Strategic Monetary Policy in Open Economies

Louphou Coulibaly
Federal Reserve Bank of New York
University of Wisconsin-Madison and NBER

Charles Engel
University of Wisconsin-Madison
NBER and CEPR

September 17, 2026

Preliminary version

Abstract

We examine optimal non-cooperative monetary policy in a two-country New Keynesian setting with complete financial markets and pricing to market for goods. In contrast to the case in which the law of one price holds for goods, currency misalignments are an important consideration even for self-oriented policymakers. Financial markets are critical in the analysis. Under "full" commitment, the optimal policy considers the effects on asset values and hence wealth distribution across countries. Optimal policy under full commitment differs from the "partial" commitment case in which the policy commitment is not revealed until after asset markets clear. Surprisingly, the optimal targeting rule under strategic policy with full commitment is the same as under cooperative policy, and we show that there are no gains from cooperation in this setting. Although a policy chosen under partial commitment may be desirable from a country's perspective, the resulting Nash equilibrium yields lower welfare.

Keywords: Strategic games, monetary policy; financial markets; currency misalignment; pricing-to-market
JEL Classifications: F41, F44

1 Introduction

In large open economies, there are spillovers from one country's policy decisions to other countries and regions. For example, Federal Reserve policy influences the European economy through its effects on aggregate demand and exchange rates, and conversely, European Central Bank choices impact the U.S. While central bankers from around the world meet and consult often, their policy mandates remain self-oriented. They aim to maximize welfare of their own residents, and do not consider policy as a joint decision process in which Foreign central bankers have a say. In this paper, we examine and characterize the key elements of strategic monetary policy in a two-country New Keynesian setting.

There are two significant dimensions to this problem that must be addressed in a realistic framework. First, we consider *full commitment* of monetary policy that takes into account the feedback from the policy rule to current asset prices and portfolios. Monetary policy therefore affects the prices at which these assets are traded and, through them, the distribution of wealth across countries. Under *partial commitment* - the implicit assumption in most of the literature - the price discovery process in asset trade takes place before the monetary policymakers reveal their objectives and policy rules. The central bank subsequently commits to a policy plan, but takes the asset positions and the associated distribution of wealth as given. This distinction is consequential for optimal monetary policy. We show that the resulting loss functions, optimal targeting rules, and gains from international monetary cooperation depend critically on which form of commitment is assumed. (We consider a setting of complete asset markets. Specifically, a complete set of state-contingent nominal bonds is traded.[1])

The second essential element is that the law of one price does not hold for goods traded across countries. That is, firms can set different prices for their good when it is sold at Home and abroad. Prices in the Home country are set in Home currency and adjust slowly, and prices in the Foreign country are set in Foreign currency, also with slow adjustment (this is referred to as local-currency pricing, or LCP). But as exchange rates adjust freely in response to exogenous disturbances, *currency misalignments*, in which the price of the same good changes in one country relative to the other as the exchange rate moves, may arise. The analysis of optimal policy differs significantly from the special case which assumes the law of one price. In that case, the optimal policy in canonical models, even when it is not cooperative, can target producer-price inflation and the output gap in the policymaker's own country, with no separate consideration for the exchange rate. The exchange rate is never at a distorted level, because under optimal inward-looking policies, the exchange rate freely adjusts to deliver efficient terms of trade movements. However, under LCP, the exchange rate does not directly influence relative prices of Home and Foreign goods that households face, since both are priced in their own local currency and do not fully adjust to shocks in the short run.

Our analysis follows the tradition of Woodford (2003), in that we consider optimal monetary policy under commitment in a linear-quadratic framework, in which policymakers maximize the utility of their residents. That utility depends on consumption and leisure, but we recast the objec-

[1] Our aim is to abstract from the separate issues that arise from imperfections in asset-market trade.

tive as a loss function whose arguments reflect the distortions imposed on the economy by sticky prices. In the familiar closed-economy analysis, these are expressed as losses from inflation and the output gap. In the open economy, they may also depend on exchange-rate misalignment, as we will show. This representation of the objective function has proven helpful for four reasons. First, it focuses attention on the economic inefficiencies that monetary policy can address. Second, it easily allows us to write the optimal policy as a *targeting rule* that shows what parameters influence the optimal tradeoff for central bank policy. Third, it yields algebraic expressions of the objectives and optimal plans for policy, so that the economic intuition of the optimal strategies is clear. And fourth, the policy recommendations can be expressed in the language of real-world policymakers and their concerns about inflation, the output gap and currency misalignments.

We begin by asking how the loss function of a self-oriented policymaker differs from the cooperative welfare criterion. Existing works (e.g., Engel, 2011, Corsetti et al., 2010) have shown that, under LCP, cooperative policy responds to deviations from the law of one price in addition to inflation and output gaps. These currency misalignments distort the distribution of consumption across countries and are thus a concern for a cooperative policymaker. But it is less obvious why a national central bank that cares only about the welfare of its own residents should seek to stabilize international consumption distribution. Does currency misalignment remain a target under non-cooperative policy? And do national policymakers face objectives that do not arise under cooperation?

The answers depend importantly on the *form of commitment*. Under full commitment, the national policymaker's loss function depends on domestic consumer-price inflation, the output gaps of both countries, currency misalignment, and a term reflecting the covariance between exchange-rate movements and world output. Thus, even though the central bank cares only about the consumption and labor of its own residents, it has an independent concern with deviations of the exchange rate from the level associated with the law of one price. This distortion operates through both goods and financial markets. When the currency moves away from its efficient level—whether it is overly appreciated or depreciated—it affects domestic consumption. Under LCP, these effects are not simply the conventional expenditure-switching effects associated with movements in relative goods prices. Complete financial markets imply that exchange-rate movements also affect the distribution of consumption through asset payoffs. Distorted fluctuations in pricing to market can thus generate excessive consumption volatility, which lowers welfare for risk-averse consumers.[2]

In addition to these concerns, the monetary authority also cares about when exchange-rate movements occur. Consider, for example, a global recession in which the world output gap is negative. A Home policymaker would like, other things equal, to redistribute wealth toward its households and away from Foreign households. A real depreciation of Home currency in such states can achieve this redistribution. Of course, because the national policymaker does not internalize Foreign welfare, it would like to raise Home consumption and reduce Home labor input at the expense of foreigners.

However, when policy is made under commitment, to a first order, policy cannot influence these real quantities on average. The relevant incentives arise through second-order effects that operate

[2] As is standard, the desirability of exchange-rate volatility depends on whether the relative risk aversion is greater than one. With low relative risk aversion (high intertemporal elasticity of substitution), households gain from substituting toward cheap states.

through firms' pricing decisions. When the demand for a firm's product becomes more volatile, it raises its *precautionary mark-up*. This arises because demand shifts away from the high-price firms towards the low-price firms, but there is a larger cost of having too low a price on average since the demand elasticity for the product is larger than one. When the Home firm sets higher prices for export due to the precautionary markup channel, the Home country benefits, but the reverse is true for the Foreign firms' choices of prices for the Home good. Once goods-market clearing and the Phillips curves governing optimal price setting are taken into account, the national loss function can be expressed in terms of consumer-price inflation (that is, including inflation in Home currency of imported goods), domestic and Foreign output distortions, and currency-misalignment terms.

The role of asset markets is important to understand how these incentives depend on the type of commitment. Under full commitment, asset prices adjust to the announced policy. Therefore, a policy designed to shift resources toward Home households also changes the prices at which Home and Foreign households purchase claims to those resources. The Home policymaker must therefore evaluate the prospective redistribution together with the endogenous adjustment in asset values and relative wealth (under partial commitment, this adjustment does not occur). Hence, the kind of commitment we assume changes the Home policymaker's welfare criterion and, in turn, optimal policy. Much of the previous literature simplifies the non-cooperative policy game in a way that effectively reduces the problem to maximizing period-by-period welfare. Once the policymaker recognizes that an ex ante commitment affects asset prices, however, the strategic problem necessarily involves the entire lifetime welfare of households. Thus, the value of commitment extends beyond the familiar benefit of influencing expectations of future inflation and output: in an open economy, it also affects the initial valuation and distribution of international wealth.

We derive a simple targeting rule in each country under full commitment, which can be shown to depend on the deviations of the consumer price level from the inherited price level, the output gap in both countries, and the currency misalignment term. In the Nash equilibrium, where each country takes the policy of the other country as given, the optimal targeting rules are the same as under cooperative policy. This implies that there are no gains from cooperation in policy setting compared to strategic policymaking. It is worth noting that our finding fundamentally differs from similar results obtained when there are no deviations from the law of one price and elasticities of substitution are set to one (see for example, Clarida et al., 2002). Indeed, though globally optimal policies are achieved through self-oriented objectives, the targets of policy are different.[3] Moreover, the conclusion that the targeting rules are the same as the cooperative case emerges under the optimal non-cooperative game (full commitment) and holds away from unitary elasticities assumption.

This latter conclusion requires comparing policy under cooperation and non-cooperation. We are looking at second-order approximations around a non-stochastic steady state, but for this comparison to be valid, it must be the same steady state in the two cases. In fact, we assume the steady state is the globally efficient one. This leads to two complications in the analysis of the non-cooperative model. The first is minor. As in much of the New Keynesian literature, we assume that there are

[3] In the law of one price with unitary elasticities case, policy targets the producer price level and domestic output gap. Under LCP with full commitment, the targets are the consumer price level, global output gap, and exchange rates.

a set of constant taxes and subsidies that would lead the economy to the efficient outcome if prices were flexible.[4] The second complication is that the globally efficient steady state is not efficient from the standpoint of the individual country policymaker. That policymaker would prefer an equilibrium in which the terms of trade are distorted in its favor, as in the optimal tariff literature. Hence, we cannot take advantage of the usually much simpler algebra that arises when the loss function is derived in terms of deviations from the steady state that is efficient from the policymaker's standpoint. Instead, we make use of Benigno and Woodford (2005) method of approximation that is valid even when the steady state is not efficient. While this approach is very algebra intensive, the loss functions and targeting rules we derive still turn out to be simple and easily interpretable.

Finally, Section 5.2 elaborates on three "takeaways" for real-world monetary policy. The first, of course, is the message from the loss function and targeting rules: *currency misalignments do matter*. Central bankers sometimes state that exchange rates matter only to the extent that they affect their ultimate targets for inflation and output. In the open economy, however, the exchange rate matters beyond these objectives because it affects the distribution of welfare between Home and Foreign residents through asset prices. In corresponding closed-economy models, distorted asset prices are not a separate concern if the underlying distortion arises from nominal price stickiness. Once the distortion arising from inflation is corrected, asset prices will be at efficient levels (abstracting from asset market imperfections, such as collateral constraints, costs of trade, and asset price bubbles). The open economy is different because asset prices influence the relative welfare of Home households compared to the rest of the world.

Second, *communication of central bank policy matters*. It is natural to assume that the policymaker must communicate its policy in order to convey to markets the policy rule it intends to commit to. If the commitment is made and announced only after asset markets trade without the benefit of knowing the intended policy rule, we are in the realm of "partial" commitment. In this case, at the time of asset market trade, markets recognize that policy is discretionary in the sense that it will take the asset prices as given when policy is set. We assume markets believe policy will be set optimally under commitment at that stage, but that is still a partial commitment ex ante — that is, it is not full commitment at the time asset trade takes place. Our result that optimal strategic policy delivers the same outcome as optimal cooperative policy depends on full commitment. In general, the outcome is worse under partial commitment, so there is value to announcing and committing to policy rather than waiting to see the outcome of asset market trades. For empirically-relevant values of elasticities of substitution, a central bank that announces and commits to its policy after assets are traded is more dovish than one under full commitment: for given output gaps and exchange rates, it accommodates more its own cost-push shocks with a higher consumer price level.

Third, our result also helps *clarify the interpretation of exchange-rate assessments* and policy recommendations by international institutions such as the International Monetary Fund. The IMF often seeks to estimate an "equilibrium" exchange rate and evaluates whether a country's currency is undervalued or overvalued. But what does "equilibrium" mean? In particular, here we can address

[4] In the closed economy, and the cooperative open economy, the assumption is that there are subsidies to monopolies that eliminate the distortion arising from underproduction when firms have market power. Here we assume there is a global fiscal authority that sets these constant subsidy rates to achieve the efficient steady state under flexible prices.

the question of whether the exchange rate that policy should aim for is the one that is desirable from the standpoint of global efficiency, or the one that the country prefers given that the rest of the world is also not setting monetary policy cooperatively? We find that these two choices are the same under full commitment. In the model, the optimal exchange rate is the one that eliminates law-of-one-price deviations, though in practice such an exchange rate is surely not easy to calculate.

Literature review. Our paper relates to several strands of the literature on optimal monetary policy in open economies: monetary policy under commitment, welfare-based targeting rules, deviations from the law of one price, international asset-market structure, and the gains from monetary cooperation. We emphasize the difference between "full commitment" monetary policy and "partial commitment", which has not yet been fully addressed by the existing literature, and characterize, in closed form, the objectives and optimal policies of central banks, rather than relying on numerical solutions.

Early open-economy New Keynesian analyses include Obstfeld and Rogoff (1995), which studies optimal responses to unexpected shocks in an otherwise non-stochastic environment. Our focus instead is on a dynamic stochastic setting in which commitment has an explicitly intertemporal dimension. In addition, we provide targeting rules, which characterize the optimal tradeoffs among welfare-relevant objectives, rather than instrument-based rules specifying how the nominal interest rate responds to observable variables. This distinguishes our approach from quantitative analyses such as Fujiwara and Wang (2017), which studies numerically non-cooperative policy under partial commitment and thus does not characterize the optimal targeting rules.

A large literature studies the gains from international monetary cooperation. Clarida et al. (2002) provides an important benchmark under producer-currency pricing (PCP), finding that nationally oriented policies targeting domestic producer-price inflation and the output gap generate no gains from cooperation. Benigno and Benigno (2003) show that this result depends on preference assumptions: with more general CES preferences, gains from cooperation can arise. Benigno and Benigno (2006) show how appropriate national targeting rules can implement the cooperative allocation. Obstfeld and Rogoff (2002) finds small gains from cooperation in a static two-country model, while Pappa (2004) reaches a similar quantitative conclusion in a dynamic stochastic PCP model.

Our analysis of the forms of commitment is directly related to the work of Devereux and Engel (2003); Sutherland (2004); Senay and Sutherland (2013); Engel (2016). Devereux and Engel (2003) compare full commitment and partial commitment, and find that both forms of commitment deliver the same results due to the special features of their model, including one-period-ahead price setting and identical Cobb–Douglas preferences across countries. Sutherland (2004) and Senay and Sutherland (2013) study the interaction between monetary policy and state-contingent asset trade. The latter considers a two-period model with equity trade and shows, through numerical evaluation of different interest-rate-based targeting rules, that the optimal rule depends on the set of assets available. Engel (2016) considers non-cooperative policy under both forms of commitment in a two-period (real) complete-markets model with flexible prices. Our analysis brings this distinction into a dynamic model with staggered price adjustment, where full and partial commitment generally lead to different welfare criteria and optimal welfare-based targeting rules.

Our analysis is particularly related to work under local-currency pricing. Betts and Devereux (2000a,b) study monetary-policy responses under LCP in non-stochastic environments. Devereux and Engel (2003) derive cooperative and non-cooperative optimal policies under both LCP and PCP with complete markets and find that there are no gains from cooperation. Sutherland (2005) extends their framework to partial exchange-rate pass-through. Engel (2011) derives cooperative optimal monetary policy under LCP with complete markets in a fully dynamic stochastic model with staggered price adjustment; Corsetti et al. (2010) extends this analysis to some forms of incomplete markets,[5] and Gong et al. (2023) introduces trade in intermediate goods. Fujiwara and Wang (2017) studies non-cooperative policy in a closely related environment, but assumes partial commitment and focuses on numerical policy analysis.

Our complete-markets assumption also connects the paper to the broader literature on international financial structure and monetary policy. Kollmann (2002) and Benigno (2009) study optimal policy with trade in non-state-contingent bonds under PCP, while Bengui and Coulibaly (2026) examines the interaction between current-account imbalances and optimal monetary policy.[6] With incomplete markets, additional policy instruments may become relevant: Prasad (2018), Basu et al. (2020), Fanelli and Straub (2021) and Itskhoki and Mukhin (2023) examine Foreign-exchange intervention and related policies, while Ottonello (2021) and Coulibaly (2023) study environments with financial constraints. By assuming complete markets, we abstract from these additional distortions and isolate the role of monetary policy in affecting asset prices and international wealth distribution.

There is also a large literature on optimal monetary policy in small open economies, including Gali and Monacelli (2005), Schmitt-Grohé and Uribe (2007), Faia and Monacelli (2008), De Paoli (2009) and Guerrieri et al. (2025). Because these models abstract from strategic interaction between two large countries, the questions of cooperation and endogenous international risk sharing that are central here are not directly comparable. Finally, recent work studies monetary policy under dominant-currency pricing (DCP). Casas et al. (2017) derives a welfare-based policy objective under DCP, while Egorov and Mukhin (2023) studies cooperative and non-cooperative monetary policy in a world with one dominant-currency country and a continuum of small economies. Relative to these strands of the literature, our paper combines LCP, complete international asset markets, staggered price adjustment, and strategic monetary policymaking in order to characterize analytically how the form of commitment affects national welfare objectives and optimal targeting rules. The distinction between full and partial commitment is crucial: it determines whether the Nash equilibrium coincides with the cooperative allocation.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 derives the welfare-based loss function and the optimal targeting rules under full commitment, and shows that the Nash equilibrium coincides with the cooperative solution. Section 4 considers the case of partial commitment. Section 5.2 draws out the implications for policy, and Section 6 concludes.

[5] Corsetti et al. (2023) considers a general degree of exchange-rate pass-through.

[6] Devereux and Sutherland (2008), Bianchi and Coulibaly (2024) and Bodenstein et al. (2025) also study monetary cooperation under alternative asset-market structures.

2 Model

The world economy consists of two equally sized countries, Home and Foreign. In each country, a continuum of goods is produced, with each good supplied by a monopolistic producer. The economy features trade in a complete set of nominally denominated contingent claims. Both monetary policy and asset trades are determined at the beginning of time, before any shock is realized. When useful, we refer to that stage as $t = -1$ and denote the associated expectation by $\mathbb{E}_{-1}$.

2.1 Households

Preferences. The Home country is populated by a continuum of households indexed by $j \in [0, 1]$. A household j has an infinite life horizon and preferences represented by

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \rho(z^t) \left[\frac{1}{1-\sigma} \left(C_{j,t}(z^t) \right)^{1-\sigma} - N_{j,t}(z^t) \right] \tag{1}$$

where z^t denotes the history of shocks through date t and $\rho(z^t)$ its probability, $\beta \in (0, 1)$ is the discount factor, $C_{j,t}$ is consumption and $N_{j,t}$ is labor supply and σ is the inverse of the intertemporal elasticity of substitution (IES). The consumption good is a CES composite of Home and Foreign goods:

$$C_{j,t}(z^t) = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{j,H,t}(z^t) \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{j,F,t}(z^t) \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

Here $C_{j,H,t}(z^t)$ and $C_{j,F,t}(z^t)$ are CES aggregates of Home and Foreign varieties, respectively, with elasticity of substitution across varieties $\varepsilon > 1$. The parameter $\eta > 0$ is the elasticity of substitution between Home and Foreign goods, while the parameter $\alpha \in (0, \frac{1}{2})$ captures the share of imports in aggregate consumption with $1 - 2\alpha$ measuring the degree of home bias.

Budget constraints. Households can trade a complete set of state-contingent claims, denominated without loss of generality in Home currency. The household's flow budget constraint is given by

$$\begin{aligned} P_t(z^t) C_{j,t}(z^t) + \sum_{z_{t+1}} \mathcal{R}(z_{t+1}|z^t) D_{j,H,t+1}(z_{t+1}, z^t) + \frac{\bar{D}_{j,t+1}(z^t)}{1 + i_t(z^t)} \\ = W_{j,t}(z^t) N_{j,t}(z^t) + \Phi_t(z^t) + T_t(z^t) + D_{j,H,t}(z^t) + \bar{D}_{j,t}(z^t) \end{aligned}$$

where P_t is the consumer price index (CPI), $W_{j,t}$ is household j's nominal wage, Φ_t denotes profits, and T_t denotes lump-sum transfers. The security price $\mathcal{R}(z_{t+1}|z^t)$ is the price of an Arrow-Debreu claim paying one unit of Home currency in state z_{t+1}, and $D_{j,H,t+1}(z_{t+1}, z^t)$ denotes the corresponding holdings. Households can also trade a risk-free nominal bond $\bar{D}_{j,t+1}(z^t)$ at price $1/(1 + i_t(z^t))$, where $i_t(z^t)$ is set by the central bank in Home. The risk-free bond is redundant given the complete set of contingent claims, but introducing it allows us to describe the central bank's instrument explicitly. Households start with zero initial assets and satisfy a no-Ponzi condition, so the sequence of flow budget constraints can be rewritten as

$$\sum_{t=0}^{\infty}\sum_{z^t}\mathcal{R}_t^0(z^t)P_t(z^t)C_{j,t}(z^t)=\sum_{t=0}^{\infty}\sum_{z^t}\mathcal{R}_t^0(z^t)\left[W_{j,t}(z^t)N_{j,t}(z^t)+\Phi_t(z^t)+T_t(z^t)\right] \tag{3}$$

Labor supply. Each household j is a monopolistically competitive supplier of a differentiated labor service and faces a CES demand schedule: $N_{j,t}(z^t)=\left(W_{j,t}(z^t)/W_t(z^t)\right)^{-\varepsilon_t^w}N_t(z^t)$, where ε_t^w is the time-varying elasticity of substitution among labor varieties, W_t is aggregate wage index and N_t is aggregate employment. Optimal wage setting results in a wage markup over the marginal disutility of working per unit of consumption,

$$\frac{W_{j,t}(z^t)}{P_t(z^t)}=\frac{\varepsilon_t^w}{\varepsilon_t^w-1}\left(C_{j,t}(z^t)\right)^{\sigma} \tag{4}$$

where $\frac{\varepsilon_t^w}{\varepsilon_t^w-1}$ is the wage markup. We denote by u_t the log deviation of the Home wage markup from its steady-state value (and u_t^* its counterpart in Foreign). These cost-push shocks are the only source of uncertainty in the model and will generate a meaningful monetary-policy tradeoff, as discussed in Clarida et al. (2002) and Engel (2011) among others. As is standard, we assume that the shocks are symmetric across Home and Foreign, that is $\{(u_t,u_t^*)\}$ and $\{(u_t^*,u_t)\}$ have the same distribution, with unconditional mean zero, so that the two countries are ex ante identical.

Foreign. Foreign households face an environment symmetric to that of Home households with Foreign variables indexed by asterisks. Households' budget constraint in Foreign is given by

$$\sum_{t=0}^{\infty}\sum_{z^t}\mathcal{R}_t^0(z^t)\mathcal{E}_t(z^t)P_t^*(z^t)C_{j,t}^*(z^t)=\sum_{t=0}^{\infty}\sum_{z^t}\mathcal{R}_t^0(z^t)\mathcal{E}_t(z^t)\left[W_{j,t}^*(z^t)N_{j,t}^*(z^t)+\Phi_t^*(z^t)+T_t^*(z^t)\right] \tag{5}$$

where $\mathcal{E}_t$ is the nominal exchange rate, defined as the domestic currency price of one unit of Foreign currency. An increase in $\mathcal{E}_t$ corresponds to a depreciation of the Home currency.

Consumption decisions. Households trade state contingent asset positions before the realization of any state and choose consumption plans to maximize (1) subject to (3). The first-order conditions in Home and in Foreign are respectively given by:

$$\lambda=\frac{\beta^t\rho(z^t)}{\mathcal{R}_t^0(z^t)}\frac{\left(C_t(z^t)\right)^{-\sigma}}{P_t(z^t)}\quad\text{and}\quad\lambda^*=\frac{\beta^t\rho(z^t)}{\mathcal{R}_t^0(z^t)}\frac{\left(C_t^*(z^t)\right)^{-\sigma}}{\mathcal{E}_t(z^t)P_t^*(z^t)} \tag{6}$$

where λ is the multiplier on (3) and represents the shadow value of a unit of Home-currency wealth to Home households and λ^* is the counterpart in Foreign. Combining the two conditions yields the international risk-sharing condition,

$$\left(\frac{C_t(z^t)}{C_t^*(z^t)}\right)^{\sigma}=\frac{\lambda^*}{\lambda}\cdot\frac{\mathcal{E}_t(z^t)P_t^*(z^t)}{P_t(z^t)} \tag{7}$$

Equation (7) highlights two determinants of relative consumption. The first is the real exchange rate, $\mathcal{E}_t P_t^* / P_t$, which measures the relative price of the two consumption baskets in a common currency. The second is λ^*/λ, which captures the relative wealth of the two countries. This term is central to our analysis because, as we will make clear below, depending on the form of commitment, monetary policy can influence the price of state-contingent claims and hence the distribution of wealth across countries.

Prices and currency misalignment. In addition to their consumption-savings decisions, households also choose how to allocate expenditures between Home and Foreign goods. This yields $C_{j,H,t} = (1-\alpha)(P_{H,t}/P_t)^{-\eta} C_t$ and $C_{j,F,t} = \alpha (P_{F,t}/P_t)^{-\eta} C_t$, where $P_{H,t}$ is the price of the domestically produced good and $P_{F,t}$ is the price of the imported good. The Home CPI is

$$P_t = \left[(1-\alpha) \left(P_{H,t}\right)^{1-\eta} + \alpha \left(P_{F,t}\right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

The Foreign CPI is defined symmetrically. Let $\mathcal{S}_t \equiv P_{F,t}/P_{H,t}$ and $\mathcal{S}_t^* \equiv P_{H,t}^*/P_{F,t}^*$ denote the relative price of imported to domestically produced goods in Home and Foreign, respectively, and let $\mathcal{Z}_t \equiv (\mathcal{S}_t \mathcal{S}_t^*)^{\frac{1}{2}}$ denote their geometric average. The real exchange rate is

$$\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$$

Under local-currency pricing, the nominal exchange rate can move without an immediate corresponding adjustment in exporters' local-currency prices. Following Engel (2011), we summarize the resulting relative deviations from the law of one price by the currency-misalignment term

$$\mathcal{M}_t \equiv \left[\frac{\mathcal{E}_t P_{H,t}^*}{P_{H,t}} \left(\frac{P_{F,t}/\mathcal{E}_t}{P_{F,t}^*} \right)^{-1} \right]^{\frac{1}{2}} = \mathcal{E}_t \left(\frac{P_{H,t}^*}{P_{H,t}} \right)^{\frac{1}{2}} \left(\frac{P_{F,t}^*}{P_{F,t}} \right)^{\frac{1}{2}}$$

When the law of one price holds in both markets, there is no currency misalignment and $\mathcal{M}_t = 1$. Therefore, $m_t \equiv \log(\mathcal{M}_t) > 0$ signals that the Home currency is relatively weak (that is, it is overly depreciated) relative to the level at which the law of one price would hold.

2.2 Firms

Technology. Final goods are CES aggregates over a continuum of goods produced domestically, with an elasticity of substitution between varieties $\varepsilon > 1$. We describe the problem of Home firms. Home firms, indexed by $l \in [0,1]$, produce differentiated goods using a linear technology $Y_t(l) = N_t(l)$, where

$$N_t(l) \equiv \left(\int_0^1 N_{j,t}(l)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w}} \, dj \right)^{\frac{\varepsilon_t^w}{\varepsilon_t^w - 1}}$$

is a composite of labor services supplied by Home households. Foreign firms face the symmetric environment, with variables defined analogously and production function given by $Y_t^*(l) = N_t^*(l)$.

Price setting. Firms operate under monopolistic competition and engage in infrequent price setting à la Calvo (1983). Prices are set in the currency of the destination market. In any given period, each firm may reset its prices with probability $1-\zeta$, independent of the time elapsed since the last adjustment. As a result, a fraction $(1-\zeta)$ of firms are able to reset prices each period. Firms that can reoptimize choose prices to maximize expected discounted profits, subject to the demand for their own varieties. The optimal pricing decision is given by

$$\mathcal{P}_{H,t}(l) = \frac{\varepsilon}{(1+\tau)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega_{h,t+j} \cdot W_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega_{h,t+j}} \quad \text{and} \quad \mathcal{P}^*_{H,t}(l) = \frac{\varepsilon}{(1+\tau)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j} \cdot \frac{W_{t+j}}{\mathcal{E}_{t+j}}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j}}$$

where τ is a constant production subsidy, $\Omega_{h,t+j} > 0$ and $\Omega^*_{h,t+j} > 0$ are defined in (A.1) and (A.2). In the limiting case $\zeta = 0$ prices are flexible, the optimal price setting reduces to a markup over current nominal marginal cost and the law of one price holds $\mathcal{P}_{H,t}(l) = \mathcal{E}_t \mathcal{P}^*_{H,t}(l)$. The pricing environment is symmetric in Foreign.

2.3 Monetary policy

The monetary authority in each country sets the nominal interest rate on its domestic risk-free bond, and commits to the state-contingent plan $\{i_t\}_{t\geq 0}$ in Home (respectively, $\{i^*_t\}_{t\geq 0}$ in Foreign) before any state is realized. We consider two forms of commitment that differ only in whether that plan is announced before or after households trade state-contingent assets.

1. Under **"full" commitment**, each central bank announces its state-contingent policy plan before asset trade. Asset markets then open and households trade state-contingent claims taking the announced plans as given. Shocks are subsequently realized, households choose consumption and labor, and markets clear for $t \geq 0$.

2. Under **"partial" commitment**, households first trade state-contingent claims. After assets are traded, each central bank announces its state-contingent policy plan. Shocks are then realized, households choose consumption and labor, and markets clear for $t \geq 0$.

In both cases, monetary policy is committed before shocks are realized. The distinction is therefore not between commitment and discretion after shocks, but whether commitment occurs in a way that allows the effect of monetary policy to be incorporated into asset prices and the initial distribution of wealth. Thus, a central bank can be perfectly committed in the traditional sense and still be operating under partial commitment.

A simple interpretation of the difference between full and partial commitment is that, under full commitment, the central bank clearly communicates its rule and financial markets take positions knowing it; while under partial commitment markets must first guess (infer) it at the time assets are traded. The object through which the distinction operates is the relative marginal value of wealth, $\tilde{\Theta} \equiv \lambda^*/\lambda$ (in logs, $\tilde{\theta} \equiv \log \tilde{\Theta}$), in the risk-sharing condition (7). Under full commitment $\tilde{\theta}$ responds to the announced plan through asset-market clearing; under partial commitment it is predetermined when the plan is chosen. Everything that follows is a consequence of that single difference.

2.4 Equilibrium

Given policy plans in Home and Foreign $\{i_t, i_t^*\}_{t=0}^{\infty}$, an equilibrium is a collection of prices and quantities such that households and firms optimize and all markets clear at every date and state.

Goods market-clearing conditions in Home and Foreign are given by

$$Y_t = \underbrace{(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t}_{C_{H,t}} + \underbrace{\alpha\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*}_{C_{H,t}^*} \quad \text{and} \quad Y_t^* = \underbrace{(1-\alpha)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*}_{C_{F,t}^*} + \underbrace{\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t}_{C_{F,t}} \quad (8)$$

The left-hand side of each equation reflects the supply of domestic goods. On the right-hand side, the first term of each equation is the demand for domestically-produced goods by domestic households and the second term is the demand by Foreign households.

Wages in Home and Foreign adjust to ensure that the aggregate labor supplied by domestic households equals the aggregate demand for labor by domestic firms, that is $N_t = \int_0^1 N_t(l)dl$ and $N_t^* = \int_0^1 N_t^*(l)dl$. Finally, assets market clearing requires every state-contingent claim to be in zero net supply.

Because households in both countries start with zero assets, asset-market clearing implies that the Home country as a whole faces an intertemporal budget constraint that takes the following form

$$\sum_{t=0}^{\infty}\sum_{z^t}\mathcal{R}_t^0(z^t)\underbrace{\left[\mathcal{E}_t(z^t)P_{H,t}^*(z^t)C_{H,t}^*(z^t) - P_{F,t}(z^t)C_{F,t}(z^t)\right]}_{\text{net exports}} = 0 \quad (9)$$

where the left-hand side is the value of Home-produced goods bought by Foreign households net of the value of Foreign-produced goods bought by Home households, both measured in terms of date-0 Home currency and discounted at the equilibrium state prices $\mathcal{R}_t^0(z^t)$. Condition (9) thus states that the present value of the country's net exports must equal the country's initial net asset position (which equals zero). Thus, trade can be unbalanced at any given date and state but must be balanced in present value. Substituting the demand functions for $C_{H,t}^*$ and $C_{F,t}$ into (9) and using the households' first-order conditions (6), the intertemporal balanced-trade condition can be rearranged (see Appendix A.2 for details) as,

$$\tilde{\Theta} \equiv \frac{\lambda^*}{\lambda} = \frac{\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{P_{H,t}^*}{P_t^*}\right)^{1-\eta}(C_t^*)^{1-\sigma}}{\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{P_{F,t}(z^t)}{P_t(z^t)}\right)^{1-\eta}(C_t)^{1-\sigma}} \quad (10)$$

Equation (10) shows that relative wealth depends on the present value of Home and Foreign income evaluated at equilibrium state prices. It is important to note that when $\sigma = \eta = 1$, the relative-wealth ratio is equal to one and is independent of policies.

2.5 Efficient allocation and production subsidies

Efficient allocation. We conclude the model description by presenting the efficient allocation. In particular, we consider a benevolent world planner who maximizes the equally weighted sum of households' welfare in both countries subject to resource constraints. As shown in Appendix A.3, because productivity is constant and markup shocks are the only source of uncertainty, consumption, output, and hours worked are constant in the first-best allocation and given by $C_t^e = Y_t^e = N_t^e = 1$ and $C_t^{*e} = Y_t^{*e} = N_t^{*e} = 1$.[7]

Optimal subsidies. We assume that the production subsidies in Home and Foreign are chosen under cooperation as the ex-ante reaction rule below and eliminate the time-inconsistency problem of policies associated with the inefficiency of the steady-state from each domestic policymaker's perspective. They are given by

$$\log(1+\tau) = \log(1+\mu) + \frac{\tilde{\theta}+v}{2} \quad \text{and} \quad \log(1+\tau^*) = \log(1+\mu) - \frac{\tilde{\theta}+v}{2}, \tag{11}$$

where $\tilde{\theta} \equiv \log(\lambda^*/\lambda)$ is the (log) relative marginal value wealth, $1+\mu \equiv \frac{\varepsilon}{\varepsilon-1}\frac{\varepsilon^w}{\varepsilon^w-1}$ is the steady-state markup, and v collects the unconditional expectation of the date-0 second-order terms defined in Appendix B.1.[8] The Lemma below describes the optimal labor subsidy in any global equilibrium.

Lemma 1 (Optimal subsidy)**.** *In any competitive equilibrium, the production subsidies* (11) *reduce to*

$$\tau = \tau^* = \mu. \tag{12}$$

Proof. See Appendix B.1. □

The subsidies reduce to $\tau = \tau^* = \mu$ because countries are ex-ante symmetric and therefore in any competitive equilibrium in the global economy $\tilde{\theta} = 0$ and $v = 0$. Thus, the allocation around which we approximate is the globally efficient one. Taken together, (11) and (12) ensure the comparability of the non-cooperative and the cooperative problems.

This characterization of the optimal subsidy in (11) is important because the globally efficient steady state is *not* necessarily efficient from each domestic policymaker's perspective: each policymaker would prefer terms of trade distorted in its favor, as in the optimal-tariff literature. As Benigno and Woodford (2005) emphasize, when the steady state is distorted from the planner's point of view, a linear-quadratic problem written in deviations from that steady state may lead to spurious welfare approximation and, most importantly, the domestic policymaker would like to move the economy away from the steady state *even absent shocks*. The second-order term v in (11) removes exactly that incentive, so that each domestic policymaker would choose zero inflation in

[7] Note that this result stems from the fact that we abstract from productivity shocks, so that the markup shocks u_t and u_t^* are the only source of uncertainty. This simplifies the notation without altering any of the results.

[8] Throughout the strategic monetary-policy game, the subsidy is given by (11). Thus, in the absence of shocks, a policy deviation that generates a nonzero output gap or inflation induces a subsidy adjustment that eliminates the perceived benefit of that deviation.

the absence of shocks, while the steady-state allocation remains globally efficient in equilibrium. Lemma 1 is thus what ensures that the non-cooperative and cooperative problems are comparable.

2.6 Linearization

This section presents first-order approximations of the key equilibrium conditions around the efficient allocation. Because the production subsidies of Lemma 1 make the deterministic steady state coincide with the efficient allocation, the log deviation of a variable from its steady state is also its gap relative to the efficient allocation. Thus, $x_t \equiv \log X_t - \log \bar{X} = \log X_t$ denotes the log-deviation of X_t from its value in the efficient allocation.[9] As in Engel (2011), we define the world output gap $y_t^W = \frac{1}{2}(y_t + y_t^*)$, the relative output gap $y_t^R \equiv \frac{1}{2}(y_t - y_t^*)$ and the relative-price gap $s_t^R \equiv \frac{1}{2}(s_t - s_t^*)$.

The demand block. Linearizing the risk-sharing condition (7) yields

$$\sigma\left(c_t - c_t^*\right) = \tilde{\theta} + q_t, \quad \text{with } \; q_t = m_t + (1-2\alpha)s_t^R \tag{13}$$

We then combine (13) with the Home and Foreign goods-market clearing conditions (8) to obtain

$$c_t = y_t + \frac{\alpha}{\sigma}\left[\tilde{\theta} - \frac{D-1}{2\alpha}s_t^R - (s_t^R - m_t)\right] \quad \text{and} \quad c_t^* = y_t^* - \frac{\alpha}{\sigma}\left[\tilde{\theta} - \frac{D-1}{2\alpha}s_t^R - (s_t^R - m_t)\right] \tag{14}$$

where $D \equiv \sigma\eta - (1-2\alpha)^2(\sigma\eta - 1) > 0$.

Conditions (14) describe the wedge between what a country consumes and what it produces, and three forces determine it. First, for a given level of Home output, an improvement in Home's terms of trade, $m_t - s_t^R = (e_t + p_{H,t}^*) - p_{F,t} > 0$, raises Home consumption since a unit of Home exports now buys more imported goods. Under local-currency pricing, a terms of trade improvement can be achieved with a currency depreciation, $m_t > 0$, which raises the Home-currency value of Home exports while leaving the Home-currency price of imports unchanged.

Second, holding the terms of trade fixed, a fall in the relative price of imports s_t^R makes imported goods cheaper relative to Home-produced goods, which in turn sets two opposing forces in motion. On the one hand, cheaper imports raise the amount of Foreign goods that a given level of Home output can buy, raising aggregate consumption with elasticity given by the intratemporal elasticity of substitution, η. On the other hand, with the terms of trade held fixed, a lower s_t^R appreciates the Home currency in real terms and, through the risk-sharing condition (13), redistributes consumption away from Home with elasticity given by the IES, $\frac{1}{\sigma}$. The net effect is given by $\frac{D-1}{2\sigma} = 2\alpha(1-\alpha)\left(\eta - \frac{1}{\sigma}\right)$, so which effect dominates depends on whether Home and Foreign goods are Hicksian substitutes $\sigma\eta > 1$ or Hicksian complements $\sigma\eta < 1$.

Third, Home consumption is higher when Home households are relatively wealthy, that is, when $\tilde{\theta}$ is large. As we show in Appendix B.3, to a first order $\tilde{\theta} = 0$ under both forms of commitment,

[9] In the presence of productivity shocks, the efficient allocation would be time-varying and would differ from the deterministic steady state. In that case, the relevant gap would be the deviation of X_t from its efficient level, and all of our findings would remain unchanged.

because its first-order component is a present value of mean-zero terms for any admissible policy plan. Combining the risk-sharing condition (13) with (14), and using $\tilde{\theta} = 0$ to a first-order, we obtain the following identity equation that relates the relative output gap with the relative international prices,

$$2\sigma y_t^R = D s_t^R + (1 - 2\alpha) m_t \tag{15}$$

This equation states that a relative increase in output in Home relative to Foreign requires either a fall in the relative price of Home-produced goods $s_t^R > 0$ or a depreciation of the Home currency $m_t > 0$. Finally, let $r^n \equiv -\log \beta$, the linearized aggregate Euler equations are

$$\sigma c_t = \sigma \mathbb{E}_t c_{t+1} - [i_t - \mathbb{E}_t \pi_{t+1} - r^n] \tag{16}$$

$$\sigma c_t^* = \sigma \mathbb{E}_t c_{t+1}^* - [i_t^* - \mathbb{E}_t \pi_{t+1}^* - r^n] \tag{17}$$

The supply block. Turning to the supply side, firms' optimal pricing conditions deliver a Phillips curve for each of the four prices set in the world economy

$$\pi_{H,t} = \kappa\, \psi_{H,t} + \beta \mathbb{E}_t \pi_{H,t+1} + \kappa u_t \quad \text{and} \quad \pi_{H,t}^* = \kappa[\psi_{H,t} - m_t - z_t] + \beta \mathbb{E}_t \pi_{H,t+1}^* + \kappa u_t \tag{18}$$

$$\pi_{F,t}^* = \kappa\, \psi_{F,t}^* + \beta \mathbb{E}_t \pi_{F,t+1}^* + \kappa u_t^* \quad \text{and} \quad \pi_{F,t} = \kappa[\psi_{F,t}^* + m_t - z_t] + \beta \mathbb{E}_t \pi_{F,t+1} + \kappa u_t^* \tag{19}$$

where $\kappa \equiv \frac{(1-\zeta)(1-\beta\zeta)}{\zeta}$ is the slope of the Phillips curve and $\psi_{H,t} = \sigma c_t + \alpha s_t$ is the real marginal cost of Home producers, expressed in units of the Home good. It corresponds to the marginal rate of substitution of Home households between consumption and leisure, σc_t(or $w_t - p_t$), deflated by the relative price of the Home good, $p_{H,t} - p_t = -\alpha s_t$. Equations (18) are the standard Phillips curves that relate current inflation of the good sold by the firm in each market with the real marginal cost of producing that good (deflated by the good's price in the market in which it is sold) and future inflation. Similarly, equations (19) are the Phillips curves in Foreign with $\psi_{F,t}^* = \sigma c_t^* + \alpha s_t^*$ denoting the real marginal cost of Foreign producers in units of the Foreign good.

Combining (18) and (19) yields two important results. First, given that $z_{-1} = 0$, the average of the relative prices of imported to locally produced goods equals zero (since $z_t = 0$ is the unique bounded solution), from which it follows that, to a first-order,

$$s_t = -s_t^* = s_t^R$$

and the currency misalignment reduces to the deviation from the law of one price, $m_t = e_t + p_{H,t}^* - p_{H,t} = p_{F,t}^* - (p_{F,t} - e_t)$. Second, the relative price of imports is independent of policies to a first-order and satisfies

$$(1 + \kappa + \beta)\, s_t - s_{t-1} - \beta \mathbb{E}_t s_{t+1} = \kappa(u_t^* - u_t). \tag{20}$$

3 Monetary Policy under Full Commitment

This section studies the optimal monetary policy under full commitment. We start by deriving the welfare-based loss function of a self-oriented policymaker and show how the response of asset prices shapes it. We then characterize the optimal targeting rule and compare the Nash equilibrium with the cooperative solution.

3.1 The welfare-based loss function

We first derive the Home monetary authority's welfare-based loss function under full commitment. Because the path of the policy rate is known before households trade state-contingent claims, the initial distribution of wealth is not predetermined when policy is chosen. In particular, the relative shadow value of wealth, $\tilde{\theta}$, adjusts endogenously to the announced policy through asset-market clearing. The objective of the Home policymaker is to maximize Home households' welfare which amounts to minimizing

$$\mathcal{L}_H \equiv \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t [U(C^e, N^e) - U(C_t, N_t)] \tag{21}$$

where $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - N_t$ is the Home households' period utility defined in (1) and $U(C^e, N^e)$ is the maximum utility achievable under the efficient allocation. A second-order approximation of (21) around the efficient steady state, yields the preliminary loss[10]

$$\begin{aligned}\mathcal{L}_H = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{\varepsilon}{2\kappa} \left[(1-\alpha) \left(\pi_{H,t}\right)^2 + \alpha \left(\pi^*_{H,t}\right)^2 \right] + \frac{\sigma}{2} \left(y_t\right)^2 - \frac{\alpha}{\sigma} \left[\tilde{\theta} - \frac{D-1}{2\alpha} s_t^R - (s_t^R - m_t) \right] \\ & + \frac{\alpha(\sigma-1)}{\sigma} \left[\tilde{\theta} - \frac{D-1}{2\alpha} s_t^R - (s_t^R - m_t) \right] y_t^W + \frac{\sigma\alpha(1-\alpha)}{2} \left[\frac{1}{\sigma}(q_t + \tilde{\theta}) - \eta(1-2\alpha) s_t^R \right]^2 \Bigg\} + \text{t.i.p}\end{aligned} \tag{22}$$

The first two terms, in squared producer price inflation and in the squared of domestic output gap, are familiar. Price dispersion across Home varieties, whether in the prices Home firms charge in the domestic market $(\pi_{H,t})^2$ or in the prices they charge abroad $(\pi^*_{H,t})^2$, acts like a negative productivity shock that raises the amount of labor inputs required to deliver a given quantity of Home goods. The two price dispersion terms have weights $1-\alpha$ and α corresponding to the steady-state shares of Home output sold domestically and abroad, respectively. The term $(y_t)^2$ is the resulting welfare cost of an inefficient level of production and thus consumption.

The three remaining terms have no closed-economy counterpart, and can be intuited by noting that they all concern a single object: the wedge between what Home consumes and what it produces, $c_t - y_t$. By (14), the linear expression in (22) is exactly $-(c_t - y_t)$, the term that follows is $(\sigma-1)(c_t - y_t)y_t^W$. What the three terms describe are the *level* of that wedge, its *timing*, and the *volatility* of the international transfer that produces it.

The linear term says that the Home policymaker would like the wedge to be large. Consuming more than one produces raises welfare, and by (14) that is achieved either by an appreciation of

[10] See Appendix B.2 for details on the derivation.

Home's terms of trade, via a depreciation of the Home currency $m_t > 0$, or by an increase in Home's relative wealth $\tilde{\theta} > 0$. This is the classical optimal-tariff motive, but transmitted through the financial market rather than through relative goods prices: under local-currency pricing, the expenditure-switching channel is muted, and what raises Home consumption for a given level of Home output is the real payoff of the claims that Home households hold. The covariance term says that when the wedge occurs matters. A transfer of consumption toward Home is worth more in some states than in others, and this depends on whether the marginal value of consumption falls strongly ($\sigma > 1$) or weakly ($\sigma < 1$) when consumption increases. When $\sigma > 1$, a positive wedge is most valuable when world output is low, $y_t^W < 0$; when $\sigma < 1$, it is most valuable when world output is high. The last term says that the volatility of the wedge is costly. Whatever raises Home consumption relative to Foreign consumption (a real depreciation, a higher relative wealth, or a movement in the relative price of imports) does so state by state. Because Home households are risk averse, the excess volatility in the international distribution of consumption is therefore costly, which is fully internalized by the Home policymaker.

What follows is about the markets' response to these motives. Two equilibrium relations discipline them: one from the supply side, which says what it takes to move the average terms of trade, and one from the asset market, which says how wealth is reallocated in response to the redistribution the policymaker plans.

The average terms-of-trade level. In a stationary model in which the unconditional mean of the shocks is zero, one might expect the unconditional expectation of the linear terms in (22) to vanish. Dropping those terms would nonetheless produce a spurious welfare approximation. As discussed in Benigno and Woodford (2005), because the steady state is not efficient from the Home policymaker's point of view, the linear terms carry information about what the policymaker would like to do, and they must be eliminated using the model's own second-order relations rather than assumed away.

In particular, the Home policymaker would like to increase $\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t(m_t - s_t^R)$, and appreciate Home terms-of-trade on average, because doing so raises the present value of Home consumption for a given path of Home output. However, the Home policymaker cannot freely increase the average terms-of-trade level, which is an equilibrium variable that depends on the pricing behavior of firms in both countries. As shown in Appendix A.4, combining the second-order approximation of aggregate supply equations of Home goods sold abroad and Foreign goods sold in Home, we get

$$
\begin{aligned}
\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left(m_t - s_t^R\right) &= \frac{\varepsilon}{2\kappa}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[(\pi_{H,t}^*)^2 - (\pi_{F,t})^2\right] \\
&\quad - \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[(m_t - s_t^R) + (1-\sigma)q_t + 2\sigma(1-\alpha)(1-\eta)s_t^R\right]y_t^W. \qquad (23)
\end{aligned}
$$

The average terms of trade is determined by two forces. The first one is price dispersion. Because the elasticity of demand for a variety exceeds one, demand shifts away from high-price firms toward low-price firms, and the profit cost of having a price set too low exceeds the cost of having a price set too high. Therefore, under staggered price setting, a firm facing more volatile aggregate prices sets a higher price on average. We refer to this as the *precautionary markup* channel, and ε governs its strength. Dispersion in the prices Home exporters charge abroad raises $p^*_{H,t}$ on average and thus appreciates Home's terms of trade, while dispersion in the prices Foreign exporters charge at Home raises $p_{F,t}$ and deteriorates Home's terms of trade. A policymaker who wants permanently better terms of trade must therefore accept more dispersion in the prices of Home exports, or engineer less dispersion in the prices of Home imports.

The second force is the covariance between world output and international relative prices: the average terms of trade that a given policy delivers depends on the states of the world in which relative prices move. Equation (23) is the equilibrium identity equation that disciplines the average terms of trade in (22).

The response of asset markets. Because policy is announced before contingent claims are traded, the redistribution motive of the Home policymaker must be evaluated together with the adjustment of the initial asset-market allocation. To a second order, the relative wealth equilibrium condition (10) can be expressed as,

$$\tilde{\theta} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} (1-\beta)\beta^t \Bigg\{ (\sigma - 1)(m_t - s_t^R) + \frac{D-1}{2\alpha} s_t^R \\ - (\sigma-1)\left[(\sigma-1)(m_t - s_t^R) + \frac{D-1}{2\alpha} s_t^R \right] y_t^W \Bigg\}, \tag{24}$$

which states that an improvement in Home's terms of trade (that is, when $e_t + p^*_{H,t} - p_{F,t} = m_t - s_t^R$ increases) in a given state has two opposing effects on the utility value of Home wealth. First, an improvement in the terms of trade raises the Home households' purchasing power: a given quantity of Home exports buys more imported goods, so Home consumption rises for a given level of Home output. This direct effect raises the value of Home income with elasticity one. Second, the associated increase in Home consumption lowers the marginal utility of consumption and thus the marginal value of an additional unit of Home wealth, with elasticity given by the inverse of the IES, σ. When $\sigma > 1$ the second effect dominates: Home's marginal value of wealth λ falls relative to λ^*, so $\tilde{\theta}$ rises. When $\sigma < 1$ the purchasing-power effect dominates, and when $\sigma = 1$ the two exactly offset.[11]

The second-order term in (24) states that when the terms-of-trade improvement occurs matters. If it arrives in a state in which world consumption is high $y_t + y_t^* > 0$, and thus marginal utility is low, the extra purchasing power is worth less in utility terms, so λ falls by less and $\tilde{\theta}$ rises by less. A terms-of-trade improvement is thus more valuable when global demand is depressed $y_t + y_t^* < 0$ and $\sigma > 1$. When $\sigma < 1$ the direction reverses. Note also that the strength of this state-contingent valuation effect, captured by the term $(\sigma-1)^2$.

[11] There is an additional effect that arises when preferences are not separable (i.e., when $D \neq 1$ or equivalently $\sigma\eta \neq 1$). In that case, a rise in the relative price of Home imports $s_t^R = p_{F,t} - p_{H,t}$ reduces import demand and depending on whether Home and Foreign goods are Hicksian substitutes ($\sigma\eta > 1$) or complements ($\sigma\eta < 1$), raises or lowers demand for Home goods as well.

A feature of (24) is that it is a present value expression: the asset market prices the entire path that the policy announcement implies, not its realization in every state. As a result, the financial market can undo, via the relative wealth, the average transfer that a policy promises, but it cannot undo the way that transfer is distributed across states. This is why, in the loss function below, the level of the redistribution motive disappears while a covariance term survives.

Proposition 1 (Loss function under full commitment). *Under full commitment, the welfare-based loss function of the Home central bank is given by*

$$\mathcal{L}_H = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}(\pi_t)^2 + \sigma(y_t^W)^2 + \frac{\sigma}{D}(y_t^R)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 + q_t y_t^W\right] + \text{t.i.p.} \tag{25}$$

where $q_t = \frac{2\sigma(1-2\alpha)}{D}y_t^R + \frac{4\alpha(1-\alpha)\sigma\eta}{D}m_t$. *The loss function of the Foreign policymaker is symmetric.*

Proof. See Appendix B.5. □

The loss function has four standard quadratic stabilization objectives: the domestic CPI inflation, the output gaps of both countries and the currency misalignment term. The cost of CPI inflation is the one discussed above. The stabilization motive of the two output gaps stems from the fact that aggregate consumption in Home depends on both Home output and Foreign output.

Why CPI inflation and not PPI inflations. To see this, note that substituting (24) into (22), so as to eliminate the internalized effects of the announced policy plan, i.e. $\tilde{\theta}$, yields

$$\begin{aligned}\mathcal{L}_H = \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{&\frac{\varepsilon}{2\kappa}\left[(1-\alpha)\left(\pi_{H,t}\right)^2 + \alpha\left(\pi_{H,t}^*\right)^2\right] + \frac{\sigma}{2}\left(y_t\right)^2 + \alpha(\sigma-1)(m_t - s_t^R)y_t^W \\ &+ \frac{\sigma\alpha\left(1-\alpha\right)}{2}\left[\frac{1}{\sigma}(q_t+\tilde{\theta}) - \eta(1-2\alpha)s_t^R\right]^2\Bigg\} - \alpha\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t(m_t - s_t^R)\end{aligned} \tag{26}$$

and plugging (23) into (26) produces two results that lie at the heart of why central banks should care about consumer price inflation.

First, *Home export price dispersion does not matter for Home's loss function.* The term $(\pi_{H,t}^*)^2$ enters (26) with weight $+\alpha\varepsilon/\kappa$ and enters through (23) with weight $-\alpha\varepsilon/\kappa$, so the two offset exactly. Intuitively, dispersion in the prices Home exporters charge abroad is costly, because more labor is required to produce the same aggregate quantity of Home goods sold abroad; but by the precautionary-markup channel it also raises the average price $p_{H,t}^*$ at which those goods sell, which improves Home's terms of trade and raises both Home households' purchasing power and the present value of Home exporters' profits. Because assets are traded after the policy is announced, that profit stream is taken into account in the price of contingent claims, and hence into the relative wealth $\tilde{\theta}$. Both the resource cost and the terms-of-trade benefit are proportional to the steady-state ratio of exports to GDP, and both are governed by the same demand elasticity ε; they therefore cancel one for one. A self-oriented policymaker has no reason to stabilize or destabilize the prices of its own exports.

Second, *import price dispersion matters for the loss function.* Dispersion in the prices that Foreign exporters charge in the Home market, $(\pi_{F,t})^2$, raises by the same precautionary-markup logic the average price Foreign firms charge at Home, and therefore worsens Home's terms of trade. Home bears that cost in full and none of the offsetting resource cost, which falls on Foreign workers.

The loss function in Home therefore depends on domestic price dispersion $(\pi_{H,t})^2$ and import price dispersion $(\pi_{F,t})^2$ with weights $1-\alpha$ and α, respectively, corresponding to the weights of Home goods and Foreign goods in Home households' consumption basket. Moreover,

$$(1-\alpha)(\pi_{H,t})^2 + \alpha(\pi_{F,t})^2 = (\pi_t)^2 + \alpha(1-\alpha)(s_t - s_{t-1})^2 \tag{27}$$

and since, by (20), s_t is independent of monetary policy to a first-order, it follows that $(s_t - s_{t-1})^2$ in (27) is a term independent of policy (t.i.p) to a second-order.

Why currency misalignment matters. The presence of the currency misalignment term $(m_t)^2$ is not obvious. Why would a domestic policymaker that cares only about the consumption and leisure of its own residents want to stabilize deviations from the law of one price? This is simply because of how such deviations affect the payoffs in the financial market. Currency misalignment distorts the distribution of consumption across countries, which, under complete markets, is governed by the payoffs on the claims Home households hold. Therefore, the extra consumption volatility that a currency misalignment generates is unambiguously costly to risk-averse Home households. It is worth noting that the currency misalignment is neither the nominal exchange rate nor the real exchange rate. The real exchange rate q_t moves with the relative output gap y_t^R and the relative price s_t, but those movements are desirable. The quadratic term in m_t calls for eliminating inefficient international price gaps, not for fixing the exchange rate.

The redistribution motive. The last term, $q_t y_t^W$, reflects the covariance between exchange-rate movements and world output. It is what remains of the redistributive motive of monetary policy under full commitment. As discussed above, given the relative wealth, the Home policymaker would like a persistently weak currency (or appreciated terms of trade), because that transfers consumption toward the Home country. Under full commitment, the price of state-contingent claims adjusts and the prospective transfer is internalized in $\tilde{\theta}$. The asset market removes the incentive to engineer a permanent transfer through terms-of-trade manipulation. What survives is a concern with the timing of a currency depreciation in real terms. A depreciation of the real exchange rate $q_t > 0$ improves welfare in Home when demand is globally depressed $y_t^W < 0$, and is costly when it occurs during a global boom $y_t^W > 0$.

3.2 The implementability constraints

Before turning to the optimal targeting rules, we present the set of implementability constraints, i.e. the set of equilibrium conditions that each policymaker's policy plan must satisfy. Because, to a first order, the relative price of imports s_t^R is independent of policies, we denote $f(\boldsymbol{u}_t) \equiv s_t^R$ and express

(15) as

$$f(\boldsymbol{u}_t) = \frac{2\sigma}{D} y_t^R - \frac{1-2\alpha}{D} m_t \tag{28}$$

Next, taking the sum and the difference of the Home and Foreign Euler equations, (16) and (17), along with (14), yields the world and relative IS equations, respectively

$$2\sigma y_t^W = 2\sigma \mathbb{E}_t y_{t+1}^W - (i_t + i_t^*) + \mathbb{E}_t \left[\pi_{t+1} + \pi_{t+1}^*\right] + 2r^n, \tag{29}$$

$$m_t = \mathbb{E}_t m_{t+1} - (i_t - i_t^*) + \mathbb{E}_t \left[\pi_{t+1} - \pi_{t+1}^*\right] + (1-2\alpha)\mathbb{E}_t \left[f(\boldsymbol{u}_{t+1}) - f(\boldsymbol{u}_t)\right], \tag{30}$$

Equation (29) describes how domestic monetary policy affects world output: holding expected future output and inflation fixed, a lower nominal rate in either country raises domestic aggregate demand and thus world demand. Equation (30) states that a decrease in the Home nominal rate relative to Foreign rate depreciates the Home currency, which, given that prices are sticky in the currency of the destination market, translates into a real depreciation and shifts consumption toward Home. With home bias, $1 - 2\alpha > 0$, this implies, by (28), an increase in Home output relative to Foreign output.

On the supply side, aggregating the Phillips curves in (18) and (19) using their respective consumption weights, along with (13) and (15), gives the CPI Phillips curves in Home and Foreign

$$\pi_t = \kappa\sigma \left[y_t^W + \frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t \right] + \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{u}_t, \tag{31}$$

$$\pi_t^* = \kappa\sigma \left[y_t^W - \frac{1-2\alpha}{D} y_t^R - \frac{2\alpha(1-\alpha)\eta}{D} m_t \right] + \beta \mathbb{E}_t \pi_{t+1}^* + \kappa \tilde{u}_t^*. \tag{32}$$

with $\tilde{u}_t \equiv (1-\alpha)u_t + \alpha u_t^*$ and $\tilde{u}_t^* \equiv (1-\alpha)u_t^* + \alpha u_t$. Taken together, (28)-(32) constitute a policymaker's set of implementability constraints.

3.3 Optimal targeting rules

We now characterize optimal policy of the Home central bank when it announces its state-contingent plans before households trade state-contingent claims, while taking the Foreign central bank's plan as given. More specifically, for given $\{i_t^*\}_{t=0}^{\infty}$ the Home policymaker chooses $\{i_t\}_{t=0}^{\infty}$ to minimize the loss function (25) subject to the set of implementability constraints describing the general equilibrium dynamics (28)–(32). The next Proposition characterizes the optimal targeting rule (i.e., the best response function) of the Home central bank.

Proposition 2 (Targeting rule under full commitment)**.** *The targeting rule of the Home policymaker is characterized by*

$$y_t^W + \frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t = -\varepsilon \left(p_t - p_{-1}\right), \tag{33}$$

where p_{-1} is the consumer price level inherited when the plans are announced.

Proof. See Appendix B.6. □

The central bank thus trades the deviation of its consumer price level from the level prevailing when the commitment was made against the output gaps in both countries and the currency-misalignment term. A first lesson from the targeting rule (33) is that, as is standard under commitment, the central bank targets the price level, rather than the inflation rate. This is because by promising to generate future deflation in response to an inflationary cost-push shock, the central bank can improve the current trade-off. Second, the optimal policy does not aim at eliminating the output gap, or currency misalignments separately. It aims at a specific combination of them, namely c_t from (14), traded against the price level.

Moreover, the Home policymaker's best response function is invariant to the aggregate demand and price levels in Foreign. This is not because the Home economy is insulated from Foreign policy, but as we show next, it follows from the fact that CPI inflation π_t and aggregate consumption c_t in Home are what really matter for Home's welfare criterion under full commitment, and the Home central bank's policy rate, i_t, moves aggregate consumption and CPI inflation in Home one for one through the Euler equation (16) and CPI Phillips curve (31).

A simplified representation. The Home policymaker's welfare-based loss function (25) can be simplified dramatically. Using $c_t = y_t^W + \frac{q_t}{2\sigma}$ from (14), the identity $2\sigma y_t^R = Ds_t^R + (1-2\alpha)m_t$ from (15), and the policy invariance of s_t, the loss function reduces to

$$\mathcal{L}_H = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}(\pi_t)^2 + \sigma(c_t)^2\right] + \text{t.i.p.} \tag{34}$$

(see Appendix B.7 for details on the derivation.) Thus, under full commitment, the loss function of a self-oriented domestic policymaker can be rewritten in terms of its own consumer-price inflation and its own consumption gap. All of the international terms (misalignment, relative output gap and the exchange-rate/world-output covariance) are exactly what is needed to convert the world output gap into the Home consumption gap.

The economic implication of the welfare-based loss function (34) is not that external variables and international prices do not matter for the Home central bank. Instead, it states that, once the asset market prices the policy, the only inefficiency the Home policymaker can affect is the aggregate consumption gap (which is a composite of Home consumption goods and Foreign consumption goods) together with the associated resource cost of consumer price dispersions. This simplified representation of the Home central bank's loss function is specific to the full commitment assumption and does not hold under partial commitment.

The problem of the central bank under full commitment therefore reduces to minimizing (34) subject to the Euler equation (16) and CPI Phillips curve (31) in Home which can be written compactly as $\pi_t = \kappa\sigma c_t + \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{u}_t$. As a result, the optimal targeting rule is simply

$$c_t = -\varepsilon\left(p_t - p_{-1}\right). \tag{35}$$

The key takeaway is that, under full commitment, a self-oriented central bank should aim at stabilizing domestic CPI inflation and consumption gap. The exchange rate matters because it affects Home consumption, and currency misalignment also matters because it affects Home consumption. Therefore, central banks should be concerned about currency misalignments and target deviations from the law of one price, not because it helps improve its *output-inflation* tradeoff, but because it is crucial to achieve efficient consumption.

3.4 Is there a need for cooperation?

In order to assess the gains from cooperation, we now characterize the Nash equilibrium and study the cooperative solution in which a global planner chooses monetary policy in both countries to maximize the average welfare of Home and Foreign households.

Nash equilibrium. The Nash equilibrium corresponds to the fixed point in the best response functions of both central banks given by (33) and the Foreign counterpart

$$y_t^W - \frac{1-2\alpha}{D} y_t^R - \frac{2\alpha(1-\alpha)\eta}{D} m_t = -\varepsilon \left(p_t^* - p_{-1}^*\right). \tag{36}$$

The cooperative solution. The cooperative planner minimizes the equally weighted sum of the two countries' loss functions given by

$$\mathcal{L}_G = \frac{1}{2}\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{2\kappa} \left((\pi_t)^2 + (\pi_t^*)^2\right) + \sigma (y_t^W)^2 + \frac{\sigma}{D}(y_t^R)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 \right] + \text{t.i.p.} \tag{37}$$

There are two important observations from (37). First, the covariance between real exchange rate and world output is absent. This is because while an exchange rate depreciation improves welfare in Home when world output is depressed, it lowers welfare in Foreign in that state and vice-versa. As a result, the global planner does not perceive any benefit from this redistribution motive. Second, unlike the country-specific welfare criteria, the welfare-based loss function of the global planner is independent of the form of the policymaker's commitment. The world planner has no incentive to move wealth across countries, so it is indifferent to whether the asset market has already priced its plan.

Proposition 3 (No gains from cooperation)**.** *Under full commitment, the targeting rules under cooperation coincide with the targeting rules in the Nash equilibrium, and are given by*

$$y_t^W = -\varepsilon(p_t^W - p_{-1}^W) \qquad \text{and} \qquad \frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t = -\varepsilon(p_t^R - p_{-1}^R) \tag{38}$$

As a result, there are no gains from international monetary cooperation.

Proof. See Appendix B.8. □

The global planner trades off the world price level against the world output gap (which is also the world consumption gap), and it also trades off the deviation of the cross-country difference in price levels against deviations of the real exchange rate from the efficient level (which is proportional to the cross-country difference in consumption gaps). But, each national central bank trades off its own consumer price level against its own consumption gap, so adding (33) and (36) reproduces the first cooperative targeting rule, and differencing them reproduces the second targeting rule. The main lesson is that delegating monetary policy to a global social planner would yield the same outcome as the one that would emerge when it is conducted by self-interested national central banks under full commitment. Therefore, there is no need for monetary policy cooperation.

Discussion and relation to Clarida-Gali-Gertler. Clarida et al. (2002) show that, under producer-currency pricing and unitary elasticities, self-oriented policy that targets domestic producer-price inflation and the domestic output gap achieves the cooperative allocation. Our findings differ from theirs in two respects. First, *the targeting rules are different*: where their national target is the producer price level and the domestic output gap, ours trades off the consumer price level against the Home consumption gap, which by (14) depends on both output gaps and the currency misalignment term. Optimal policy is therefore not inward-looking in the standard sense.

Second, the equivalence between the Nash equilibrium and the cooperative solution does not rely on unitary elasticities. Under the Cole-Obstfeld parametrization that Clarida et al. (2002) adopt, terms-of-trade movements provide perfect insurance, the relative wealth $\tilde{\theta}$ is policy-invariant, and the national policymakers' objectives therefore coincide with the global planner's. Benigno and Benigno (2003) show that their result is special to that parametrization: with general CES preferences over Home and Foreign bundles, the Nash equilibrium under producer-currency pricing is inefficient and there are gains from cooperation. In our environment the equivalence survives away from unitary elasticities because of the timing of the commitment relative to asset trade, which lets the financial market price the relative wealth, and with it the national policymaker's incentive to manipulate international relative prices in its own favor. Benigno and Benigno instead solve the non-cooperative problem taking the initial distribution of wealth as given, which is partial commitment in our terminology. As Coulibaly and Engel (2026) show, absent home bias in preferences – the case considered in Benigno and Benigno (2003) and Clarida et al. (2002) – the Nash targeting rules under full commitment again coincide with the cooperative ones for *any* values of the elasticities of substitution σ and η, and there are no gains from cooperation. What generates gains from cooperation under producer-currency pricing is therefore not the departure from unitary elasticities on its own, but its combination with the partial-commitment assumption. We turn next to how that assumption shapes the loss function and the optimal targeting rule in our environment.

4 Monetary Policy under Partial Commitment

We now consider the case of partial commitment in which each central bank announces its state-contingent plan after households trade a complete set of state-contingent claims. Monetary policy is thus discretionary from the perspective of the financial market under this form of commitment, even though the policymaker *commits* after assets are traded. This is the form of commitment that the previous literature has, usually implicitly, adopted.[12]

The implication of this assumption for the policy problem is that the relative marginal value of wealth is no longer an endogenous variable for the domestic policymaker, so we now have $\tilde{\theta} = 0$.

4.1 The welfare criterion under partial commitment

We note that the preliminary loss function (22) holds independently of whether the relative wealth is endogenous to policy or exogenous, so we use it as our starting point. Given that $\tilde{\theta}$ is now exogenous, we can use the second-order approximation of aggregate supply equations for goods sold in both Home and Foreign, and express the difference between the welfare-based loss function under partial commitment and the one under full commitment in terms of covariance terms reflecting the additional incentives that arise under partial commitment.

$$\begin{aligned}\mathcal{L}_H^P = \mathcal{L}_H + \frac{\alpha}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t \Bigg\{ & \underbrace{\frac{1-\sigma}{\sigma}\left[\frac{\varepsilon}{\kappa}\left((\pi_{F,t})^2 - (\pi_{H,t}^*)^2\right) + 2q_t y_t^W\right]}_{\text{wealth-valuation channel}} \\ & + \underbrace{4(1-\alpha)\left[\sigma\eta(1-\eta)\, s_t y_t^W + \frac{\varepsilon(\sigma\eta-1)}{\sigma\kappa}\left(s_t - s_{t-1}\right)\pi_t^W\right]}_{\text{static and dynamic ``expenditure-switching'' channels}}\Bigg\} + \text{t.i.p.} \qquad (39)\end{aligned}$$

where $\pi_t^W \equiv \frac{1}{2}(\pi_t + \pi_t^*)$. Equation (39) shows that the welfare criteria under both forms of commitment coincide in the closed-economy limit, $\alpha \to 0$, and under the Cole-Obstfeld unitary elasticities parametrization $\sigma = \eta = 1$. The gap between the welfare criterion under partial and full commitment originates from two distinct channels.

The first channel is the wealth-valuation channel by which a higher Home export price dispersion $(\pi_{H,t}^*)^2$ (or a lower import price dispersion $(\pi_{F,t})^2$) leads to an appreciation of the terms of trade through the precautionary mark-up effect in (23). This increases the present value of Home exporters' profits which raises the relative value of Home income with elasticity one, while the associated increase in consumption in Home lowers the marginal value of an additional unit of Home wealth with elasticity σ. Therefore, when $\sigma < 1$, the Home policymaker likes dispersion in export prices and dislikes dispersion in import prices as it increases welfare in Home. When $\sigma > 1$ the opposite is true. The additional term $q_t y_t^W$ is the covariance term that captures the state-dependency of the benefit of a Home currency depreciation in real terms. Because the policymaker announces and commits to its policy after assets are traded, this valuation effect is not priced in $\tilde{\theta}$, in contrast to the full commitment case.

The second line of (39) captures both the static and dynamic "expenditure-switching" channels, often referred to as terms-of-trade manipulation channels under PCP. The static channel is well-understood from the optimal tariff literature and depends on $\eta - 1$, which captures the elasticity of the relative expenditure share on import (i.e., home import-to-domestic expenditure ratio) to a change in the relative price of imports s_t. Under LCP, the Home policymaker cannot affect s_t, but it

[12] Benigno and Benigno (2003) state the assumption explicitly. Fujiwara and Wang (2017) adopt it in a setting close to ours. Egorov and Mukhin (2023) also assumes the initial wealth is policy-invariant.

can run the world economy hotter $y_t^W > 0$ or cooler $y_t^W < 0$. The dynamic channel, first highlighted by Costinot et al. (2014), refers to how the change in the relative price of imports over time can improve or reduce welfare at Home (which depends here on whether Home and Foreign goods are substitutes or complements in utility) when the Home country's net exports are expected to grow. Again, under LCP, the policymaker cannot alter the relative price of imports over time, but can raise the growth rate of world output which can be achieved by increasing world inflation $\pi_t^W > 0$.

4.2 Targeting rules under partial commitment

The Home central bank again chooses $\{i_t\}_{t=0}^{\infty}$ to minimize its loss function (39) subject to the implementability constraints (28)–(32), taking $\{i_t^*\}_{t=0}^{\infty}$ as given. The proposition below shows that the best response of a policymaker under partial commitment corresponds to the full-commitment rule plus an additional term in the relative price of imports.

Proposition 4 (Targeting rule under partial commitment)**.** *Under partial commitment, the Home policymaker targets*

$$\underbrace{y_t^W + \frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t}_{c_t} + \Lambda s_t = -\varepsilon\left(p_t - p_{-1}\right) \tag{40}$$

where $\Lambda \equiv \frac{\alpha(1-\alpha)}{1-\alpha+\alpha\sigma^{-1}}\left(\eta-1\right)\left(\varepsilon-\eta\right)$.

Proof. See Appendix B.10. □

Comparing (40) with the full-commitment rule (33), the rule under partial commitment features one additional term, Λs_t. We refer to Λ as the *partial-commitment wedge*. It measures the self-interested benefit that the Home policymaker attaches to a movement in the relative price of imports, beyond its effect on the stabilization of Home consumption, once the distribution of wealth across countries is fixed and can no longer be repriced. Under full commitment that value is zero and the rule collapses to the price-level rule in the Home consumption gap of Section 3.3.

The wedge depends on the product of two factors. The factor $\eta - 1$ is the elasticity of the Home expenditure share on imports with respect to the relative price of imports. When $\eta = 1$ nominal expenditure shares are constant. Thus, a movement in s_t redistributes nothing across countries, and the two forms of commitment deliver the same targeting rule.

The factor $\varepsilon - \eta$ nets two channels through which a relative-price movement is valued once asset have already been traded. The first, with weight ε, operates through price dispersion. As discussed in Section 3, dispersion in the prices of individual varieties raises the average price those firms charge, and ε governs the strength of this precautionary-markup channel. When the commitment is made after financial assets are traded, a policymaker is willing to accommodate movements in relative prices that increase dispersion in the prices of Home exports since it raises the present value of Home producers' profits and hence makes Home households wealthier.[13] The second channel, with weight η, comes from the fact that a higher relative price of imports reduces imports with

[13] It operates through both the wealth valuation effect and the dynamic 'expenditure switching' effect in (39).

elasticity η, and the resulting loss of purchasing power transfers real income away from Home households. The two channels work in opposite directions and the net effect is $\varepsilon - \eta$.

For empirically relevant parameters, $\varepsilon > \eta$ and $\eta > 1$, hence $\Lambda > 0$. Consider a Home cost-push shock, $u_t > 0$ and $u_t^* = 0$. Home producer prices rise relative to Foreign producer prices, so the relative price of imports in Home falls, $s_t < 0$, and the targeting rule (40) then implies

$$\varepsilon (p_t - p_{-1}) + c_t = -\Lambda s_t > 0.$$

It calls for a higher Home price level for a given consumption gap or, equivalently, a smaller contraction in Home consumption for a given price level, than the full-commitment rule (33) would call for. Therefore, a central bank under partial commitment tends to accommodate cost-push shocks more. It is *more dovish*.[14]

The Nash equilibrium. Combining (40) with its counterpart in Foreign, and using $s_t^* = -s_t$, gives the targeting rules that characterize the Nash equilibrium under partial commitment,

$$y_t^W = -\varepsilon \left(p_t^W - p_{-1}^W \right), \tag{41}$$

$$\frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t + \Lambda s_t = -\varepsilon \left(p_t^R - p_{-1}^R \right). \tag{42}$$

Comparing (41) and (42) with (38) in Proposition 3 isolates what the partial commitment assumption changes. The world target criterion (41) is the same as the world target criterion under full commitment and therefore the same as the world target criterion under cooperation. What it distorts is the relative target criterion (42), which now trades the relative price level against the real exchange rate and the relative price term, s_t. The additional motives that the domestic policymaker acquires under partial commitment are redistributive, as shown in (39), and the redistributive terms enter the two countries' objectives with opposite signs. Partial commitment distorts the relative stance of the two central banks.

4.3 Welfare ranking

Because the joint shock process are symmetric, that is (u_t, u_t^*) and (u_t^*, u_t) have the same distribution, and both countries are ex-ante identical, the Nash equilibria yield the same ex-ante losses in Home and Foreign. In particular, given an allocation $a^F \equiv \{\pi_t, \pi_t^*, y_t^W, y_t^R, m_t\}$ in the Nash equilibrium under full commitment, we have that $\mathcal{L}_H(a^F) = \mathcal{L}_F(a^F) = \mathcal{L}_G(a^F)$. Similarly, given an allocation a^P in the Nash equilibrium under partial commitment, we have $\mathcal{L}_H^P(a^P) = \mathcal{L}_F^P(a^P) = \mathcal{L}_G(a^P)$. Thus, the two Nash equilibria can be ranked. The proposition below establishes that partial commitment is, in general, welfare dominated.

[14] If instead of this empirically relevant configuration, we consider that Home and Foreign goods are poor substitutes, $\eta < 1$, or if $\eta > \varepsilon$, then the policymaker under partial commitment is more hawkish than under full commitment.

Proposition 5 (The welfare cost of late commitment)**.** *Let a^F and a^P denote the allocations implemented by the Nash equilibria under full and partial commitment. Then*

$$\mathcal{L}_G(a^P) \;\geq\; \mathcal{L}_G(a^F),$$

with strict inequality whenever $\eta \notin \{1, \varepsilon\}$ and $\operatorname{var}(u_t - u_t^*) > 0$*, in which case a^P is strictly Pareto dominated by a^F.*

Proof. See Appendix B.11. □

To understand this result, it is important to first recall that the two forms of commitment share the same feasible set. The implementability constraints (28)–(32) do not involve $\tilde{\theta}$, because relative wealth is a second-order object, so any allocation attainable under one form of commitment is attainable under the other. Second, the two forms of commitment share the same global criterion. The global planner has no redistribution motive and is therefore indifferent to whether the asset market has already priced its plan. Third, by Proposition 3 the Nash allocation under full commitment is the minimizer of the global planner's loss function over that set. The Nash allocation under partial commitment is a feasible point of the same problem, hence weakly worse, and strictly worse whenever the two allocations differ.

The size of the loss. The structure of the problem allows for a closed form characterization of the welfare cost of partial commitment. Let $\hat{\pi}_t \equiv \pi_t(a^P) - \pi_t(a^F)$ and $\hat{c}_t \equiv c_t(a^P) - c_t(a^F)$ denote the difference between the Home paths implemented by the two Nash equilibria (with the difference Foreign defined symmetrically). As shown in Appendix B.12, to a second-order, we have that

$$\mathcal{L}_G(a^P) - \mathcal{L}_G(a^F) = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}\left(\hat{\pi}_t\right)^2 + \sigma\left(\varepsilon\sum_{k=0}^{t}\hat{\pi}_k + \Lambda s_t\right)^2\right] \tag{43}$$

The two terms in (43) reflects the distortions introduced under partial commitment. The first is the excess volatility of consumer prices generated by the more accommodative response to cost-push shocks. The second is the induced distortion of the consumption gap, $\hat{c}_t = -[\varepsilon\sum_{k=0}^{t}\hat{\pi}_k + \Lambda s_t]$, which has two components: the redistributive term Λs_t that the partial-commitment policymaker acts upon, and the drift in the consumer price level that it requires. Under the assumption cost-push shocks in Home and Foreign, u_t and u_t^* are i.i.d with unconditional mean 0 and variance σ_u^2, the loss can be rewritten as[15]

$$\mathcal{L}_G(a^P) - \mathcal{L}_G(a^F) = \frac{\Omega}{2\left(1-\beta\right)}\Lambda^2 \operatorname{var}\left(s_t\right) \tag{44}$$

where $\Omega > 0$ is defined in (B.63) in Appendix B.13 and $\operatorname{var}(s_t)$ is the variance of s_t. There are two properties of the welfare cost of partial commitment (43) that are worth noting. First, the welfare cost proportional to Λ^2, so it rises with the magnitude of the partial-commitment wedge. Second, it is proportional to the variance of s_t, which in turn rises with increasing volatility of cost-push

[15]As shown in Appendix B.13, in that case, s_t is an AR(1): $s_t = \rho_s s_{t-1} + \kappa\rho_s(u_t - u_t^*)$ with $\rho_s \in (0,1)$ and $\operatorname{var}(s_t) = \frac{2\kappa^2\rho_s^2}{1-\rho_s^2}\sigma_u^2$.

shocks. For instance, trade policy shocks may act as cost-push shocks for firms, so the relative gain from full commitment is greater during times of greater trade policy uncertainty.

5 Extension and Implications for Policy

5.1 Dominant currency invoicing at the border matters

Our analysis so far assumes that the price a household pays is sticky in the currency of the country in which the good is sold. A large body of evidence documents, however, that international trade is invoiced at the border in a dominant currency, most often the U.S. dollar (Gopinath et al., 2020). This subsection shows that our analysis extends to the case in which goods are invoiced in a dominant currency at the borders. We show that the currency in which the good is invoiced when it crosses the dock is irrelevant for the optimal policy analysis: what matters is in which currency the price *paid by the consumer* is sticky.

Consider the following extension of the baseline model under dominant currency pricing (DCP) at the border. Home firms no longer set a price in Foreign currency. A firm l in Home that can reset its price at date t now sets an export price $\mathcal{P}^X_{H,t}(l)$ denominated in *Home* currency (the dominant currency) and keeps it until its able to reset it.[16] A competitive sector of Foreign distributors then buys Home goods at the border at that price and resells them to Foreign households at a price $\mathcal{P}^D_{H,t}(l)$ denominated in Foreign currency, which they reset on the same dates. Distribution is costless and there is free entry. The latter implies that the present value of distributors' profits over the life of a price is zero. Everything else remains unchanged.

Proposition 6 (Irrelevance of border invoicing)**.** *In the economy with border invoicing, the price paid by Foreign households for Home goods satisfies $\mathcal{P}^D_{H,t}(l) = \mathcal{P}^*_{H,t}(l)$ for every firm l and every date t where $\mathcal{P}^*_{H,t}(l)$ is the price paid under LCP. As a result, the equilibrium allocation, the welfare criteria and targeting rules are identical to the one under LCP.*

Proof. See Appendix B.14. □

This equivalence emerges because the free entry condition guarantees the retail price to the border price moves one for one. A distributor charges the Home-currency border price converted at a weighted average of the exchange rates expected to prevail over the life of the price, with using the same discount rate as Home exporter's (once converted in the same currency). Moreover, since distributors break even, the present value of Home exporters' foreign revenue, and hence of Home income, is the same as under LCP and no wealth is transferred across countries.

It is important to note that the economies with DCP at the border differ from the ones under LCP in who bears the exchange rate risk. Under LCP, the Home exporter's revenue moves with $\mathcal{E}_{t+j}$, whereas with border invoicing their revenue is fixed in Home currency and the distributor absorbs the exchange-rate risk. This difference corresponds to a stream with zero present value that is

[16] In Foreign, a firm l that can reset its price at date t now sets an export price continue to set its price in Home currency.

spanned by the claims traded. It is therefore undone under complete markets and the consumption allocation and relative wealth $\tilde{\Theta}$ remain unchanged.

5.2 Implications for Policy

Three important policy implications can be derived from our analysis.

Currency misalignments do matter. Central bankers often say that they care about the exchange rate only to the extent that it affects their ultimate objectives for inflation and employment. We show that even for a self-oriented central bank, the currency misalignment matters beyond those domestic goals. This is because of the role of the financial market in the cross-country distribution of consumption. When the law of one price fails, movements in the exchange rate change the real payoffs on the international claims a country's residents hold, and hence change their consumption. The closed-economy intuition does not carry over. In a closed economy, once the distortion created by sticky prices is corrected, asset prices are efficient too, and a policymaker who targets inflation and the output gap has no separate reason to care about asset valuations. In the open economy, asset prices determine the relative welfare of domestic and foreign households, and a self-oriented policymaker cares about that. What a central bank should target, however, is the deviation from the law of one price, not the level of the exchange rate, and not its volatility. The real exchange rate ought to move. It moves with relative supply conditions, and those movements are efficient. It is the misalignment component that should be stabilized.

While our analysis is under the assumption that there are traded a complete set of nominal state-contingent claims, the general insight applies to trade in non-state-contingent assets. If the home country holds claims on the foreign country that are denominated in foreign currency, the home value of those claims increases with a home depreciation - and, conversely, a home depreciation lowers the foreign-currency value of foreign home-currency claims. When gross asset holdings are large, the wealth redistribution effect of exchange-rate changes may be a significant consideration for monetary policy.

Communication of central bank policy matters. The distinction between full and partial commitment is not an abstraction about information sets. It is about when a central bank tells the market what it will do. If a central bank communicates clearly its rule, reaction function, or state-contingent path, and does so before financial markets have taken positions, then those positions are taken at prices that already embody the policy. The average redistribution the policy would have delivered is priced away by the financial market, so the central bank is left with nothing to gain from beggar-thy-neighbor behavior. If instead the announcement comes after markets have guessed and taken positions, the policymaker inherits a wealth distribution it can exploit and, knowing this, it will. The resulting equilibrium is worse for everybody, including the country doing the exploiting.

In other words, there is a value of commitment in the open economy that has nothing to do with the anchoring of inflation expectations. It is the value of letting the financial market price the policy before it becomes binding. For empirically relevant elasticities, a central bank that announces late is more dovish than one that announces early: it accommodates its own cost-push shocks with more consumer price inflation.

On the "equilibrium" exchange rates. International institutions such as the International Monetary Fund routinely estimate an equilibrium exchange rate and assess whether a currency is over- or undervalued. Such an assessment assumes an answer to a question that is rarely posed: equilibrium from whose perspective? One natural benchmark is the exchange rate that would prevail under globally efficient policy. Another, and the one relevant to a country deciding what to do, is the exchange rate that country would want given that the rest of the world are also setting policy in their own interest. These are conceptually different objects.

Our analysis says that under full commitment they coincide, and that in both cases the equilibrium exchange rate is the one at which the law of one price holds. An overvalued exchange rate therefore has an unambiguous meaning, and it does not require to take a stand on whether the rest of the world is cooperating or not. Under partial commitment, this is no longer the case. A country's preferred exchange rate is, in general, different from the globally efficient one. Thus, whether these notions of equilibrium exchange rates coincide in practice is also a question about central bank communication.

6 Conclusion

This paper shows that financial markets and the form of policy commitment are central to strategic monetary policy in open economies. Under full commitment, when policy plans are announced before assets are traded, financial markets incorporate those plans into asset prices. Each central bank then targets domestic consumer-price (CPI) inflation and the consumption gap, which depends on the output gaps in both countries and on currency misalignment, and there are no gains from monetary cooperation. Under partial commitment, by contrast, policy is announced only after asset trading has taken place without knowledge of the intended policy rule, leaving initial relative wealth predetermined from the policymaker's perspective. We show that the resulting allocation is Pareto dominated by the one that emerges under full commitment. Finally, the currency of invoicing at the border does not affect these conclusions as long as consumer prices remain sticky in local currency.

References

Basu, Suman Sambha, Emine Boz, Gita Gopinath, Francisco Roch, and Filiz Unsal, "A Conceptual Model for the Integrated Policy Framework," 2020. Mimeo, IMF.

Bengui, Julien and Louphou Coulibaly, "Destabilizing Capital Flows amid Global Inflation," *Review of Economic Studies*, 2026, p. rdag007.

Benigno, Gianluca and Michael Woodford, "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economic Association*, 2005, *3*, 1185–1236.

_ and Pierpaolo Benigno, "Price stability in open economies," *The Review of Economic Studies*, 2003, *70* (4), 743–764.

_ and _ , "Designing targeting rules for international monetary policy cooperation," *Journal of Monetary Economics*, 2006, *53* (3), 473–506.

Benigno, Pierpaolo, "Price Stability with Imperfect Financial Integration," *Journal of Money, Credit and Banking*, 2009, *41* (s1), 121–149.

Betts, Caroline and Michael B Devereux, "Exchange rate dynamics in a model of pricing-to-market," *Journal of international Economics*, 2000, *50* (1), 215–244.

_ and _ , "International monetary policy coordination and competitive depreciation: A reevaluation," *Journal of money, credit and banking*, 2000, pp. 722–745.

Bianchi, Javier and Louphou Coulibaly, "Financial integration and monetary policy coordination," Technical Report, National Bureau of Economic Research 2024.

Bodenstein, Martin, Giancarlo Corsetti, and Luca Guerrieri, "The elusive gains from nationally oriented monetary policy," *Review of Economic Studies*, 2025, *92* (2), 873–908.

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Casas, Camila, Mr Federico Diez, Gita Gopinath, and Pierre-Olivier Gourinchas, *Dominant currency paradigm: A new model for small open economies*, International Monetary Fund, 2017.

Clarida, Richard, Jordi Galı, and Mark Gertler, "A simple framework for international monetary policy analysis," *Journal of monetary economics*, 2002, *49* (5), 879–904.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc, "Optimal Monetary Policy in Open Economies," in "Handbook of monetary economics," Vol. 3, Elsevier, 2010, pp. 861–933.

_ , _ , and _ , "Exchange rate misalignment and external imbalances: What is the optimal monetary policy response?," *Journal of International Economics*, 2023, *144*, 103771.

Costinot, Arnaud, Guido Lorenzoni, and Ivan Werning, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77–128.

Coulibaly, Louphou, "Monetary policy in sudden stop-prone economies," *American Economic Journal: Macroeconomics*, 2023, *15* (4), 141–176.

_ **and Charles Engel**, "Are they gains from international monetary policy coordination?," Technical Report, Mimeo, University of Wisconsin 2026.

Devereux, Michael B and Alan Sutherland, "Financial globalization and monetary policy," *Journal of Monetary Economics*, 2008, *55* (8), 1363–1375.

_ **and Charles Engel**, "Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility," *The review of economic studies*, 2003, *70* (4), 765–783.

Egorov, Konstantin and Dmitry Mukhin, "Optimal policy under dollar pricing," *American Economic Review*, 2023, *113* (7), 1783–1824.

Engel, Charles, "Currency Misalignments and Optimal Monetary Policy: A Reexamination," *American Economic Review*, 2011, *101* (6), 2796–2822.

_ , "Policy Cooperation, Incomplete Markets and Risk Sharing," *IMF Economic Review*, 2016, *64* (1), 103–133.

Faia, Ester and Tommaso Monacelli, "Optimal monetary policy in a small open economy with home bias," *Journal of Money, credit and Banking*, 2008, *40* (4), 721–750.

Fanelli, Sebastián and Ludwig Straub, "A theory of foreign exchange interventions," *The Review of Economic Studies*, 2021, *88* (6), 2857–2885.

Fujiwara, Ippei and Jiao Wang, "Optimal monetary policy in open economies revisited," *Journal of International Economics*, 2017, *108*, 300–314.

Gali, Jordi and Tommaso Monacelli, "Monetary policy and exchange rate volatility in a small open economy," *The Review of Economic Studies*, 2005, *72* (3), 707–734.

Gong, Liutang, Chan Wang, Liyuan Wu, and Heng fu Zou, "Currency misalignments, international trade in intermediate inputs, and inflation targeting," *Journal of International Money and Finance*, 2023, *134*, 102843.

Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller, "Dominant Currency Paradigm," *American Economic Review*, 2020, *110* (3), 677–719.

Guerrieri, Veronica, Guido Lorenzoni, and Iván Werning, "Global price shocks and international monetary coordination," Technical Report, National Bureau of Economic Research 2025.

Itskhoki, Oleg and Dmitry Mukhin, "Optimal exchange rate policy," Technical Report, National Bureau of Economic Research 2023.

Kollmann, Robert, "Monetary policy rules in the open economy: effects on welfare and business cycles," *Journal of Monetary Economics*, 2002, *49* (5), 989–1015.

Obstfeld, Maurice and Kenneth Rogoff, "Exchange rate dynamics redux," *Journal of political economy*, 1995, *103* (3), 624–660.

_ and _ , "Global implications of self-oriented national monetary rules," *The Quarterly journal of economics*, 2002, *117* (2), 503–535.

Ottonello, Pablo, "Optimal exchange-rate policy under collateral constraints and wage rigidity," *Journal of International Economics*, 2021, *131*, 103478.

Paoli, Bianca De, "Monetary policy and welfare in a small open economy," *Journal of international Economics*, 2009, *77* (1), 11–22.

Pappa, Evi, "Do the ECB and the Fed really need to cooperate? Optimal monetary policy in a two-country world," *Journal of Monetary Economics*, 2004, *51* (4), 753–779.

Prasad, Nalini, "Sterilized interventions and capital controls," *Journal of International Money and Finance*, 2018, *88*, 101–121.

Schmitt-Grohé, Stephanie and Martin Uribe, "Optimal simple and implementable monetary and fiscal rules," *Journal of monetary Economics*, 2007, *54* (6), 1702–1725.

Senay, Ozge and Alan Sutherland, "The timing of asset trade and optimal policy in dynamic open economies," *Macroeconomic Dynamics*, 2013, *17* (8), 1543–1573.

Sutherland, Alan, "International monetary policy coordination and financial market integration," *Available at SSRN 510183*, 2004.

_ , "Incomplete pass-through and the welfare effects of exchange rate variability," *Journal of international economics*, 2005, *65* (2), 375–399.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy* 2003.

Appendix

A Model details

A.1 Firms' optimal pricing decisions

A firm l in Home that can reset at date t chooses $\mathcal{P}_{H,t}(l)$ and $\mathcal{P}^*_{H,t}(l)$ to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \cdot \beta^j \frac{P_t}{P_{t+j}} \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} \left\{(1+\tau)\Big[\mathcal{P}_{H,t}(l) C_{H,t+j}(l) + \mathcal{E}_{t+j}\mathcal{P}^*_{H,t}(l) C^*_{H,t+j}(l)\Big] - W_{t+j}N_{t+j}(l)\right\}$$

subject to

$$C_{H,t+j}(l) = \left(\frac{\mathcal{P}_{H,t}(l)}{P_{H,t+j}}\right)^{-\varepsilon} C_{H,t+j} \quad \text{and} \quad C^*_{H,t+j}(l) = \left(\frac{\mathcal{P}^*_{H,t}(l)}{P^*_{H,t+j}}\right)^{-\varepsilon} C^*_{H,t+j}$$

The first-order condition for $\mathcal{P}_{H,t}(l)$ and $\mathcal{P}^*_{H,t}(l)$ are given by

$$\begin{aligned}
0 &= \mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \beta^j \frac{(C_{t+j})^{-\sigma}}{P_{t+j}} C_{Ht+j} \left(P_{H,t+j}\right)^{\varepsilon} \left[(1+\tau)\left(\varepsilon - 1\right)\mathcal{P}_{H,t}(l) - \varepsilon W_{t+j}\right] \\
0 &= \mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \beta^j \frac{(C_{t+j})^{-\sigma}}{P_{t+j}} C^*_{H,t+j}\mathcal{E}_{t+j} \left(P^*_{H,t+j}\right)^{\varepsilon} \left[(1+\tau)\left(1-\varepsilon\right)\mathcal{P}^*_{H,t}(l) + \varepsilon \frac{W_{t+j}}{\mathcal{E}_{t+j}}\right]
\end{aligned}$$

Rearranging it, we arrive at

$$\mathcal{P}_{H,t}(l) = \frac{\varepsilon}{(1+\tau)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega_{h,t+j} W_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega_{h,t+j}}, \ \text{with} \ \Omega_{h,t+j} \equiv \beta^j \frac{(C_{t+j})^{-\sigma}}{P_{t+j}} C_{H,t+j} (P_{Ht+j})^{\varepsilon} \tag{A.1}$$

$$\mathcal{P}^*_{H,t}(l) = \frac{\varepsilon}{(1+\tau)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j} \frac{W_{t+j}}{\mathcal{E}_{t+j}}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j}}, \ \text{with} \ \Omega^*_{h,t+j} \equiv \beta^j \frac{(C_{t+j})^{-\sigma}}{P_{t+j}} C^*_{H,t+j}\mathcal{E}_{t+j} (P^*_{H,t+j})^{\varepsilon} \tag{A.2}$$

Similarly, a firm l in Foreign that can reset at date t chooses $\mathcal{P}^*_{F,t}(l)$ and $\mathcal{P}_{F,t}(l)$ to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \cdot \beta^j \frac{P^*_t}{P^*_{t+j}} \left(\frac{C^*_{t+j}}{C^*_t}\right)^{-\sigma} \left\{(1+\tau^*)\left[\mathcal{P}^*_{F,t}(l) C^*_{F,t+j}(l) + \frac{1}{\mathcal{E}_{t+j}}\mathcal{P}_{F,t}(l) C_{F,t+j}(l)\right] - W^*_{t+j}N^*_{t+j}(l)\right\}$$

subject to

$$C^*_{F,t+j}(l) = \left(\frac{\mathcal{P}^*_{F,t}(l)}{P^*_{F,t+j}}\right)^{-\varepsilon} C^*_{F,t+j} \quad \text{and} \quad C_{F,t+j}(l) = \left(\frac{\mathcal{P}_{F,t}(l)}{P_{F,t+j}}\right)^{-\varepsilon} C_{F,t+j}$$

and we get

$$\mathcal{P}^*_{F,t}(l) = \frac{\varepsilon}{(1+\tau^*)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{f,t+j} W^*_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{f,t+j}}, \ \text{with} \ \Omega^*_{f,t+j} \equiv \beta^j \frac{(C^*_{t+j})^{-\sigma}}{P^*_{t+j}} C^*_{F,t+j} (P^*_{F,t+j})^{\varepsilon} \tag{A.3}$$

$$\mathcal{P}_{F,t}(l) = \frac{\varepsilon}{(1+\tau^*)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega_{f,t+j} \mathcal{E}_{t+j} W^*_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega_{f,t+j}}, \ \text{with} \ \Omega_{f,t+j} \equiv \beta^j \frac{(C^*_{t+j})^{-\sigma}}{P^*_{t+j}} \frac{C_{F,t+j} (P_{F,t+j})^{\varepsilon}}{\mathcal{E}_{t+j}} \tag{A.4}$$

A.2 The initial relative wealth equation (10)

Using the Home households' optimality condition for $C_{F,t}(z^t)$ and Foreign households' optimality condition for $C^*_{H,t}(z^t)$ which are respectively given by

$$C_{F,t}\left(z^t\right) = \alpha \left(\frac{P_{F,t}\left(z^t\right)}{P_t\left(z^t\right)}\right)^{-\eta} C_t\left(z^t\right), \quad \text{and} \quad C^*_{H,t}\left(z^t\right) = \alpha \left(\frac{P^*_{H,t}\left(z^t\right)}{P^*_t\left(z^t\right)}\right)^{-\eta} C^*_t\left(z^t\right) \tag{A.5}$$

and substituting them into (9), we get

$$\begin{aligned} 0 &= \sum_{t=0}^{\infty}\sum_{z^t}\mathcal{R}^0_t(z^t)\left[\alpha\mathcal{E}_t(z^t)\left(\frac{P^*_{H,t}(z^t)}{P^*_t(z^t)}\right)^{1-\eta}P^*_t(z^t)C^*_t(z^t) - \alpha\left(\frac{P_{F,t}(z^t)}{P_t(z^t)}\right)^{1-\eta}P_t(z^t)C_t(z^t)\right] \\ 0 &= \sum_{t=0}^{\infty}\sum_{z^t}\beta^t\rho(z^t)\left[\frac{1}{\lambda^*}\left(\frac{P^*_{H,t}(z^t)}{P^*_t(z^t)}\right)^{1-\eta}\left(C^*_t(z^t)\right)^{1-\sigma} - \frac{1}{\lambda}\left(\frac{P_{F,t}(z^t)}{P_t(z^t)}\right)^{1-\eta}\left(C_t(z^t)\right)^{1-\sigma}\right] \end{aligned} \tag{A.6}$$

where the second equality uses (6). Rearranging (A.6), we arrive at (10).

A.3 Efficient allocation

The social planner chooses $\{C_{H,t}, C_{F,t}, C^*_{H,t}, C^*_{F,t}, N_t, N^*_t\}$ to maximize the equally weighted sum of Home and Foreign lifetime utilities. The Lagrangian for its problem can be written as

$$\begin{aligned} \mathcal{L}^{\text{fb}} = \sum_{t=0}^{\infty}\sum_{z^t}\beta^t\rho(z^t)\Bigg[&\frac{1}{1-\sigma}\left((1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}(z^t)\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}(z^t)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\sigma)}{\eta-1}} - N_t(z^t) \\ &+ \frac{1}{1-\sigma}\left((1-\alpha)^{\frac{1}{\eta}}\left(C^*_{F,t}(z^t)\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C^*_{H,t}(z^t)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta(1-\sigma)}{\eta-1}} - N^*_t(z^t) \\ &+ \vartheta_t(z^t)\left(N_t(z^t) - C_{H,t}(z^t) - C^*_{H,t}(z^t)\right) + \vartheta^*_t(z^t)\left(N^*_t(z^t) - C_{F,t}(z^t) - C^*_{F,t}(z^t)\right)\Bigg] \end{aligned}$$

The first-order condition with respect to $N_t(z^t)$ and $N^*_t(z^t)$ yields $\vartheta_t(z^t) = \vartheta^*_t(z^t) = 1$. Using this, the optimality conditions for $C_{H,t}(z^t)$ and $C_{F,t}(z^t)$ can be expressed as

$$(1-\alpha)^{\frac{1}{\eta}}(C_{H,t}(z^t))^{-\frac{1}{\eta}}(C_t(z^t))^{\frac{1}{\eta}-\sigma} = 1 \quad \text{and} \quad \alpha^{\frac{1}{\eta}}(C_{F,t}(z^t))^{-\frac{1}{\eta}}(C_t(z^t))^{\frac{1}{\eta}-\sigma} = 1 \tag{A.7}$$

and the optimality conditions for $C^*_{F,t}(z^t)$ and $C^*_{H,t}(z^t)$ can be expressed as

$$(1-\alpha)^{\frac{1}{\eta}}(C^*_{F,t}(z^t))^{-\frac{1}{\eta}}(C^*_t(z^t))^{\frac{1}{\eta}-\sigma} = 1 \quad \text{and} \quad \alpha^{\frac{1}{\eta}}(C^*_{H,t}(z^t))^{-\frac{1}{\eta}}(C^*_t(z^t))^{\frac{1}{\eta}-\sigma} = 1 \tag{A.8}$$

Combining (A.7) and (A.8), we arrive at

$$C_{H,t}(z^t) = C^*_{F,t}(z^t) = 1-\alpha, \qquad C^*_{H,t}(z^t) = C_{F,t}(z^t) = \alpha$$

which implies that $N_t(z^t) = N^*_t(z^t) = C_t(z^t) = C^*_t(z^t) = 1$. □

A.4 Second-order approximation of aggregate supply equations

Home goods sold domestically. Using (4), we can express (A.1) as

$$\overline{\mathcal{P}}_{H,t}(l) \equiv \frac{\mathcal{P}_{H,t}(l)}{P_{H,t}} = \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\zeta)^j \left(\frac{P_{H,t+j}}{P_{H,t}}\right)^{\varepsilon} X_{H,t+j} \cdot \frac{1+\mu_{t+j}}{1+\mu} \cdot \Psi_{H,t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\zeta)^j \left(\frac{P_{H,t+j}}{P_{H,t}}\right)^{\varepsilon-1} X_{H,t+j}}$$

where $X_{H,t} \equiv (P_{H,t}/P_t)^{1-\eta} C_t^{1-\sigma}$ and $\Psi_{H,t} \equiv \frac{1+\mu}{1+\tau}(P_t/P_{H,t})C_t^{\sigma}$ is the real marginal cost, while $1+\mu_t$ is the time-varying gross markup, with steady-state value $1+\mu$, so that $u_t \equiv \log[(1+\mu_t)/(1+\mu)]$ and $\frac{1+\mu_t}{1+\mu} = 1 + u_t + \frac{1}{2}u_t^2 + \mathrm{o}(||\boldsymbol{u}||^3)$. We can then rewrite it recursively as $\overline{\mathcal{P}}_{H,t}(l) = K_{H,t}/F_{H,t}$ with

$$K_{H,t} = X_{H,t}\Psi_{H,t}\frac{1+\mu_t}{1+\mu} + \beta\zeta\mathbb{E}_t\left[\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\varepsilon} K_{H,t+1}\right], \qquad F_{H,t} = X_{H,t} + \beta\zeta\mathbb{E}_t\left[\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\varepsilon-1} F_{H,t+1}\right]$$

A second-order approximation around the efficient steady-state allocation yields

$$\begin{aligned}\overline{p}_{H,t} + \frac{1}{2}\overline{p}_{H,t}\ell_{H,t} = (1-\beta\zeta)&\left[\psi_{H,t} + \psi_{H,t}\left(x_{H,t} + \frac{1}{2}\psi_{H,t}\right) + (\psi_{H,t} + x_{H,t})\, u_t + \frac{1}{2}u_t^2 + u_t\right] \\ &+ \beta\zeta\mathbb{E}_t\left[\pi_{H,t+1} + \overline{p}_{H,t+1} + \frac{1}{2}\left(\pi_{H,t+1} + \overline{p}_{H,t+1}\right)\left((2\varepsilon-1)\,\pi_{H,t+1} + \ell_{H,t+1}\right)\right]\end{aligned} \tag{A.9}$$

where note that using (11) we have

$$\psi_{H,t} = \sigma c_t + \alpha s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\,(s_t)^2 - \frac{\tilde{\theta} + v}{2} + \mathrm{o}(||\boldsymbol{u}||^3) \tag{A.10}$$

$$x_{H,t} = \alpha\,(\eta-1)\,s_t + (1-\sigma)\,c_t + \mathrm{o}(||\boldsymbol{u}||^2) \tag{A.11}$$

where for clarity we introduce $\mathrm{o}(||\boldsymbol{u}||^2)$, which indicates that second-order and higher terms are left out, and $\mathrm{o}(||\boldsymbol{u}||^3)$, which indicates that third-order and higher terms are left out. Moreover we defined $\ell_{H,t} \equiv k_{H,t} + f_{H,t}$ in (A.9) where to a first-order

$$\ell_{H,t} = (1-\beta\zeta)\,[\psi_{H,t} + 2x_{H,t}] + \beta\zeta\mathbb{E}_t\,[(2\varepsilon-1)\pi_{H,t+1} + \ell_{H,t+1}]$$

Next recall that $P_{H,t} = [\zeta(P_{H,t-1})^{1-\varepsilon} + (1-\zeta)(\mathcal{P}_{H,t})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$ and to a second order we have

$$\overline{p}_{H,t} = \frac{\zeta}{1-\zeta}\pi_{H,t} + \frac{\zeta(\varepsilon-1)}{2(1-\zeta)^2}(\pi_{H,t})^2 + \mathrm{o}(||\boldsymbol{u}||^3) \tag{A.12}$$

Defining

$$V_{H,t} \equiv \pi_{H,t} + \frac{1}{2}\left[\left(\varepsilon + \frac{\varepsilon-1}{1-\zeta}\right)(\pi_{H,t})^2 + \pi_{H,t}\ell_{H,t} + u\ell_{H,t}\right] \tag{A.13}$$

where $u\ell_{H,t}$ satisfies the following recursive equation, $u\ell_{H,t} = -2\kappa\,[\psi_{H,t} + x_{H,t}]\,u_t + \beta\mathbb{E}_t u\ell_{H,t+1}$. and expanding $k_{H,t}$ and $f_{H,t}$ to second order, one obtains the aggregate supply relation

$$V_{H,t} = \kappa\left[\psi_{H,t} + \frac{1}{2}\psi_{H,t}^2 + \psi_{H,t}x_{H,t} + u_t + \frac{1}{2}u_t^2\right] + \frac{\varepsilon}{2}(\pi_{H,t})^2 + \beta\mathbb{E}_t V_{H,t+1} \tag{A.14}$$

Home goods sold abroad. Using (4) and (7), we can express (A.2) as

$$\overline{\mathcal{P}}^*_{H,t}(l) \equiv \frac{\mathcal{P}^*_{H,t}(l)}{P^*_{H,t}} = \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\zeta)^j \left(\frac{P^*_{H,t+j}}{P^*_{H,t}}\right)^{\varepsilon} X^*_{H,t+j} \cdot \frac{1+\mu_{t+j}}{1+\mu} \cdot \Psi^*_{H,t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\zeta)^j \left(\frac{P^*_{H,t+j}}{P^*_{H,t}}\right)^{\varepsilon-1} X^*_{H,t+j}}$$

where $X^*_{H,t} \equiv \left(P^*_{H,t}/P^*_t\right)^{1-\eta} (C^*_t)^{1-\sigma}$ and $\Psi^*_{H,t} \equiv \Psi_{H,t} P_{H,t}/(\mathcal{E}_t P^*_{H,t})$ and we have

$$\psi^*_{H,t} = \sigma c_t + \alpha s_t - m_t - z_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)(s_t)^2 - \frac{\tilde{\theta}+v}{2} + \mathrm{o}(||\boldsymbol{u}||^3) \tag{A.15}$$

$$x^*_{H,t} = (1-\alpha)(1-\eta)s^*_t + (1-\sigma)\, c^*_t + \mathrm{o}(||\boldsymbol{u}||^2) \tag{A.16}$$

We can then express $\overline{\mathcal{P}}^*_{H,t}(l)$ recursively and, proceeding similarly to the above, we get

$$V^*_{H,t} = \kappa\left[\psi^*_{H,t} + \frac{1}{2}(\psi^*_{H,t})^2 + \psi^*_{H,t}x^*_{H,t} + u_t + \frac{1}{2}u_t^2\right] + \frac{\varepsilon}{2}(\pi^*_{H,t})^2 + \beta\mathbb{E}_t V^*_{H,t+1} \tag{A.17}$$

where we define

$$V^*_{H,t} \equiv \pi^*_{H,t} + \frac{1}{2}\left[\left(\varepsilon + \frac{\varepsilon-1}{1-\zeta}\right)(\pi^*_{H,t})^2 + \pi^*_{H,t}\ell^*_{H,t} + u\ell^*_{H,t}\right] \tag{A.18}$$

and $u\ell^*_{H,t}$ and $\ell^*_{H,t}$ satisfy the following recursive equation, $u\ell^*_{H,t} = -2\kappa[\psi^*_{H,t} + x^*_{H,t}]u_t + \beta\mathbb{E}_t u\ell^*_{H,t+1}$ and $\ell^*_{H,t} = (1-\beta\zeta)[\psi^*_{H,t} + 2x^*_{H,t}] + \beta\zeta\mathbb{E}_t[(2\varepsilon-1)\pi^*_{H,t+1} + \ell^*_{H,t+1}]$.

Foreign. The second-order approximations of (A.3) an (A.4) in Foreign are symmetric

$$V^*_{F,t} = \kappa\left[\psi^*_{F,t} + \frac{1}{2}(\psi^*_{F,t})^2 + \psi^*_{F,t}x^*_{F,t} + u^*_t + \frac{1}{2}(u^*_t)^2\right] + \frac{\varepsilon}{2}(\pi^*_{F,t})^2 + \beta\mathbb{E}_t V^*_{F,t+1} \tag{A.19}$$

$$V_{F,t} = \kappa\left[\psi_{F,t} + \frac{1}{2}(\psi_{F,t})^2 + \psi_{F,t}x_{F,t} + u^*_t + \frac{1}{2}(u^*_t)^2\right] + \frac{\varepsilon}{2}(\pi_{F,t})^2 + \beta\mathbb{E}_t V_{F,t+1} \tag{A.20}$$

where

$$V^*_{F,t} \equiv \pi^*_{F,t} + \frac{1}{2}\left[\left(\varepsilon + \frac{\varepsilon-1}{1-\zeta}\right)(\pi^*_{F,t})^2 + \pi^*_{F,t}\ell^*_{F,t} + u\ell^*_{F,t}\right] \tag{A.21}$$

$$V_{F,t} \equiv \pi_{F,t} + \frac{1}{2}\left[\left(\varepsilon + \frac{\varepsilon-1}{1-\zeta}\right)(\pi_{F,t})^2 + \pi_{F,t}\ell_{F,t} + u\ell_{F,t}\right] \tag{A.22}$$

and where we have

$$x^*_{F,t} = \alpha(\eta-1)s^*_t + (1-\sigma)c^*_t + \mathrm{o}(||\boldsymbol{u}||^2), \qquad x_{F,t} = (1-\alpha)(1-\eta)s_t + (1-\sigma)c_t + \mathrm{o}(||\boldsymbol{u}||^2) \tag{A.23}$$

$$\psi^*_{F,t} = \sigma c^*_t + \alpha s^*_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)(s_t)^2 + \frac{\tilde{\theta}+v}{2} + \mathrm{o}(||\boldsymbol{u}||^3), \qquad \psi_{F,t} = \psi^*_{F,t} + m_t - z_t \tag{A.24}$$

B Proofs

B.1 Proof of Lemma 1

Define

$$v^{\text{bef}} \equiv \frac{1}{2}\mathbb{E}\left\{\left(\varepsilon+\frac{\varepsilon-1}{1-\zeta}\right)\left[(\pi_{F,0})^2-(\pi^*_{H,0})^2\right]+(\pi_{F,0}\ell_{F,0}-\pi^*_{H,0}\ell^*_{H,0})+(u\ell_{F,0}-u\ell^*_{H,0})\right\} \tag{B.1}$$

$$v^{\text{ex}} \equiv \frac{1}{2}\mathbb{E}\left\{\left(\varepsilon+\frac{\varepsilon-1}{1-\zeta}\right)\left[(\pi^*_{F,0})^2-(\pi_{H,0})^2\right]+(\pi^*_{F,0}\ell^*_{F,0}-\pi_{H,0}\ell_{H,0})+(u\ell^*_{F,0}-u\ell_{H,0})\right\} \tag{B.2}$$

Noting that $\mathbb{E}(\pi_{H,0})=\mathbb{E}(\pi^*_{H,0})=\mathbb{E}(\pi^*_{F,0})=\mathbb{E}(\pi_{F,0})=0$ since the unconditional mean of the shocks is zero, we can use (A.13), (A.18), (A.21), (A.22) to rewrite (B.1) and (B.2) as

$$v^{\text{bef}}=\mathbb{E}\left[V_{F,0}-V^*_{H,0}\right] \quad \text{and} \quad v^{\text{ex}}=\mathbb{E}\left[V^*_{F,0}-V_{H,0}\right]$$

Because the shocks u_t and u^*_t are drawn from the same distribution, it follows that in any competitive equilibrium $v^{\text{bef}}=v^{\text{ex}}=0$. Under full commitment, we have

$$v=\frac{1-\beta}{\kappa}v^{\text{bef}} \tag{B.3}$$

and under partial commitment we have

$$v=\frac{1-\beta}{\kappa}\left[v^{\text{bef}}+\frac{D-1}{2(D-1+2\alpha)}\left(v^{\text{ex}}-v^{\text{bef}}\right)\right] \tag{B.4}$$

It follows that in any competitive equilibrium, $v=0$ since $v^{\text{bef}}=v^{\text{ex}}=0$. Moreover, we have that $\tilde{\Theta}=1$ in any competitive equilibrium, which implies that $\tilde{\theta}=0$. Plugging $\tilde{\theta}=v=0$ into (11), we arrive at $\tau=\tau^*=\mu$. □

B.2 Derivation of (22)

Let $U(C_t,N_t)\equiv\frac{1}{1-\sigma}(C_t)^{1-\sigma}-N_t$ denote period utility in Home. A second-order expansion around the non-stochastic efficient steady state, where $C=N=1$, gives

$$\begin{aligned}U(C_t,N_t)&=\frac{1}{1-\sigma}(C)^{1-\sigma}-N+\left[c_t-n_t+\frac{1-\sigma}{2}(c_t)^2-\frac{1}{2}(n_t)^2\right]N+\text{o}(||\boldsymbol{u}||^3)\\&=U(C^e,N^e)+\left[c_t-n_t+\frac{1-\sigma}{2}(c_t)^2-\frac{1}{2}(n_t)^2\right]N+\text{o}(||\boldsymbol{u}||^3)\end{aligned} \tag{B.5}$$

where recall that $C^e_t=C$ and $N^e_t=N$. Defining $\mathcal{L}_H=\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t[U(C^e,N^e)-U(C_t,N_t)]$ as in (21), we get

$$\mathcal{L}_H=\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[-c_t+n_t+\frac{\sigma-1}{2}(c_t)^2+\frac{1}{2}(y_t)^2\right]+\text{o}(||\boldsymbol{u}||^3), \tag{B.6}$$

where we used $n_t=y_t+\text{o}(||\boldsymbol{u}||^2)$.

Next, from $Y_t(l) = C_{H,t}(l) + C^*_{H,t}(l)$ and the labor market clearing, $N_t = \int_0^1 N_t(l)dl$, we have

$$N_t = \left[\int_0^1 \left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon} dl\right] C_{H,t} + \left[\int_0^1 \left(\frac{P^*_{H,t}(l)}{P^*_{H,t}}\right)^{-\varepsilon} dl\right] C^*_{H,t}$$

and recalling that $Y_t = C_{H,t} + C^*_{H,t}$ from (8), we obtain to a second-order

$$\sum_{t=0}^{\infty} \beta^t n_t = \sum_{t=0}^{\infty} \beta^t \left\{ y_t + \frac{\varepsilon}{2\kappa} \left[(1-\alpha) \left(\pi_{H,t}\right)^2 + \alpha \left(\pi^*_{H,t}\right)^2 \right] \right\} + \mathrm{o}(||\boldsymbol{u}||^3) \tag{B.7}$$

where we used the fact that to a second-order the price dispersions, denoted $\delta_{H,t}$ and $\delta^*_{H,t}$ are given by $\delta_{H,t} = \frac{\varepsilon}{2}\mathrm{var}(p_{H,t}) + \mathrm{o}(||\boldsymbol{u}||^3)$ and $\delta^*_{H,t} = \frac{\varepsilon}{2}\mathrm{var}(p^*_{H,t}) + \mathrm{o}(||\boldsymbol{u}||^3)$ where "var" is our notation for variances (see Engel, 2011 for details), and from the fact that (following Woodford, 2003, Chapter 6) $\sum_{t=0}^{\infty} \mathrm{var}(p_{H,t}) = \frac{1}{\kappa}\sum_{t=0}^{\infty}(\pi_{H,t})^2$ and $\sum_{t=0}^{\infty} \mathrm{var}(p^*_{H,t}) = \frac{1}{\kappa}\sum_{t=0}^{\infty}(\pi^*_{H,t})^2$.

Next, combining market clearing for Home goods (8) with the risk sharing condition (7), we get

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha \left(\mathcal{M}_t \mathcal{Z}_t\right)^{-\eta} \tilde{\theta}^{-\frac{1}{\sigma}} \mathcal{Q}_t^{\frac{\sigma\eta-1}{\sigma}} \right] C_t$$

and the second-order approximation yields

$$\begin{aligned} y_t = {} & c_t + \alpha\eta\left(s_t - z_t\right) - \alpha\eta m_t - \frac{\alpha}{\sigma}\tilde{\theta} + \alpha\frac{\sigma\eta-1}{\sigma}\left[m_t + \frac{1}{2}(1-2\alpha)\left(s_t - s^*_t\right)\right] + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta s_t^2 \\ & + \frac{1}{2}\alpha(1-\alpha)\left\{\eta m_t + \eta z_t + \frac{1}{\sigma}\tilde{\theta}_t - \frac{\sigma\eta-1}{\sigma}\left[m_t + \frac{1}{2}(1-2\alpha)\left(s_t - s^*_t\right)\right]\right\}^2 + \mathrm{o}(||\boldsymbol{u}||^3) \end{aligned}$$

Using $z_t = \frac{1}{2}(s_t + s^*_t)$ and $s_t^2 = (s^R_t)^2 + \mathrm{o}(||\boldsymbol{u}||^3)$ since $z_t = 0 + \mathrm{o}(||\boldsymbol{u}||^2)$ to a first-order we arrive at

$$\begin{aligned} c_t = {} & y_t + \frac{\alpha}{\sigma}\left(m_t + \tilde{\theta}\right) - \frac{D-1+2\alpha}{2\sigma}s^R_t - \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta\left(s^R_t\right)^2 \\ & - \frac{\alpha(1-\alpha)}{2\sigma^2}\left[(1-2\alpha)\left(\sigma\eta-1\right)s^R_t - \left(m_t + \tilde{\theta}\right)\right]^2 + \mathrm{o}(||\boldsymbol{u}||^3) \end{aligned} \tag{B.8}$$

Moreover, from (B.8), it follows that

$$\begin{aligned} (c_t)^2 = {} & (y_t)^2 + \frac{\alpha(1-\alpha)}{\sigma^2}\left[m_t + \tilde{\theta} - (1-2\alpha)\left(\sigma\eta-1\right)s^R_t\right]^2 \\ & - \alpha(1-\alpha)\eta^2\left(s^R_t\right)^2 + \left[\frac{\alpha}{\sigma}\left(m_t + \tilde{\theta}\right) - \frac{D-1+2\alpha}{2\sigma}s^R_t\right]\left(y_t + y^*_t\right) + \mathrm{o}(||\boldsymbol{u}||^3) \end{aligned} \tag{B.9}$$

Finally, substituting (B.7), (B.8) and (B.9) into (B.6) and using (15), we arrive at

$$\begin{aligned} \mathcal{L}_H = {} & \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{\kappa}\left[(1-\alpha)(\pi_{H,t})^2 + \alpha(\pi^*_{H,t})^2\right] + \sigma(y_t)^2 - \frac{2\alpha}{\sigma}(m_t + \tilde{\theta}) + \frac{D-1+2\alpha}{\sigma}s^R_t \right. \\ & \left. + \frac{\alpha(1-\alpha)\eta}{D}(m_t + \tilde{\theta})^2 + \frac{1-D}{D}\sigma(y^R_t)^2 + (\sigma-1)\left[\frac{\alpha}{\sigma}(m_t + \tilde{\theta}) - \frac{D-1+2\alpha}{2\sigma}s^R_t\right]\left(y_t + y^*_t\right)\right\}. \quad \square \end{aligned}$$

B.3 Derivations of (24)

From (10), we have

$$\tilde{\Theta} = \frac{\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left((1-\alpha) \left(\mathcal{S}_t^*\right)^{\eta-1} + \alpha \right)^{-1} \left(C_t^*\right)^{1-\sigma}}{\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left((1-\alpha) \left(\mathcal{S}_t\right)^{\eta-1} + \alpha \right)^{-1} \left(C_t\right)^{1-\sigma}} \tag{B.10}$$

To a first-order, we have

$$\tilde{\theta} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} (1-\beta)\, \beta^t \left[(1-\sigma)\left(c_t^* - c_t\right) + (1-\alpha)(1-\eta)\left(s_t^* - s_t\right) \right] + o(||\boldsymbol{u}||^2) \tag{B.11}$$

and thus $\tilde{\theta} = 0 + o(||\boldsymbol{u}||^2)$. To a second order, we have

$$\begin{aligned}\tilde{\theta} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} (1-\beta)\, \beta^t \Big[& (1-\sigma)\left(c_t^* - c_t\right) + \frac{1}{2}(1-\sigma)^2 \left(c_t^{*2} - c_t^2\right) - (1-\alpha)(1-\eta)\left(s_t - s_t^*\right) \\ & + (1-\alpha)(1-\eta)\,(1-\sigma)\left(s_t^* c_t^* - s_t c_t\right) + \frac{\sigma-1}{2}\tilde{\theta}\left(c_t + c_t^*\right) \Big] + o(||\boldsymbol{u}||^3)\end{aligned}$$

where we also used

$$\left(\mathbb{E}_{-1} \sum_{t=0}^{\infty} (1-\beta)\, \beta^t \left[(1-\sigma)\left(c_t^* - c_t\right) + (1-\alpha)(1-\eta)\left(s_t^* - s_t\right) \right] \right)^2 = 0 + o(||\boldsymbol{u}||^3) \tag{B.12}$$

Then, using $\sigma(c_t - c_t^*) = \tilde{\theta} + m_t + (1-2\alpha)s_t^R$ from (13) and noting that $\tilde{\theta}\mathbb{E}_{-1}\sum_{t=0}^{\infty}(1-\beta)\beta^t(c_t + c_t^*) = 0 + o(||\boldsymbol{u}||^3)$ along with $s_t^* c_t^* - s_t c_t = -s_t^R(c_t + c_t^*) + o(||\boldsymbol{u}||^3)$ from $z_t = 0$ to a first-order, we get

$$\begin{aligned}\tilde{\theta} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} (1-\beta)\, \beta^t \Big\{ & (\sigma-1)m_t + \left(\frac{D-1}{2\alpha} - (\sigma-1) \right) s_t^R \\ & - \frac{1}{2}(\sigma-1) \left[(\sigma-1)m_t + \left(\frac{D-1}{2\alpha} - (\sigma-1) \right) s_t^R \right] \left(c_t + c_t^*\right) \Big\} + o(||\boldsymbol{u}||^3)\end{aligned} \tag{B.13}$$

Finally, noting that $c_t + c_t^* = y_t + y_t^*$ and rearranging (B.13) yields (24). $\square$

B.4 Derivation of (23)

Iterating forward (A.17) and (A.20), we get

$$\frac{V_{H,0}^*}{\kappa} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \psi_{H,t}^* + \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\psi_{H,t}^* \left(\psi_{H,t}^* + 2x_{H,t}^* \right) + 2u_t + u_t^2 \right] + \frac{\varepsilon}{2\kappa}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_{H,t}^*\right)^2 + o(||\boldsymbol{u}||^3)$$

$$\frac{V_{F,0}}{\kappa} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \psi_{F,t} + \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\psi_{F,t} \left(\psi_{F,t} + 2x_{F,t} \right) + 2u_t^* + u_t^{*2} \right] + \frac{\varepsilon}{2\kappa}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_{F,t}\right)^2 + o(||\boldsymbol{u}||^3)$$

Then using (A.15), (A.16), (A.23), (A.24) to substitute for $\psi^*_{H,t}$, $x^*_{H,t}$, $x_{F,t}$ and $\psi_{F,t}$ we get

$$\begin{aligned}\frac{1}{\kappa}\mathbb{E}_{-1}\left[V^*_{H,0}-V_{F,0}\right] = &-\frac{v}{1-\beta}+\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left(s^R_t-m_t\right)+\frac{\varepsilon}{2\kappa}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\left(\pi^*_{H,t}\right)^2-\left(\pi_{F,t}\right)^2\right]\\ &-\frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{m_t+(1-\sigma)\left(q_t+\tilde{\theta}\right)-\left[1+2\sigma(1-\alpha)\left(\eta-1\right)\right]s^R_t\right\}\left(y_t+y^*_t\right)\\ &+\frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left(2u_t+u_t^2-2u^*_t-u^{*2}_t\right)+\mathrm{o}(||\boldsymbol{u}||^3)\end{aligned} \tag{B.14}$$

where it should be noted that we also used $\sigma(c_t-c^*_t)=\tilde{\theta}+m_t+(1-2\alpha)s^R_t$ from (13). Finally, noting from (B.3) that $v=\frac{1-\beta}{\kappa}\mathbb{E}_{-1}[V_{F,0}-V^*_{H,0}]$, so that the left-hand side of (B.14) cancels the term $-v/(1-\beta)$ on its right-hand side, and rearranging (B.14), we arrive at (23) where t.i.p stands for the terms independent of policies, $\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t(2u_t+u_t^2-2u^*_t-u^{*2}_t)$. □

B.5 Proof of Proposition 1

Substituting (23) into (26) and noting from (B.13), that is

$$\tilde{\theta}=\mathbb{E}_{-1}\sum_{t=0}^{\infty}(1-\beta)\,\beta^t\left[(\sigma-1)m_t+\left(\frac{D-1}{2\alpha}-(\sigma-1)\right)s^R_t\right]+\mathrm{o}(||\boldsymbol{u}||^2)=0+\mathrm{o}(||\boldsymbol{u}||^2) \tag{B.15}$$

that $(\tilde{\theta})^2=\mathrm{o}(||\boldsymbol{u}||^3)$ and $\tilde{\theta}\ \mathbb{E}_{-1}\sum_{t=0}^{\infty}(1-\beta)\beta^t(y_t+y^*_t)=\mathrm{o}(||\boldsymbol{u}||^3)$, we get

$$\begin{aligned}\mathcal{L}_H=\frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{&\frac{\varepsilon}{\kappa}\left[(1-\alpha)\left(\pi_{H,t}\right)^2+\alpha\left(\pi_{F,t}\right)^2\right]+\sigma\left(y_t\right)^2+\frac{1-D}{4D}\sigma\left(y_t-y^*_t\right)^2\\ &+\frac{\alpha(1-\alpha)\eta}{D}\left(m_t\right)^2+\left[\alpha m_t-\frac{D-1+2\alpha}{2}s^R_t\right]\left(y_t+y^*_t\right)\Bigg\}+\mathrm{o}(||\boldsymbol{u}||^3)+\text{t.i.p}\end{aligned} \tag{B.16}$$

where we also used $q_t=m_t+(1-2\alpha)s^R_t$. Then, note from (15) that we have

$$\alpha m_t-\frac{D-1+2\alpha}{2}s^R_t=\frac{1}{2}\left[-\sigma\left(y_t-y^*_t\right)+q_t\right]$$

which we plug back into (B.16) to obtain

$$\begin{aligned}\mathcal{L}_H=\frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{&\frac{\varepsilon}{\kappa}\left[(1-\alpha)\left(\pi_{H,t}\right)^2+\alpha\left(\pi_{F,t}\right)^2\right]+\frac{\sigma}{2}\left[\left(y_t\right)^2+\left(y^*_t\right)^2\right]\\ &+\frac{1-D}{D}\sigma\left(y^R_t\right)^2+\frac{\alpha(1-\alpha)\eta}{D}\left(m_t\right)^2+q_ty^W_t\Bigg\}+\mathrm{o}(||\boldsymbol{u}||^3)+\text{t.i.p}\end{aligned} \tag{B.17}$$

Finally we use (27) and

$$\frac{\sigma}{2}\left[\left(y_t\right)^2+\left(y^*_t\right)^2\right]+\frac{1-D}{D}\sigma\left(y^R_t\right)^2=\sigma(y^W_t)^2+\frac{\sigma}{D}(y^R_t)^2 \tag{B.18}$$

to arrive at (25). □

B.6 Proof of Propositions 2

Optimal policy. The problem of the Home policymaker consists in choosing $\{i_t, \pi_t, \pi_t^*, y_t^W, y_t^R, m_t\}$ to minimize (25) subject to (28)-(32). The associated Lagrangian is given by

$$\begin{aligned}
\mathbb{H} = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t & \left[\frac{\varepsilon}{\kappa}(\pi_t)^2 + \sigma(y_t^W)^2 + \frac{\sigma}{D}(y_t^R)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 + 2\sigma\left(\frac{1-2\alpha}{D}y_t^R + \frac{2\alpha(1-\alpha)\eta}{D}m_t\right)y_t^W\right] \\
& + \mu_{h,t}\left[\beta\mathbb{E}_t\pi_{t+1} - \pi_t + \kappa\sigma\left(y_t^W + \frac{1-2\alpha}{D}y_t^R + \frac{2\alpha(1-\alpha)\eta}{D}m_t\right) + \kappa\tilde{u}_t\right] \\
& + \mu_{h,t}^*\left[\beta\mathbb{E}_t\pi_{t+1}^* - \pi_t^* + \kappa\sigma\left(y_t^W - \frac{1-2\alpha}{D}y_t^R - \frac{2\alpha(1-\alpha)\eta}{D}m_t\right) + \kappa\tilde{u}_t^*\right] \\
& + \xi_{h,t}\left[2\sigma\mathbb{E}_t y_{t+1}^W - 2\sigma y_t^W - (i_t + i_t^*) + \mathbb{E}_t\left(\pi_{t+1} + \pi_{t+1}^*\right) + 2r^n\right] \\
& + \xi_{h,t}^*\left[\mathbb{E}_t m_{t+1} - m_t - (i_t - i_t^*) + \mathbb{E}_t\left(\pi_{t+1} - \pi_{t+1}^*\right) + (1-2\alpha)\mathbb{E}_t\left(f\left(\boldsymbol{u}_{t+1}\right) - f\left(\boldsymbol{u}_t\right)\right)\right] \\
& + \vartheta_{h,t}\left[\frac{2\sigma}{D}y_t^R - \frac{1-2\alpha}{D}m_t - f\left(\boldsymbol{u}_t\right)\right]
\end{aligned}$$

with $\mu_{h,-1} = \mu_{h,-1}^* = 0$. The first-order conditions for i_t, y_t^W, y_t^R and m_t are given by

$$[i_t] :: \xi_{h,t}^* = -\xi_{h,t} \tag{B.19}$$

$$\left[y_t^W\right] :: y_t^W + \frac{1-2\alpha}{D}y_t^R + \frac{2\alpha(1-\alpha)\eta}{D}m_t + \kappa(\mu_{h,t} + \mu_{h,t}^*) - 2\left(\xi_{h,t} - \frac{1}{\beta}\xi_{h,t-1}\right) = 0 \tag{B.20}$$

$$\left[y_t^R\right] :: \frac{1}{D}y_t^R + \frac{1-2\alpha}{D}y_t^W + \kappa\frac{1-2\alpha}{D}\left(\mu_{h,t} - \mu_{h,t}^*\right) + \frac{2}{D}\vartheta_{h,t} = 0 \tag{B.21}$$

$$[m_t] :: \frac{\alpha(1-\alpha)\eta}{D}m_t + \frac{2\alpha(1-\alpha)\sigma\eta}{D}y_t^W + \kappa\frac{2\alpha(1-\alpha)\sigma\eta}{D}(\mu_{h,t} - \mu_{h,t}^*) - \left(\xi_{h,t}^* - \frac{1}{\beta}\xi_{h,t-1}^*\right) - \frac{1-2\alpha}{D}\vartheta_{h,t} = 0 \tag{B.22}$$

Combining (B.21) and (B.22), we obtain

$$\frac{1-2\alpha}{D}y_t^R + \frac{2\alpha(1-\alpha)\eta}{D}m_t + y_t^W + \kappa\left(\mu_{h,t} - \mu_{h,t}^*\right) + 2\left(\xi_{h,t} - \frac{1}{\beta}\xi_{h,t-1}\right) = 0 \tag{B.23}$$

where we also used $\xi_{h,t}^* = -\xi_{h,t}$ from (B.19). Then, using (B.23) to substitute for $(\xi_{h,t} - \beta^{-1}\xi_{h,t-1})$ in (B.20), we arrive at

$$y_t^W + \frac{1-2\alpha}{D}y_t^R + \frac{2\alpha(1-\alpha)\eta}{D}m_t + \kappa\mu_{h,t} = 0 \tag{B.24}$$

Next, using (B.19), the first-order condition for π_t can be expressed as

$$\frac{\varepsilon}{\kappa}\pi_t = \mu_{h,t} - \mu_{h,t-1} \quad \Longrightarrow \quad \varepsilon(p_t - p_{-1}) = \kappa\mu_{h,t} \tag{B.25}$$

Finally, we substitute (B.25) into (B.24) to obtain (33). □

B.7 The simplified representation: derivations of (34)

Thus, from (14) we have that $c_t = y_t^W + \frac{q_t}{2\sigma}$ which we use to obtain

$$\sigma(y_t^W)^2 + q_t y_t^W = \sigma(c_t)^2 - \frac{(q_t)^2}{4\sigma}. \tag{B.26}$$

Moreover, using $\frac{q_t}{2\sigma} = \frac{2\alpha(1-\alpha)\eta}{D} m_t + \frac{1-2\alpha}{D} y_t^R$ and $D = 4\alpha(1-\alpha)\sigma\eta + (1-2\alpha)^2$ we obtain that

$$\begin{aligned} -\frac{(q_t)^2}{4\sigma} + \frac{\sigma}{D}(y_t^R)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 &= \frac{\alpha(1-\alpha)\eta}{D^2}\left[4\sigma^2(y_t^R)^2 - 4\sigma(1-2\alpha)m_t y_t^R + (1-2\alpha)^2(m_t)^2\right] \\ &= \frac{\alpha(1-\alpha)\eta}{D^2}\left[2\sigma y_t^R - (1-2\alpha)m_t\right]^2 = \alpha(1-\alpha)\eta\,(s_t^R)^2 \end{aligned} \tag{B.27}$$

Combining (B.26) and (B.27), we get that

$$\sigma(y_t^W)^2 + q_t y_t^W + \frac{\sigma}{D}(y_t^R)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 = \sigma(c_t)^2 + \alpha(1-\alpha)\eta\,(s_t^R)^2 \tag{B.28}$$

Plugging (B.28) into (25) and recalling that $(s_t^R)^2$ is independent of policy (t.i.p), we arrive at (34). □

B.8 Proof of Propositions 3

The Nash equilibrium. First, note that, by symmetry with the Home policymaker's loss function, that the loss function of the Foreign policymaker is given by

$$\mathcal{L}_F = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}(\pi_t^*)^2 + \sigma(y_t^W)^2 + \frac{\sigma}{D}(y_t^R)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 - q_t y_t^W\right] + t.i.p. \tag{B.29}$$

The problem of the Foreign policymaker thus consists in choosing $\{i_t^*, \pi_t, \pi_t^*, y_t^W, y_t^R, m_t\}$ to minimize (B.29) subject to (28)-(32). Proceeding as in Appendix B.6, the targeting rule in Foreign is given, by symmetry, by

$$y_t^W - \frac{1-2\alpha}{D}y_t^R - \frac{2\alpha(1-\alpha)\eta}{D}m_t = -\varepsilon\left(p_t^* - p_{-1}^*\right). \tag{B.30}$$

Taking the sum of (33) and (B.30), and the difference between (33) and (B.30), we obtain that the targeting rules in the Nash equilibrium can be characterized by

$$y_t^W = -\varepsilon(p_t^W - p_{-1}^W) \qquad \text{and} \qquad \frac{1-2\alpha}{D}y_t^R + \frac{2\alpha(1-\alpha)\eta}{D}m_t = -\varepsilon(p_t^R - p_{-1}^R) \tag{B.31}$$

The loss function of the global planner. The global planner seeks to maximize the average of households' welfare in Home and Foreign or, alternatively, minimize

$$\mathcal{L}_G = \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\left[U\left(C^e, N^e\right) - U\left(C_t, N_t\right)\right] + \frac{1}{2}\left[U\left(C^e, N^e\right) - U\left(C_t^*, N_t^*\right)\right]\right\} \tag{B.32}$$

and from (B.5) and its counterpart in Foreign, we can rewrite it as

$$\mathcal{L}_G = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t \left\{ -(c_t+c_t^*)+(n_t+n_t^*)+\frac{\sigma-1}{2}\left[(c_t)^2+(c_t^*)^2\right]+\frac{1}{2}\left[(n_t)^2+(n_t^*)^2\right]\right\}+o(||\boldsymbol{u}||^3) \quad \text{(B.33)}$$

Then, plugging in (B.8), (B.9), (B.7) and their counterparts in Foreign, we obtain

$$\begin{aligned}\mathcal{L}_G = \frac{1}{4}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t \Bigg\{ & \frac{\varepsilon}{\kappa}\left[(1-\alpha)\left(\pi_{H,t}\right)^2+\alpha\left(\pi_{F,t}\right)^2+(1-\alpha)\left(\pi_{F,t}^*\right)^2+\alpha\left(\pi_{H,t}^*\right)^2\right] \\ & +\sigma\left[(y_t)^2+(y_t^*)^2\right]+\frac{2\alpha(1-\alpha)\eta}{D}m_t^2+\frac{1-D}{2D}\sigma\left(y_t-y_t^*\right)^2\Bigg\}+o(||\boldsymbol{u}||^3) \quad \text{(B.34)}\end{aligned}$$

Finally, using (B.18), (27), $(1-\alpha)(\pi_{F,t}^*)^2+\alpha(\pi_{H,t}^*)^2 = (\pi_t^*)^2+\alpha(1-\alpha)(s_t^*-s_{t-1}^*)^2$, and noting that the term $(s_t^*-s_{t-1}^*)^2$ is independent of policy, we arrive at (37).

The cooperative solution. The global planner chooses $\{i_t, i_t^*, \pi_t, \pi_t^*, y_t^W, y_t^R, m_t\}$ to minimize (37) subject to (28)-(32). The Lagrangian is

$$\begin{aligned}\mathbb{H} = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg[& \frac{\varepsilon}{2\kappa}\left(\pi_t^2+(\pi_t^*)^2\right)+\sigma(y_t^W)^2+\frac{\sigma}{D}(y_t^R)^2+\frac{\alpha(1-\alpha)\eta}{D}m_t^2\Bigg] \\ & +\mu_t\left[\beta\mathbb{E}_t\pi_{t+1}-\pi_t+\kappa\sigma\left(y_t^W+\frac{1-2\alpha}{D}y_t^R+\frac{2\alpha(1-\alpha)\eta}{D}m_t\right)+\kappa\tilde{u}_t\right] \\ & +\mu_t^*\left[\beta\mathbb{E}_t\pi_{t+1}^*-\pi_t^*+\kappa\sigma\left(y_t^W-\frac{1-2\alpha}{D}y_t^R-\frac{2\alpha(1-\alpha)\eta}{D}m_t\right)+\kappa\tilde{u}_t^*\right] \\ & +\xi_t\left[2\sigma\mathbb{E}_t y_{t+1}^W-2\sigma y_t^W-(i_t+i_t^*)+\mathbb{E}_t\left(\pi_{t+1}+\pi_{t+1}^*\right)+2r^n\right] \\ & +\xi_t^*\left[\mathbb{E}_t m_{t+1}-m_t-(i_t-i_t^*)+\mathbb{E}_t\left(\pi_{t+1}-\pi_{t+1}^*\right)+(1-2\alpha)\mathbb{E}_t\left(f\left(\boldsymbol{u}_{t+1}\right)-f\left(\boldsymbol{u}_t\right)\right)\right] \\ & +\vartheta_t\left[\frac{2\sigma}{D}y_t^R-\frac{1-2\alpha}{D}m_t-f\left(\boldsymbol{u}_t\right)\right]\end{aligned}$$

with $\mu_{-1}=\mu_{-1}^*=0$. The optimality conditions for i_t and i_t^* imply that $\xi_t=\xi_t^*=0$.

Using $\xi_t=\xi_t^*=0$, we get that the optimality conditions for π_t and π_t^* imply

$$\varepsilon(p_t-p_{-1}) = 2\kappa\mu_t \quad \text{and} \quad \varepsilon(p_t^*-p_{-1}^*) = 2\kappa\mu_t^* \quad \text{(B.35)}$$

The first-order condition for y_t^W is

$$y_t^W+\kappa(\mu_t+\mu_t^*) = 0 \quad \text{(B.36)}$$

Next, from combining the optimality conditions for y_t^R and m_t we have

$$\frac{1-2\alpha}{D}y_t^R+\frac{2\alpha(1-\alpha)\eta}{D}m_t = -\kappa\left(\mu_t-\mu_t^*\right) \quad \text{(B.37)}$$

Finally, we substitute (B.35) into (B.36) and (B.37) to arrive at

$$y_t^W = -\varepsilon(p_t^W - p_{-1}^W) \qquad \text{and} \qquad \frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t = -\varepsilon(p_t^R - p_{-1}^R)$$

which corresponds to the targeting rules in the Nash equilibrium. □

B.9 Derivation of (39)

Starting with the preliminary loss function (22) and setting $\tilde{\theta} = 0$ gives

$$\mathcal{L}_H^P = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{\frac{\varepsilon}{\kappa}\left[(1-\alpha)(\pi_{H,t})^2 + \alpha(\pi_{H,t}^*)^2\right] + \sigma(y_t)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 + \frac{1-D}{D}\sigma(y_t^R)^2$$
$$+ (\sigma-1)\left[\frac{\alpha}{\sigma}m_t - \frac{D-1+2\alpha}{2\sigma}s_t^R\right](y_t + y_t^*)\Bigg\} + \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{D-1+2\alpha}{2\sigma}s_t^R - \frac{\alpha}{\sigma}m_t\right] \tag{B.38}$$

To determine the present value linear terms, $\Xi \equiv \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{D-1+2\alpha}{2\sigma}s_t^R - \frac{\alpha}{\sigma}m_t\right]$ in (B.38), we proceed as in Appendix B.4, by first iterating forward (A.14) and (A.19) and combining them to get

$$\frac{1}{\kappa}\mathbb{E}_{-1}\left[V_{H,0} - V_{F,0}^*\right] = -\frac{v}{1-\beta} + \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left(s_t^R + m_t\right) + \frac{\varepsilon}{2\kappa}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[(\pi_{H,t})^2 - (\pi_{F,t}^*)^2\right]$$
$$+ \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{(2-\sigma)\,m_t - [2-\sigma+2\alpha\,(\sigma\eta-1)]\,s_t^R\right\}(y_t + y_t^*)$$
$$+ \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left(2u_t + u_t^2 - 2u_t^* - u_t^{*2}\right) \tag{B.39}$$

We then multiply equation (B.39) by $\frac{D-1}{2\sigma}$ and equation (B.14) by $\frac{D-1}{2\sigma} + \frac{2\alpha}{\sigma}$ and use (B.4) to obtain

$$\Xi = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{\frac{\varepsilon}{2\kappa}\left[-\frac{D-1}{2\sigma}\left((\pi_{H,t})^2 - (\pi_{F,t}^*)^2\right) + \left(\frac{D-1}{2\sigma} + \frac{2\alpha}{\sigma}\right)\left((\pi_{F,t})^2 - (\pi_{H,t}^*)^2\right)\right]$$
$$+ \left[-(1-\sigma+\sigma\eta)\left(\frac{D-1+2\alpha}{2\sigma}s_t^R - \frac{\alpha}{\sigma}m_t\right) - \frac{\alpha}{\sigma}(\sigma\eta-1)\,q_t\right](y_t + y_t^*) + \text{t.i.p}\Bigg\} \tag{B.40}$$

We then substitute (B.40) into (B.38), and use (15), which yields

$$\mathcal{L}_H^P = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{\frac{\varepsilon}{\kappa}(\pi_t)^2 + \sigma\left(y_t^W\right)^2 + \frac{\sigma}{D}\left(y_t^R\right)^2 + \frac{\alpha(1-\alpha)\eta}{D}(m_t)^2 + \left((1-2\alpha)\eta + \frac{2\alpha}{\sigma}\right)q_t y_t^W$$
$$+ \alpha\frac{\varepsilon}{\kappa}\left[\frac{\sigma\eta-1}{\sigma}\left((\pi_t^*)^2 - (\pi_t)^2\right) + (\eta-1)\left((\pi_{F,t})^2 - (\pi_{H,t}^*)^2\right)\right] + 2\sigma(1-\eta)y_t^R y_t^W\Bigg\} + \text{t.i.p} \tag{B.41}$$

where we used $(1-\alpha)(\pi_{H,t})^2+\alpha(\pi_{F,t})^2 = (\pi_t)^2+\text{t.i.p}$ and $(1-\alpha)(\pi^*_{F,t})^2+\alpha(\pi^*_{H,t})^2 = (\pi^*_t)^2+\text{t.i.p.}$ We then use two identities. First, from $\pi_t+\pi^*_t=\pi_{F,t}+\pi^*_{H,t}$ and $\pi_t-\pi^*_t=\pi_{F,t}-\pi^*_{H,t}-2(1-\alpha)\Delta s_t$,

$$(\pi_t)^2-(\pi^*_t)^2 = (\pi_{F,t})^2-\left(\pi^*_{H,t}\right)^2-4(1-\alpha)\Delta s_t\,\pi^W_t \tag{B.42}$$

where $\Delta s_t \equiv s_t - s_{t-1}$. Second, using (15) and $q_t = m_t+(1-2\alpha)s_t$, we have that

$$\begin{aligned}\left[(1-2\alpha)\eta+\frac{2\alpha}{\sigma}-1\right]q_t y^W_t+2\sigma(1-\eta)y^R_t y^W_t &= -\frac{2\alpha(\sigma-1)}{\sigma}m_t y^W_t-\left[\frac{2\alpha(\sigma-1)(1-2\alpha)}{\sigma}+4\alpha(1-\alpha)\sigma\eta(\eta-1)\right]s_t y^W_t\\ &= -\frac{2\alpha(\sigma-1)}{\sigma}q_t y^W_t-4\alpha(1-\alpha)\sigma\eta(\eta-1)s_t y^W_t \qquad\qquad (\text{B.43})\end{aligned}$$

Finally, we substitute (B.42) and (B.43) to obtain

$$\begin{aligned}\mathcal{L}^P_H = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{&\left[\frac{\varepsilon}{\kappa}(\pi_t)^2+\sigma(y^W_t)^2+\frac{\sigma}{D}(y^R_t)^2+\frac{\alpha(1-\alpha)\eta}{D}(m_t)^2+q_t y^W_t\right]+\text{t.i.p.}\\ &+\frac{\alpha(1-\sigma)}{\sigma}\left[\frac{\varepsilon}{\kappa}\left((\pi_{F,t})^2-(\pi^*_{H,t})^2\right)+2q_t y^W_t\right]-4\alpha(1-\alpha)\left[\sigma\eta(\eta-1)\,s_t y^W_t+\frac{\varepsilon(1-\sigma\eta)}{\sigma\kappa}\Delta s_t\pi^W_t\right]\Bigg\}\end{aligned}$$

which corresponds to (39). □

B.10 Proof of Propositions 4

Consider the characterization of the loss function under partial commitment (B.41). Using (B.42) to replace for $(\pi_{F,t})^2-(\pi^*_{H,t})^2$ in (B.41) gives

$$\begin{aligned}\mathcal{L}^P_H = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\Bigg\{&\frac{\varepsilon}{\kappa}\left[\left(1-\alpha+\frac{\alpha}{\sigma}\right)(\pi_t)^2+\left(\alpha-\frac{\alpha}{\sigma}\right)(\pi^*_t)^2+2\alpha(1-\alpha)(\eta-1)\,\Delta s_t\cdot(\pi_t+\pi^*_t)\right]+\sigma\left(y^W_t\right)^2\\ &+\frac{\sigma}{D}\left(y^R_t\right)^2+\frac{\alpha(1-\alpha)\eta}{D}(m_t)^2+2\sigma(1-\eta)y^R_t y^W_t+2\left[(1-2\alpha)\sigma\eta+2\alpha\right]\left(\frac{1-2\alpha}{D}y^R_t+\frac{2\alpha(1-\alpha)\eta}{D}m_t\right)y^W_t\Bigg\}\end{aligned}$$

where we also used $\frac{q_t}{2\sigma} = \frac{1-2\alpha}{D}y^R_t+\frac{2\alpha(1-\alpha)\eta}{D}m_t$.

The problem of the Home policymaker thus consists in choosing $\{i_t, \pi_t, \pi^*_t, y^W_t, y^R_t, m_t\}$ to minimize the loss function $\mathcal{L}^P_H$ subject to (28)-(32). As in Appendix B.6, we denote by ϑ_t, $\xi_{h,t}$, $\xi^*_{h,t}$, $\mu_{h,t}$ and $\mu^*_{h,t}$ are the Lagrange multipliers on (28), (29), (30), (31) and (32) respectively with $\mu_{h,-1}=\mu^*_{h,-1}=0$. First, note that the first-order condition for i_t yields $\xi_{h,t}=-\xi^*_{h,t}$ which we use to express the first-order condition with respect to π_t as

$$\frac{\varepsilon}{\kappa}\left[\left(1-\alpha+\frac{\alpha}{\sigma}\right)\pi_t+\alpha(1-\alpha)(\eta-1)\,\Delta s_t\right] = \mu_{h,t}-\mu_{h,t-1} \tag{B.44}$$

Next, we have that the first-order conditions for $[y_t^W]$, $[y_t^R]$ and $[m_t]$ are respectively given by

$$0 = y_t^W + (1-\eta) y_t^R + \left[(1-2\alpha)\eta + 2\alpha\sigma^{-1}\right] \left(\frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t\right) + \kappa\left(\mu_{h,t} + \mu_{h,t}^*\right) - 2\left[\xi_{h,t} - \frac{1}{\beta}\xi_{h,t-1}\right]$$
$$0 = \frac{1}{D} y_t^R + (1-\eta) y_t^W + \left[(1-2\alpha)\eta + 2\alpha\sigma^{-1}\right] \frac{1-2\alpha}{D} y_t^W + \kappa \frac{1-2\alpha}{D}\left(\mu_{h,t} - \mu_{h,t}^*\right) + \frac{2}{D}\vartheta_{h,t}$$
$$0 = \frac{\alpha(1-\alpha)\eta}{D} m_t + [(1-2\alpha)\sigma\eta + 2\alpha] \frac{2\alpha(1-\alpha)\eta}{D} y_t^W + \kappa \frac{2\alpha(1-\alpha)\sigma\eta}{D}\left(\mu_{h,t} - \mu_{h,t}^*\right) + \left[\xi_{h,t} - \frac{1}{\beta}\xi_{h,t-1}\right] - \frac{1-2\alpha}{D}\vartheta_{h,t}$$

which we combine to eliminate for $\vartheta_{h,t}$ and $[\xi_{h,t} - \frac{1}{\beta}\xi_{h,t-1}]$ and obtain

$$2\kappa\mu_{h,t} + (1-\eta) y_t^R + 2\left(1-\alpha+\frac{\alpha}{\sigma}\right) y_t^W + \left[1 + (1-2\alpha)\eta + \frac{2\alpha}{\sigma}\right]\left(\frac{2\alpha(1-\alpha)\eta}{D} m_t + \frac{1-2\alpha}{D} y_t^R\right) = 0$$

and using (15) can be rewritten as

$$\kappa\mu_{h,t} - \alpha(1-\alpha)\eta\,(\eta-1)\, s_t + \left(1-\alpha+\frac{\alpha}{\sigma}\right)\left(y_t^W + \frac{2\alpha(1-\alpha)\eta}{D} m_t + \frac{1-2\alpha}{D} y_t^R\right) = 0 \tag{B.45}$$

Moreover, from (B.44), we have that

$$\kappa\mu_{h,t} = \varepsilon\left(1-\alpha+\frac{\alpha}{\sigma}\right)(p_t - p_{-1}) + \alpha(1-\alpha)\varepsilon\,(\eta-1)\, s_t \tag{B.46}$$

We then substitute (B.46) into (B.45), to eliminate for $\kappa\mu_{h,t}$, and obtain

$$y_t^W + \frac{1-2\alpha}{D} y_t^R + \frac{2\alpha(1-\alpha)\eta}{D} m_t + \frac{\alpha(1-\alpha)}{1-\alpha+\alpha\sigma^{-1}}(\eta-1)(\varepsilon-\eta)\, s_t = -\varepsilon\,(p_t - p_{-1})$$

which corresponds to (40). □

B.11 Proof of Propositions 5

We start by noting that the set of implementability constraints (i.e., the set of constraints that a policymaker faces) is invariant to the form of commitment. Therefore, for each domestic policymaker, any allocation $a \equiv \{\pi_t, \pi_t^*, y_t^W, y_t^R, m_t\}_{t=0}^{\infty}$ that is feasible under full commitment is also feasible under partial commitment.

We now proceed in three steps. First, let a^P denote the allocation in the Nash equilibrium under partial commitment and a^{P*} the optimal allocation chosen by the global policymaker under partial commitment. Then, it must be

$$\mathcal{L}_G(a^{P*}) \leq \mathcal{L}_G(a^P) \tag{B.47}$$

with strict inequality when $a^P \neq a^{P*}$. Second, because the objective function of the global planner is the same under both forms of commitment, it follows that

$$a^{F*} = a^{P*} \tag{B.48}$$

where a^{F*} is the cooperative allocation under full commitment. Third, by Proposition 3, we have $a^F = a^{F*}$ with a^F denoting the allocation in the Nash equilibrium under full commitment. Then, by (B.48) and (B.47), it must be that

$$\mathcal{L}_G(a^F) \leq \mathcal{L}_G(a^P) \tag{B.49}$$

with strict inequality when $a^F \neq a^P$. □

B.12 Derivation of (43)

Using the simplified representation (34) and its Foreign counterpart, we have that

$$\mathcal{L}_G(a) = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{2\kappa}\left((\pi_t)^2 + (\pi_t^*)^2\right) + \frac{\sigma}{2}\left((c_t)^2 + (c_t^*)^2\right)\right] + \text{t.i.p.} \tag{B.50}$$

For $a^P = a^F + \hat{a}$ with $\hat{a} \equiv \{\hat{\pi}_t, \hat{\pi}_t^*, \hat{c}_t, \hat{c}_t^*\}_{t=0}^{\infty}$, the second-order expansion yields

$$\mathcal{L}_G(a^P) - \mathcal{L}_G(a^F) = \mathcal{D}(\hat{a}) + \mathcal{Q}(\hat{a}) \tag{B.51}$$

where

$$\begin{aligned}
\mathcal{D}(\hat{a}) &\equiv \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}\left(\pi_t(a^F)\hat{\pi}_t + \pi_t^*(a^F)\hat{\pi}_t^*\right) + \sigma\left(c_t(a^F)\hat{c}_t + c_t^*(a^F)\hat{c}_t^*\right)\right] \\
\mathcal{Q}(\hat{a}) &\equiv \frac{1}{4}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}\left((\hat{\pi}_t)^2 + (\hat{\pi}_t^*)^2\right) + \sigma\left((\hat{c}_t)^2 + (\hat{c}_t^*)^2\right)\right]
\end{aligned}$$

The first-order term $\mathcal{D}(\hat{a})$ vanishes because a^F satisfies the first-order conditions of each policymaker's problem, so that the objective is locally orthogonal to any feasible perturbation, as we show next. So, we have that $\hat{a}$ satisfies

$$\kappa\sigma\hat{c}_t = \hat{\pi}_t - \beta\mathbb{E}_t\hat{\pi}_{t+1} \qquad \text{and} \qquad \kappa\sigma\hat{c}_t^* = \hat{\pi}_t^* - \beta\mathbb{E}_t\hat{\pi}_{t+1}^* \tag{B.52}$$

Then, note that the Home block of the allocation a^F solves the problem of Home policymaker under full commitment. It minimizes (34) subject to the Home CPI Phillips curve, $\pi_t = \kappa\sigma c_t + \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{u}_t$. The associated Lagrangian is

$$\mathbb{H}_H = \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\left[\frac{\varepsilon}{\kappa}(\pi_t)^2 + \sigma(c_t)^2\right] + \mu_t\left[\kappa\sigma c_t + \beta\mathbb{E}_t\pi_{t+1} - \pi_t + \kappa\tilde{u}_t\right]\right\}$$

with $\mu_{-1} = 0$, and the first-order conditions with respect to c_t and π_t are

$$c_t(a^F) = -\kappa\mu_t \qquad \text{and} \qquad \frac{\varepsilon}{\kappa}\pi_t(a^F) = \mu_t - \mu_{t-1} \tag{B.53}$$

which together deliver the targeting rule $c_t = -\varepsilon(p_t - p_{-1})$. Substituting (B.53) and then (B.52) into the Home block of $\mathcal{D}(\hat{a})$,

$$\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}\pi_t(a^F)\hat{\pi}_t + \sigma c_t(a^F)\hat{c}_t\right] = \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\left(\mu_t - \mu_{t-1}\right)\hat{\pi}_t - \mu_t\left(\hat{\pi}_t - \beta\mathbb{E}_t\hat{\pi}_{t+1}\right)\right]$$
$$= -\mathbb{E}_{-1}\sum_{t=1}^{\infty}\beta^t\mu_{t-1}\hat{\pi}_t + \mathbb{E}_{-1}\sum_{t=1}^{\infty}\beta^t\mu_{t-1}\hat{\pi}_t = 0$$

where the second equality uses $\mu_{-1} = 0$ and the law of iterated expectations. The Foreign block is symmetric. It thus follows that $\mathcal{D}(\hat{a}) = 0$.

Next, substituting the targeting rule $c_t = -\varepsilon(p_t - p_{-1}) - \Lambda s_t$ into the CPI Phillips curve $\pi_t = \kappa\sigma c_t + \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{u}_t$, and normalizing $p_{-1} = p^*_{-1} = 0$, the Home consumer price level satisfies

$$\beta\mathbb{E}_t p_{t+1} - \left(1 + \beta + \kappa\sigma\varepsilon\right)p_t + p_{t-1} = \kappa\sigma\Lambda s_t - \kappa\tilde{u}_t$$

with $\Lambda = 0$ under full commitment. Taking differences across the two forms of commitment, yields

$$\beta\mathbb{E}_t\hat{p}_{t+1} - \left(1 + \beta + \kappa\sigma\varepsilon\right)\hat{p}_t + \hat{p}_{t-1} = \kappa\sigma\Lambda s_t \tag{B.54}$$

Similarly, for the Foreign price we have

$$\beta\mathbb{E}_t\hat{p}^*_{t+1} - \left(1 + \beta + \kappa\sigma\varepsilon\right)\hat{p}^*_t + \hat{p}^*_{t-1} = -\kappa\sigma\Lambda s_t \tag{B.55}$$

where we used $s^*_t = -s_t$. It follows that $\hat{p}^*_t = -\hat{p}_t$, and therefore $\hat{\pi}^*_t = -\hat{\pi}_t$ and $\hat{c}^*_t = -\hat{c}_t$, so that

$$\mathcal{Q}(\hat{a}) = \frac{1}{2}\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\kappa}(\hat{\pi}_t)^2 + \sigma(\hat{c}_t)^2\right] \tag{B.56}$$

Finally, the targeting rules under the two forms of commitment,

$$c_t(a^P) = -\varepsilon\, p_t(a^P) - \Lambda s_t$$

imply $\hat{c}_t = -\left[\varepsilon\hat{p}_t + \Lambda s_t\right]$ with $\hat{p}_t = \sum_{k=0}^{t}\hat{\pi}_k$. Substituting this into (B.56) yields (43). □

B.13 Derivation of (44)

The relative price. Guess that the solution of (20) takes the form

$$s_t = \rho_s s_{t-1} + \rho_2(u^*_t - u_t)$$

Substituting it into (20) and matching coefficients gives $\rho_2 = \kappa\rho_s$ since shocks u_t and u_t^* are i.i.d. and $\rho_s = [1+\kappa+\beta(1-\rho_s)]^{-1}$. And solving for the quadratic equation in ρ_s yields

$$s_t = \rho_s\, s_{t-1} + \kappa\rho_s\,(u_t^* - u_t)\,, \quad \text{with} \quad \rho_s = \frac{1+\kappa+\beta-\sqrt{(1+\kappa+\beta)^2-4\beta}}{2\beta} \in (0,1) \tag{B.57}$$

The relative price is therefore an AR(1) with

$$\operatorname{var}(s_t) = \frac{2\kappa^2\rho_s^2}{1-\rho_s^2}\,\sigma_u^2 \qquad \text{and} \qquad V_s \equiv \mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t s_t^2 = \frac{1-\rho_s^2}{1-\beta\rho_s^2}\,\frac{\operatorname{var}(s_t)}{1-\beta} \tag{B.58}$$

The expression for V_s uses the normalization $s_{-1} = 0$, consistent with $\hat{p}_{-1} = 0$ below. Had s_{-1} instead been drawn from the stationary distribution, $V_s = \operatorname{var}(s_t)/(1-\beta)$. More generally an extra term in s_{-1}^2 appears. None of these initializations affects the sign of Ω or the welfare ranking.

The welfare cost. By (B.54), the gap between the consumer price levels implemented under full and partial commitment satisfies $\beta\mathbb{E}_t\hat{p}_{t+1} - (1+\beta+\kappa\sigma\varepsilon)\hat{p}_t + \hat{p}_{t-1} = \kappa\sigma\Lambda s_t$ with $\hat{p}_{-1} = 0$. Let $\phi \in (0,1)$ denote the stable root of $\beta z^2 - (1+\beta+\kappa\sigma\varepsilon)z + 1 = 0$. Since $\mathbb{E}_t s_{t+j} = \rho_s^j s_t$ by (B.57), the unique bounded solution is

$$\hat{p}_t = -\frac{\lambda\,\Lambda}{\varepsilon}\sum_{i=0}^{t}\phi^i s_{t-i} \quad \text{with} \quad \lambda \equiv \frac{\varepsilon\kappa\sigma\phi}{1-\beta\phi\rho_s}$$

which can be rewritten as

$$\hat{p}_t = -\frac{\lambda\,\Lambda}{\varepsilon}\,g_t, \quad \text{with} \quad g_t \equiv \phi\, g_{t-1} + s_t \tag{B.59}$$

from which we get

$$\hat{\pi}_t = -\frac{\Lambda\lambda}{\varepsilon}\,(g_t - g_{t-1}) \quad \text{and} \quad \hat{c}_t = \Lambda\,(\lambda g_t - s_t)$$

Then substituting into (43) yields

$$\mathcal{L}_G(a^P) - \mathcal{L}_G(a^F) = \frac{\Lambda^2}{2}\,\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\lambda^2}{\varepsilon\kappa}\,(g_t-g_{t-1})^2 + \sigma\,(\lambda g_t - s_t)^2\right] \tag{B.60}$$

Moreover, noting that

$$\frac{\mathbb{E}_{-1}\sum_t\beta^t g_t s_t}{V_s} = \frac{1}{1-\beta\phi\rho_s}$$

$$\frac{\mathbb{E}_{-1}\sum_t\beta^t g_t^2}{V_s} = \frac{1+\beta\phi\rho_s}{(1-\beta\phi\rho_s)(1-\beta\phi^2)} \equiv \Gamma \tag{B.61}$$

$$\frac{\mathbb{E}_{-1}\sum_t\beta^t\,(g_t-g_{t-1})^2}{V_s} = \frac{(1-\beta\phi)(1-\beta\rho_s)+\beta(1-\phi)(1-\rho_s)}{(1-\beta\phi\rho_s)(1-\beta\phi^2)} \equiv \Gamma_\Delta \tag{B.62}$$

Substituting (B.61) and (B.62) into (B.60) and using (B.58) to replace V_s delivers (44), with

$$\Omega \equiv \frac{1-\rho_s^2}{1-\beta\rho_s^2}\left[\frac{\lambda^2}{\varepsilon\kappa}\Gamma_\Delta + \sigma\left(\lambda^2\Gamma - \frac{2\lambda}{1-\beta\phi\rho_s} + 1\right)\right] > 0 \tag{B.63}$$

Note that $\Omega > 0$ by (B.60). Note that one can also rewrite it as

$$\Omega = \frac{1-\rho_s^2}{1-\beta\rho_s^2}\left[\frac{\lambda^2}{\varepsilon\kappa}\Gamma_\Delta + \sigma\Gamma\left(\lambda-\lambda^*\right)^2\right] + \frac{\sigma\beta\phi^2\left(1-\rho_s^2\right)}{1-\beta^2\phi^2\rho_s^2}, \quad \text{with} \quad \lambda^* \equiv \frac{1-\beta\phi^2}{1+\beta\phi\rho_s} \tag{B.64}$$

and every term is positive since $\beta, \phi, \rho_s \in (0,1)$. □

B.14 Proof of Proposition 6

Write the Home households' nominal stochastic discount factor, which appears in the firms' objective in Appendix A, as

$$\Theta_{t,t+j} \equiv \beta^j \frac{P_t}{P_{t+j}}\left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} \quad \text{so that} \quad \Omega^*_{h,t+j} = \frac{1}{P_t C_t^\sigma}\Theta_{t,t+j}\,\mathcal{E}_{t+j}\left(P^*_{H,t+j}\right)^{\varepsilon} C^*_{H,t+j}$$

as defined in (A.2). Let $\Theta^*_{t,t+j}$ denote the Foreign counterpart. The risk-sharing condition (7) implies

$$\Theta^*_{t,t+j} = \Theta_{t,t+j}\,\frac{\mathcal{E}_{t+j}}{\mathcal{E}_t} \tag{B.65}$$

Under complete markets the two discount factors are the same object expressed in two currencies, so it is immaterial whose discount factor is used to value a stream of profits.

The distribution stage. A distributor resetting at t sets $\mathcal{P}^D_{H,t}(l)$ so that

$$\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Theta^*_{t,t+j}\left[\mathcal{P}^D_{H,t}(l) - \frac{\mathcal{P}^X_{H,t}(l)}{\mathcal{E}_{t+j}}\right] C^*_{H,t+j}(l) = 0, \qquad C^*_{H,t+j}(l) = \left(\frac{\mathcal{P}^D_{H,t}(l)}{P^*_{H,t+j}}\right)^{-\varepsilon} C^*_{H,t+j}$$

It is important to note that zero profits hold in present value over the life of the price rather than state by state. Substituting (B.65) and cancelling $\mathcal{E}_t$ and $\left(\mathcal{P}^D_{H,t}(l)\right)^{-\varepsilon}$,

$$\mathcal{P}^D_{H,t}(l) = \mathcal{X}_t\,\mathcal{P}^X_{H,t}(l), \qquad \text{with} \qquad \mathcal{X}_t \equiv \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j}/\mathcal{E}_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j}} \tag{B.66}$$

The conversion factor $\mathcal{X}_t$ is common across firms and does not depend on the firm's own price.

The export stage. Taking (B.66) as given, and using $Y_t(l) = N_t(l)$, firm l chooses $\mathcal{P}^X_{H,t}(l)$ to maximize the export component of its profits,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Theta_{t,t+j} \Big[(1+\tau)\mathcal{P}^X_{H,t}(l) - W_{t+j} \Big] C^*_{H,t+j}(l), \qquad C^*_{H,t+j}(l) = \left(\frac{\mathcal{X}_t \mathcal{P}^X_{H,t}(l)}{P^*_{H,t+j}} \right)^{-\varepsilon} C^*_{H,t+j}$$

The first-order condition is

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Theta_{t,t+j} \left(P^*_{H,t+j} \right)^{\varepsilon} C^*_{H,t+j} \Big[(1+\tau)(1-\varepsilon)\mathcal{P}^X_{H,t}(l) + \varepsilon W_{t+j} \Big]$$

and, since $\Theta_{t,t+j} \left(P^*_{H,t+j} \right)^{\varepsilon} C^*_{H,t+j} \propto \Omega^*_{h,t+j} / \mathcal{E}_{t+j}$,

$$\mathcal{P}^X_{H,t}(l) = \frac{\varepsilon}{(1+\tau)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \frac{\Omega^*_{h,t+j}}{\mathcal{E}_{t+j}} W_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \frac{\Omega^*_{h,t+j}}{\mathcal{E}_{t+j}}} \tag{B.67}$$

Multiplying (B.67) by $\mathcal{X}_t$ from (B.66), the term $\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j} / \mathcal{E}_{t+j}$ cancels and

$$\mathcal{P}^D_{H,t}(l) = \frac{\varepsilon}{(1+\tau)(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j} \frac{W_{t+j}}{\mathcal{E}_{t+j}}}{\mathbb{E}_t \sum_{j=0}^{\infty} \zeta^j \Omega^*_{h,t+j}} = \mathcal{P}^*_{H,t}(l)$$

which is (A.2).

Allocations. Because the price paid by Foreign households is the same in the both economies under LCP and under DCP with distribution sector, and so are $P^*_{H,t}$, demands $C^*_{H,t+j}(l)$, employment, and market-clearing conditions. As for the distribution of wealth, the present value of a Home exporter's foreign revenue is $\mathbb{E}_t \sum_j \zeta^j \Theta_{t,t+j} \mathcal{P}^X_{H,t}(l) C^*_{H,t+j}(l)$ with border invoicing and $\mathbb{E}_t \sum_j \zeta^j \Theta_{t,t+j} \mathcal{E}_{t+j} \mathcal{P}^*_{H,t}(l) C^*_{H,t+j}(l)$ under LCP; by (B.66) and $\mathcal{P}^D_{H,t}(l) = \mathcal{P}^*_{H,t}(l)$ the two coincide (distributors break even). The two revenue streams differ state by state, but their difference has zero present value and is spanned by the traded claims, so with complete markets the equilibrium consumption allocation, and therefore λ, λ^* and $\tilde{\Theta}$, are unchanged. □