the characteristics of top-listed schools: school quality, distance, and fees all increase with parental education. Table 5 shows that 97% of all students were enrolled in the public school system. Among those assigned to their first choice, 2.2% chose not to enroll, which can be rationalized by ex post shocks.[50]

[45] We know their priority scores and applications, which enables us to include them in the calculation of admission thresholds.

[46] In the estimation, we exclude 748 parents who reported their education as "high school or above." In policy simulations, we include this subsample so as to make equilibrium assignments. We estimate the probability of each of them as being college educated as a flexible function of all the other observable characteristics, by comparing them with those who reported exactly high school or college education. The model fit for this subsample is as good as that for the estimation sample and is available on request.

[47] School quality is measured by average student test scores on a scale from 0 to 10.

[48] Following the literature on child development, we use mother's education as the definition of parental education if the mother is present in the household; otherwise, we use the father's education.

[49] Other studies also find that most households were assigned to their first choices under manipulable mechanisms, e.g., Abdulkadiroğlu et al. (2006), Hastings, Kane, and Staiger (2009), Lavy (2010), and Agarwal and Somaini (2018).

[50] The online appendix shows that the probabilities of being assigned in round 1 were lower for nonenrollees, suggesting that households that took higher risks might have better outside options.

TABLE 1
School Characteristics

	Public	Semipublic	All
Quality	7.4 (.8)	8.0 (.5)	7.7 (.7)
Fees (€100)	0	12.8 (5.7)	6.4 (7.5)
No. of classes	1.4 (.5)	1.8 (1.0)	1.6 (.8)
Observations	158	159	317

Note.—Standard deviations are in parentheses.

For suggestive evidence of strategic behavior, table 6 compares a household's top-listed school with other schools. A school is labeled "better in three aspects" if it had higher quality and lower tuition and was at a shorter distance than one's top-listed school and "better in two aspects" if it failed one of the three conditions. Of all households, 41% had at least one "better-in-three" school, with the average number being 5.2. Almost all households had a considerable number of "better-in-two" schools. Of course, unobservable tastes and/or school characteristics, both of which are incorporated into our model, may drive these choices. However, these unobservables are unlikely to suffice. First, "better" schools overlap very little across households, suggesting a very limited role for school unobservables. Second, these "better" schools were disproportionally unlikely to be schools for which the household had higher admission probabilities. For example, for an average household with some "better-in-three" schools, for only 14% of such schools did the household have higher chances than for its top choice. The same pattern holds if we exclude those who top-listed a sibling school. These facts are hard to rationalize with truthful reporting, unless households' unobserved tastes vary systematically

TABLE 2
Household Characteristics

Education < HS (%)	29.8
Education = HS (%)	30.4
Education > HS (%)	39.8
Single parent (%)	15.8
Have school-age older sibling(s) (%)	42.2
No. of schools in zone (standard deviation)	22.3 (7.9)
Average school quality in zone (standard deviation)	7.8 (.3)
Observations	6,836

Note.—In all tables, "education" represents the mother's education if she is present and the father's education otherwise.

TABLE 3
NUMBER OF SCHOOLS LISTED AND ASSIGNMENT (%)

	A. Number of Schools Listed				B. Assignment Round			
	1	2	3	4 or more	First	Second	Third–Tenth	Unassigned
All	46.9	12.4	16.9	23.8	93.0	2.8	1.5	2.7
Education < HS	49.8	15.0	19.5	15.7	93.2	2.7	1.6	2.5
Education = HS	43.4	12.1	18.4	26.2	92.0	3.5	1.5	3.0
Education > HS	47.4	10.6	13.9	20.1	93.7	2.3	1.3	2.7
Single parent	43.3	14.7	16.8	25.2	94.0	1.9	1.4	2.7

TABLE 4
TOP-LISTED SCHOOLS

	Quality	Distance (100 m)	Fees (€100)
All	7.9 (.6)	7.1 (8.7)	8.1 (7.7)
Education < HS	7.6 (.7)	5.2 (6.2)	5.4 (6.6)
Education = HS	7.9 (.5)	7.0 (8.6)	8.1 (7.5)
Education > HS	8.2 (.4)	8.7 (9.9)	9.9 (8.1)
Single parent	8.0 (.6)	8.1 (9.9)	8.6 (8.4)

NOTE.—Standard deviations are in parentheses.

TABLE 5
ENROLLMENT IN PUBLIC SYSTEM (%)

All	96.7
Education < HS	97.0
Education = HS	97.1
Education > HS	96.3
Single parent	96.1
Assigned in round 1	97.8

TABLE 6
SCHOOLS "BETTER" THAN THE TOP-LISTED ONE

	Households (%) (1)	No. of Better Schools (2)	Better w/ Higher p (%) (3)
All households ($N = 6{,}836$):			
Have school better in 3 aspects	40.7	5.2 (9.9)	14.1
Have school better in 2 aspects	99.8	75.9 (45.1)	10.5
Sibling's school not top-listed ($N = 4{,}025$):			
Have school better in 3 aspects	39.3	4.6 (8.7)	24.8
Have school better in 2 aspects	99.8	77.3 (44.2)	17.8

NOTE.—Column 1 shows the percentage of households that satisfy the condition specified in each row, col. 2 the average (standard deviation) number of better schools for households with such schools, and col. 3 the percentage of better schools with higher admission probability than one's top choice.

with the household-specific admission probabilities.[51] These data facts provide information for the identification of household type and preference distribution.

VI. Results

A. Parameter Estimates

Table 7 presents the estimated parameters governing household preferences. Panel A reports structural parameters governing parts of the utility function that vary within an education group and the dispersions of tastes (σ_ϵ) and postapplication shocks (σ_η). The cost of distance is convex, although the square term is not precisely estimated.[52] We also allow for two jumps in the cost of distance. The first jump is set at 500 m, an easy-to-walk distance even for a 3-year-old; the second is at 1 km, a long yet manageable walking distance. As households may have to use other transportation methods beyond these distances, the cost of distance jumps significantly at the thresholds. The parameter on sibling schools adjusts such that most (97%) households with sibling schools top-listed them. We find a high σ_η, which explains why a household would give up its assigned school, especially if it is its first choice.[53] Taste dispersion σ_ϵ is relatively small, which is consistent with households' low willingness to take risks (table 6).

It would be noninformative to report the over 300 parameter estimates (κ_j) of school values. Instead, we use an ordinary least squares (OLS) regression of these estimates on observables as a summary (panel B of table 7).[54] These OLS estimates will be unbiased only if the unobserved school characteristics are uncorrelated with the observables. This potential correlation does not affect our policy analyses, which use the consistently estimated school values. However, one should be cautious when relating school values and utils to w_j. With caution, we have the following findings that are consistent with data facts in tables 3–5. (1) HS-educated parents value schools more than the others, especially the college-educated group, for whom the outside option may be more affordable. (2) Higher-educated parents value school quality more and are less sensitive to fees.[55] (3) Households prefer schools with larger capacity, which tend to have

[51] The online appendix shows further evidence via regression analyses.

[52] As in other discrete choice models, we need to normalize one coefficient in the utility function in order to identify σ_ϵ and σ_η, and we normalize the linear term on distance to -1.

[53] The ex ante value of the outside option is normalized to zero, while the average value of schools (κ_j) is estimated to be 3,391.

[54] The form of the OLS follows model specifications in sec. IV.A and app. A.

[55] Preferences for quality peak beyond the maximum school quality for the high-education group, at the 99th percentile for the middle-education group, and around the 60th percentile for the low-education group. Other studies also find that parents of different education value school characteristics differently, e.g., Burgess et al. (2009), Hastings, Kane, and Staiger (2009), He (2014), and Abdulkadiroğlu, Agarwal, and Pathak (2017).

TABLE 7
PREFERENCE PARAMETERS

A. STRUCTURAL PREFERENCE PARAMETER ESTIMATES[a]		B. SUMMARIZE SCHOOL FIXED EFFECTS[b]	Education < HS	Education = HS	Education > HS
Distance^2	−.05 (.04)	Constant	2,766.3	2,783.3	2,423.0
Distance > 500 m	−55.3 (7.1)	Quality	152.0	176.4	187.8
Distance > 1 km	−46.5 (7.9)	Fee	−1.0	−.6	−.3
Sibling school	1,339.0 (86.5)	Semipublic	−.6	6.5	.8
Single parent	−404.3 (12.2)	Capacity	.7	.7	.7
σ_ϵ (taste dispersion)	66.3 (6.2)	Capacity^2	−.001	−.001	−.001
σ_η (postapplication shock)	1,937.8 (18.7)	Quality^2	−9.9	−9.9	−9.9

[a] Standard errors are in parentheses.
[b] OLS regression of the estimated school value parameters on observables.

more resources. (4) Everything else being equal, semipublic schools are more preferable, except for the low-educated group.

Table 8 presents the estimated type distribution parameters. Single parents and parents with higher education levels are more likely to be strategic. Although strategic households are not more likely to live in zones with

TABLE 8
TYPE DISTRIBUTION

Constant	−18.9 (2.3)
Single parent	.3 (.5)
Education < HS	−.1 (.2)
Education > HS	.7 (.3)
No. schools in zone	−.1 (.2)
Average school quality in zone	3.1 (1.1)
Have an older sibling	49.0 (24.7)

NOTE.—Standard errors are in parentheses.

TABLE 9
STRATEGIC VERSUS NONSTRATEGIC TYPE: SIMULATION

	Strategic (%)	Strategic	Nonstrategic
All	96.3		
Education < HS	94.7		
Education = HS	95.8		
Education > HS	97.8		
Single parent	96.6		
Have an older sibling	97.1		
Schools in zone:			
No. of schools		22.3	21.8
Average quality		7.9	7.7

more schools, they are more likely to live in zones with better schools.[56] Households that have older children and therefore have already gone through the process are more likely to be strategic.[57] Based on these estimates, table 9 shows the simulated type distribution in our sample. Consistent with data facts such as those in table 6, 96% of households were strategic, and the fraction increases with education.[58] To obtain further insights on our findings, we have reestimated our model with the restriction that only 80% of households were strategic. The fit of this restricted model is significantly worse, as shown in the online appendix.

B. Model Fits and Out-of-Sample Validation

The 2007 redefinition of priority zones abruptly changed the school-household-specific priorities: priority schools became those that surrounded each home location.[59] We show model fits for both the 2006 and the 2007 samples.[60] To simulate the 2007 outcomes, we first calculate the admission probabilities in 2007 via the same procedure as we do for 2006. Then we use the 2007 sample to conduct an out-of-sample validation.[61] Because the reform came as a surprise and households were unlikely to relocate before submitting their applications in 2007, we simulate the distribution of 2007 household types, using the characteristics of their residential zones according to the 2006 definition.[62]

Considered as the most informative test of the model, the first two rows of table 10 explore the changes in the definition of priority zones. The reform led to situations where some schools were in the priority zone for a household in one year but not in the other, which would affect the behavior of a strategic household. In 2006, 24% of households top-listed a school that was in their priority zone by the 2006 definition but not by the 2007 definition. In 2007, the fraction of households top-listing

[56] We allow for the correlation between zone characteristics and types. The estimates are consistent with our intuition that strategic households may choose home locations to utilize the residence-based priority structure.

[57] One extension is to incorporate the dynamic considerations by households with multiple children.

[58] We find a much smaller fraction of nonstrategic households than Abdulkadiroğlu et al. (2006). The main reason is our incorporation of the outside option and the leftover schools, which rationalizes the choices by a substantial fraction of households that might be categorized as nonstrategic otherwise. Another reason is the long history of BM in Barcelona, where parents have become very familiar with the mechanism.

[59] In 2007, the average (standard deviation) number of schools to which a household had priority became 7.0 (1.5).

[60] More fitness tables are in app. A.

[61] In 2007, 12,335 Barcelona households participated, 7,437 of whom are selected into our validation sample, under the same selection rule as before.

[62] We also assume that strategic households had rational expectation about admission probabilities in 2007. As shown below, we can fit the data in both years, suggesting that our assumptions are not unreasonable.

TABLE 10
MODEL FITS

	2006		2007	
	Data	Model	Data	Model
Top-listed schools:				
In zone 2006 only (%)	24.1	24.1	12.0	14.6
In zone 2007 only (%)	3.0	4.5	12.0	10.9
Quality	7.9	7.9	7.9	7.9
Distance (100 m)	7.1	7.2	6.6	7.0
Fee (€100)	8.1	8.1	7.9	7.8
Assignment round (%):				
1	93.0	91.3	92.0	92.6
2	2.8	4.0	3.1	3.6
≥3	1.5	.9	1.8	.6
Unassigned	2.7	3.8	3.1	3.2
Enrollment in the public system (%):				
All	96.7	96.5	97.6	96.6
Assigned in round 1	97.8	97.1	98.3	97.1

these schools dropped to 12%. On the other hand, the fraction of households that top-listed schools in their priority zone only by the 2007 definition but not by the 2006 definition increased from 3% to 12% over the two years. The model is able to replicate such behaviors and predicts the changes as being from 24% to 14.6% for the first case and from 4.5% to 11% for the second case. The model also replicates the fact that top-listed schools in 2007 were of similar quality and at a shorter distance and had lower tuition, relative to those in 2006. The model slightly underpredicts the fraction of households assigned in round 1 for 2006 but closely replicates the enrollment rate.

VII. BM versus DA versus TTC

Using the estimated model, we compare the baseline BM with DA and TTC. In a different experiment, presented in appendix A, we assess the impacts of the 2007 reform. In both experiments, households' welfare refers to their evaluations of their assignment outcomes relative to their outside options, that is, v_{ij}.[63]

A. *Theoretical Background*

The DA procedure is similar to the BM; however, students are only temporarily assigned to schools in each round, and a student's chance of being finally admitted to a school does not depend on the ranking of the

[63] All simulations use the school-household-specific priority scores given by eq. (1), following official rules in the relevant year.

school on her application. TTC creates cycles of trade between individuals in each round. Each individual in a cycle trades off a seat in her highest-priority school for a seat in her announced most preferred school among those with open seats. Whenever such a cycle is formed the allocation is final.

Three properties are considered desirable but cannot hold simultaneously in a mechanism: Pareto efficiency, truth revealing, and the elimination of justified envy (aka stability).[64] BM satisfies none of the properties. DA and TTC, with the standard priority structure, are both truth revealing, which are the cases we consider.[65] Between the other two conflicting properties, DA eliminates justified envy, while TTC achieves Pareto efficiency. The welfare comparison between BM and its alternatives is ambiguous because of two competing forces. On the one hand, BM can lead to potential misallocations because households hide their true preferences, which is absent in DA and TTC. On the other hand, BM may better "respect" households' cardinal preferences than DA and TTC (Abdulkadiroğlu, Che, and Yasuda 2011). BM-induced household behaviors increase the chance of a "right match" in that a school is matched to households that value it more. Under a truth-revealing mechanism, households that share the same ordinal preferences will rank schools the same way and have the same chance of being allocated to various schools, regardless of who will gain the most from each school. Given that it is theoretically inconclusive, the welfare comparison between these mechanisms becomes an empirical question, one that we answer below.

B. *Results*

Under both DA and TTC with the standard priority structure, all households will list their true preferences.[66] We simulate each household's application list accordingly, assign them first with DA and then with TTC, and compare the results with those from the baseline Barcelona case, that is, BM with constant priority and a single lottery number.[67] We present our results under the more recent (2007) priority-zone structure.[68]

[64] Stability requires that there be no unmatched student-school pair (i, j) where student i prefers school j to her assignment and she has higher priority at j than some other student who is assigned a seat at school j.

[65] For example, one's priority score in round 1 does not carry over to future rounds.

[66] To simulate DA and TTC, it is sufficient to know household preferences. However, to compare DA or TTC with the baseline, one needs to know the distribution of household strategic types.

[67] All these mechanisms use random lotteries to break ties. For a given set of random lottery numbers, we simulate the allocation procedure and obtain the outcomes for all students. We repeat this process many times to obtain the expected (average) outcomes for each simulated student.

[68] The 2006 results are similar.

REMARK 4. We report total household welfare and the distribution of winners and losers among different subgroups of households, as well as the assignment outcomes. Total household welfare is not necessarily the criterion for social welfare, which may involve different weights across households. Given that we can calculate the welfare at the household level, our results can be used to calculate any weighted social welfare. Given a social objective, our results can be easily used for policy-making purposes, although we do not necessarily recommend one mechanism over another in this paper.

1. Household Welfare Comparison

The first two columns of table 11 show the averages and the cross-household standard deviations of welfare under BM. The next four columns show welfare changes when BM is replaced by DA, both in utils and in euros.[69] The impacts differ across households, where there are both winners and losers, leading to a wide dispersion of welfare changes across households. Therefore, we present both the means and the cross-household standard deviations of these changes. Average welfare decreases by 5.4 utils or €1,020, with a cross-household standard deviation of 30.9 utils or €7,180. Average welfare decreases more for nonstrategic households. Although the welfare loss in utils decreases with education, the decreasing price sensitivity across education groups yields a different ranking of euros lost. Clearly, one should not compare the losses directly across education groups, because they view the same euro amount differently.[70] The last four columns of table 11 compare BM with TTC. For an average household, a change from BM to TTC would increase the welfare by 1.9 utils or €460. Consequently, TTC leads to the highest total household welfare among all three alternatives. The gains are especially large for the nonstrategic households, measured at €1,970.

RESULT 1. In terms of total household welfare, the three mechanisms are ranked TTC > BM > DA. There are more losers than winners from a

[69] The translation of utils to euros uses the education-specific coefficients for fees, as in table 7.

[70] Although the households we study face a much larger choice set and a more complicated problem under BM as a result of the special priority rule in Barcelona, our findings are not peculiar. Hwang (2015) and Agarwal and Somaini (2018), who study BM with standard priority rules, also find that DA would yield lower welfare. Assuming that households have equal priority and the same ordinal preferences, Abdulkadiroğlu, Che, and Yasuda (2011) shows that DA may decrease welfare for both strategic and nonstrategic households. Given the different priority structure and the rich preference heterogeneity in our data, it is not clear whether their result would hold in Barcelona. However, the main intuition behind their result, i.e., BM better respects cardinal preferences, applies in our setting as well.

TABLE 11
HOUSEHOLD WELFARE: BM VERSUS DA VERSUS TTC

	Welfare under BM (utils)		Change from BM to DA				Change from BM to TTC			
			(Δutils)		(Δ€100)		(Δutils)		(Δ€100)	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
All	3,811	633	−5.4	30.9	−10.2	71.8	1.9	40.5	4.6	96.3
Strategic	3,810	633	−5.3	30.3	−10.2	69.9	1.6	39.9	3.9	94.3
Nonstrategic	3,818	629	−5.7	41.0	−19.7	131.8	8.3	51.5	19.7	131.8
Education < HS	3,690	600	−9.9	28.3	−10.0	28.6	.1	31.4	.1	31.7
Education = HS	3,934	619	−5.7	33.0	−18.3	106.3	2.5	44.1	8.0	141.9
Education > HS	3,792	647	−2.1	30.3	−3.7	54.6	2.7	42.8	4.8	77.2

NOTE.—Means and cross-household standard deviations of welfare changes in utils (Δutils) and hundreds of euros (Δ€100).

TABLE 12
WINNERS AND LOSERS (%)

	Change from BM to DA		Change from BM to TTC	
	Winners	Losers	Winners	Losers
All	11.7	33.0	25.0	21.5
Strategic	11.6	32.8	24.6	21.7
Nonstrategic	15.3	36.7	32.3	17.5
Education < HS	7.8	37.0	23.1	22.1
Education = HS	13.0	35.5	27.4	23.0
Education > HS	13.3	28.2	24.3	20.0

change of BM to DA and more winners than losers from a change of BM to TTC (table 12).

2. Cross-Zone Inequality

A household's welfare can be significantly affected by the school quality within its zone not only because of the quality-distance trade-off but also because of the quality-risk trade-off created by the priority structure. For equity concerns, a replacement of BM will be more desirable if it is more likely to benefit those living in poor-quality zones. Table 13 tests whether each of the counterfactual reforms meets this goal. In the change from BM to DA, winners live in better zones than losers, which is against the equity goal. The difference in zone quality between these two groups is almost 20% of a standard deviation of quality across all zones. In changes from BM to TTC, the average zone quality is similar across winners and losers.

RESULT 2. Welfare dependence on zone quality increases with a change from BM to DA and remains unaffected by a change from BM to TTC.

The cost of the elimination of justified envy.—Underlying the results in table 13 are the residence-based priority and the high respect DA has for priorities that enables it to eliminate justified envy. The first three

TABLE 13
ZONE QUALITY: WINNERS VERSUS LOSERS

	Change from BM to DA		Change from BM to TTC	
	Winners	Losers	Winners	Losers
All	7.83	7.78	7.80	7.80
Education < HS	7.72	7.67	7.69	7.68
Education = HS	7.79	7.77	7.79	7.79
Education > HS	7.90	7.88	7.89	7.91

TABLE 14
Cost of the Elimination of Justified Envy

	Assigned in Zone (%)			Assigned to Favorite (%)		
	BM	DA	TTC	BM	DA	TTC
All households	65.1	70.1	58.4	68.3	64.1	67.8
Favorite is out of zone	28.4	38.4	18.8	47.2	41.8	58.9

columns of table 14 show the fraction of households assigned to schools in their own school zones under alternative mechanisms among all households and among those whose favorite schools are out of their zones. The last three columns of table 14 show households' chances of being assigned to their favorite schools.

Result 3. DA assigns the largest fraction of households to in-zone schools, followed by BM and then TTC. In terms of enabling households to get out of their zones to attend their desired schools, the three mechanisms are ranked TTC > BM > DA.

Remark 5. Like most studies on school choice mechanisms, our cross-mechanism comparisons takes the priority structure as given.[71] These structures differ across cities, and they play an essential role. We leave it for future research to understand the trade-offs and social objectives underlying these different priority structures.

3. School Assignment

Table 15 presents changes in the characteristics of schools households are assigned to.[72] When BM is replaced by DA, households are assigned to schools with higher quality and higher fees at a shorter distance.[73] The low-education group sees the smallest increase in quality and reduction in distance but the largest increase in fees, which explains why the average welfare (utils) decreases the most for this group (table 11). When BM is replaced by TTC, households are assigned to schools with higher quality and higher fees at a longer distance. That is, BM leads to misallocation because people hide their true preferences and inefficiently apply for in-zone schools for which they have higher priorities while giving up higher-quality out-of-zone schools.

[71] See Kominers and Sönmez (2016) and Dur et al. (2013) for examples of theoretical studies on priority structures.

[72] Table A5 shows the characteristics of schools households are assigned to under BM.

[73] A nonzero average change in quality is possible because there are more school seats than students citywide.

TABLE 15
School Assignment

	Quality		Distance (100 m)		Fees (€)	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
From BM to DA:						
All	.04	.4	−.4	4.7	7.8	333.2
Education < HS	.02	.3	−.1	4.2	8.8	284.6
Education = HS	.05	.5	−.5	5.1	6.3	356.0
Education > HS	.05	.5	−.5	4.7	8.3	343.9
From BM to TTC:						
All	.05	.5	.6	5.5	12.4	427.0
Education < HS	.03	.4	.5	4.7	2.4	360.4
Education = HS	.06	.5	.7	5.9	17.1	461.1
Education > HS	.06	.5	.7	5.6	15.3	438.8

Result 4. Compared to TTC, both BM and DA inefficiently assign students to schools that are closer but of lower quality.

VIII. Conclusion

We have developed a model of households' choices of schools under BM and estimated the joint distribution of household preferences and their strategic types, using data before a drastic change in household-school priorities. The estimated model has been validated with data after this drastic change. We have developed an efficient method to fully solve household problems even when the choice set is large. This method is applicable to a broad class of choice mechanisms, which may expand the scope of empirical studies in this literature beyond what has been feasible with the traditional solution method.

We have quantified the welfare impacts of replacing the BM with its two alternatives, DA and TTC. A change from BM to DA decreases household welfare and exacerbates inequalities across residential zones. A change from BM to TTC increases welfare but does not affect cross-zone inequalities.

The methods developed in this paper and the main empirical findings are promising for future research. One particularly interesting extension is to incorporate households' residential choices into the framework of this paper. Individual households may relocate in order to take advantage of changes in school choice mechanisms and/or in residence-based priority structures. Such individual incentives will in turn affect the housing market. There is a large literature on the capitalization of school quality for housing prices, as reviewed by Black and Machin (2011) and Gibbons and Machin (2008).[74] An important yet challenging research project

[74] Ries and Somerville (2010) exploit changes in the catchment areas of public schools in Vancouver and find significant effects of school performance on housing prices. Epple

involves combining this literature and the framework proposed in our paper, in order to form a more comprehensive view of the equilibrium impacts of school choice mechanisms on households' choices of schools and residential areas, and on the housing market.

Appendix A

A1. Detailed Functional Forms

Household. $x_i = [x_{i1}, \ldots, x_{i5}]$; $x_{i1} = I(\text{edu}_i < \text{HS})$, $x_{i2} = I(\text{edu}_i = \text{HS})$, $x_{i3} = I(\text{edu}_i > \text{HS})$, $x_{i4} = I(\text{single parent}_i = 1)$, $x_{i5} =$ sibling's school (0 if outside school, $\in \{1, \ldots J\}$ if nonprivate school, -9 if no sibling).

School. $w_j = [w_{j1}, w_{j2}, w_{j3}, w_{j4}]$; w_{j1} represents school quality, w_{j2} tuition, and w_{j3} capacity; $w_{j4} = 1$ if semipublic, 0 otherwise.

Zone. N_z is the number of schools in zone z, and $\bar{q}_z$ the average school quality in z.

A1.1. Utility

Preference heterogeneity is captured via three channels: (1) general school values relative to the outside option vary with x; (2) trade-offs among distance, quality, tuition, and unobserved school characteristics vary with education; and (3) idiosyncratic tastes. Define $g^*(\cdot)$ and $C(\cdot)$ such that $U(w_j, x_i, d_{ij}, \zeta_j) = g^*(w_j, x_i, \zeta_j) - C(d_{ij})$,

$$\begin{aligned} C(d_{ij}) &= \left(d_{ij} + c_1 d_{ij}^2 + c_2 I(d_{ij} > 5) + c_3 I(d_{ij} > 10)\right), \\ g^*(w_j, x_i, \zeta_j) &= \tau_1 x_{i4} + \tau_2 (I(x_{i5} = j) - I(x_{i5} = 0)) \\ &\quad + \sum_{m=1}^{3} x_{im}\left(\delta_{0m} + \delta_{1m}\zeta_j\right) + w_{j1}\left(\sum_{m=1}^{3} a_m x_{im}\right) + w_{j2}\left(\sum_{m=1}^{3} a_{3+m} x_{im}\right) \\ &\quad + \alpha_7 w_{j3} + \alpha_8 w_{j3}^2 + \alpha_9 w_{j1}^2 + w_{j1}\left(\sum_{m=1}^{3} a_{9+m} x_{im}\right). \end{aligned}$$

The last two rows are education-specific preference for w_j and ζ_j, with the form of

$$\sum_e \left(\delta_{0e} + \delta_{1e}\zeta_j + w_j \alpha_e\right) I(\text{edu}_i = e).$$

A1.2. Type Distribution

$$\lambda(x_i, l_i) = \lambda(x_i, z_{l_i}) = \frac{\exp(\beta_0 + \sum_{m=1}^{4} \beta_m x_{im} + \beta_5 I(x_{i5} \geq 0) + \beta_6 N_{z_{l_i}} + \beta_7 \bar{q}_{z_{l_i}})}{1 + \exp(\beta_0 + \sum_{m=1}^{4} \beta_m x_{im} + \beta_5 I(x_{i5} \geq 0) + \beta_6 N_{z_{l_i}} + \beta_7 \bar{q}_{z_{l_i}})}.$$

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and Romano (2003) conjecture that school choice systems can eliminate the capitalization of school quality on the housing market. Machin and Salvanes (2016) exploit policy reforms in Oslo that allowed students to attend schools without having to live in the school's catchment area, and find a significant decrease in the correlation between a school's quality and housing prices.

A2. Policy Evaluation: The 2007 Reform

We simulate the outcomes for the 2007 applicants had they lived under the 2006 regime, taking as given the 2006 admission probabilities. The results can be interpreted in two ways. (1) They are at the individual level; that is, "What would have happened to a 2007 applicant had she applied in 2006?" (2) Assuming that the 2006 and 2007 cohorts are two i.i.d. random samples from the same distribution, the results tell us what would have happened to all 2007 households without the reform if they had played the same equilibrium as the 2006 cohort. Table A6 shows that 17% of households gained and 7% of them lost from the reform, with more nonstrategic households affected. Overall, the gain from the reform was equivalent to €1,430.

A3. Additional Tables

As mentioned in section III, there can be multiple lists that are payoff equivalent and imply the same allocation results. All these lists have identical ordered elements that are allocation relevant, which is what our model can explain. For example, consider a list of length four, the third element of which was a leftover school. Our model is designed to replicate the first three elements of that list, not how many schools would be listed beyond that point. Table A1 presents the model fit for the length of the allocation-relevant part of household application lists.

TABLE A1
Model Fit: Relevant List Length (%)

	2006		2007	
List Length	Data	Model	Data	Model
1	85.8	83.1	86.1	83.2
2	11.5	14.5	11.7	10.3
≥3	2.7	3.4	2.2	6.5

TABLE A2
Model Fit: Assignment Round (%)

	Education < HS		Education = HS		Education > HS		Single Parents	
Round	Data	Model	Data	Model	Data	Model	Data	Model
2006:								
1	93.2	91.8	92.0	91.0	93.7	91.2	94.0	93.4
2	2.7	2.8	3.5	4.0	2.3	4.8	1.9	2.8
Unassigned	2.5	4.2	3.0	3.9	2.7	3.3	2.7	3.6
2007:								
1	91.0	92.0	90.6	91.8	93.7	93.7	91.3	92.4
2	3.7	3.8	3.2	3.8	2.2	3.1	2.9	3.1
Unassigned	3.5	3.0	4.0	4.0	2.5	2.8	3.3	3.1

TABLE A3
MODEL FIT: ENROLLMENT IN PUBLIC SYSTEM

	2006		2007	
	Data	Model	Data	Model
Education < HS	96.9	96.2	96.4	96.2
Education = HS	97.1	97.0	98.4	97.0
Education > HS	96.3	96.4	97.8	96.7
Single parents	96.1	95.5	97.0	95.8

TABLE A4
MODEL FIT: TOP-LISTED SCHOOLS

	QUALITY		DISTANCE (100 m)		TUITION (€100)	
	Data	Model	Data	Model	Data	Model
2006:						
Education < HS	7.6	7.6	5.2	6.0	5.4	5.7
Education = HS	7.9	7.9	7.0	7.1	8.1	8.3
Education > HS	8.2	8.2	8.7	8.2	9.9	9.6
Single parents	8.0	8.0	8.1	8.0	8.6	8.6
2007:						
Education < HS	7.5	7.6	5.2	5.9	5.3	5.4
Education = HS	8.0	7.9	6.3	6.8	8.2	8.0
Education > HS	8.2	8.2	7.8	8.2	9.7	9.5
Single parents	8.0	8.0	6.8	7.2	8.2	8.0

TABLE A5
SCHOOL ASSIGNMENT: BM

	Quality	Distance (100 m)	Fees (€100)
All	7.8 (.7)	7.3 (7.9)	7.7 (7.4)
Education < HS	7.4 (.7)	6.6 (6.9)	5.2 (6.2)
Education = HS	7.8 (.6)	7.1 (7.2)	7.7 (7.1)
Education > HS	8.0 (.7)	8.1 (8.3)	9.3 (8.0)

NOTE.—Standard deviations are in parentheses.

TABLE A6
IMPACT OF THE 2007 REFORM

	Winners (%)	Losers (%)	ΔUtils	Δ€100
All	16.7	6.7	6.7 (29.5)	14.3 (68.4)
Strategic	16.6	6.5	6.6 (28.7)	14.1 (65.9)
Nonstrategic	20.8	11.9	8.2 (43.8)	18.9 (108.4)
Education < HS	14.7	7.0	4.9 (25.7)	5.0 (26.0)
Education = HS	18.1	7.7	7.1 (31.0)	22.9 (99.8)
Education > HS	17.1	5.9	7.6 (30.6)	13.7 (55.2)

NOTE.—This table compares the welfare of a 2007 household under the 2007 regime with its would-be welfare under the 2006 regime. Winners have higher welfare under the 2007 regime. Standard deviations are in parentheses.

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