U.S. Liquid Government Liabilities and Emerging Market Capital Flows *

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Abstract

Empirical work finds that flows of investments from the U.S. and other high income countries to emerging markets increase during times of quantitative easing by the U.S. Federal Reserve, and the reverse movement occurs under quantitative tightening. We offer new evidence to confirm these findings, and then propose a theory based on the liquidity of U.S. government liabilities held by the public. We hypothesize that QE, by increasing liquidity, offers greater flexibility for investors that might be concerned their funds will be tied up when shocks to income or investment opportunities arise. With the assurance that some of their portfolio can be readily sold in liquid markets, rich country investors are more willing to increase investments in illiquid loans to emerging markets. The effect of increasing the liquidity of U.S. government liabilities on investments in EMs may even be stronger during times of greater uncertainty.

JEL Classification Codes: F32, F41, G15

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1 Introduction

Government bonds issued by the U.S. are safe, liquid, and offer a convenience yield.¹ Policymakers and academics have emphasized the importance of liquidity in the market for U.S. government liabilities in facilitating efficient functioning of financial markets.² Short-term T-bills or reserves held at the Federal Reserve appear to be the ultimate prize for investors that require liquidity, and the effects of insufficient liquidity have been extensively studied. However, the provision of greater liquidity to the markets has implications for the demand for other types of assets. We ask how funding to emerging market firms and governments is affected by changes in the supply of liquid U.S. debt.

Quantitative easing is an example of liquidity provision. Under quantitative easing, the Federal Reserve buys less liquid debt of the government – medium- and long-term Treasury bonds and agency debt. It creates reserves held at the Fed. Following Rogoff (2017), ch. 8, “pure” quantitative easing in which the Fed buys long-term Treasury bonds in exchange for money which creates reserves is equivalent to changing the maturity and liquidity of U.S. government debt without changing the quantity outstanding. Since reserves pay interest (and in recent years, the interest rate on reserves has usually been nearly identical to the interest rate on 30-day Treasury bills), the Fed is in essence swapping short-term debt for long-term debt. The Fed’s balance sheet is part of the consolidated U.S. government balance sheet (interest the Fed earns on its government bond portfolio is repatriated to the general Treasury account), so quantitative easing is effectively an operation that changes the maturity structure of the outstanding Treasury debt held by the public. In fact, Nagel (2016) refers to short-term T-bills as “near-money assets”, implying that short-term T-bills and reserves are close substitutes. Reserves and short-term Treasury bills are prized for their high liquidity, as recent events such as the March 2020 “dash for cash” have verified.³

Our study is, therefore, related to work that has investigated the impact of quantitative easing on reserves and bank lending. Most of that literature has centered on the ways in which domestic lending has been influenced.⁴ However, Bhattarai et al. (2021) and Burger et al. (2018) are empirical studies whose findings are similar to ours – that quantitative easing in the U.S. appears to increase capital flows to emerging markets. This finding at first glance is puzzling because quantitative easing supplies the markets with liquid U.S. government assets during times in which they are in high demand, which, ceteris paribus, would tend to redirect asset demand toward these U.S.

²See Duffie (2023), Copeland et al. (2021), Acharya and Rajan (2022), Acharya et al. (2023), Diamond et al. (2023) and Afonso et al. (2022).
³See Vissing-Jorgensen (2021), Barone et al. (2022) and Cesa-Bianchi and Eguren Martin (2021).
⁴Diamond et al. (2023), Kandrac et al. (2021), Kandrac and Schlusche (2021), Li et al. (2019), Rodnyansky and Darmouni (2017), Chakraborty et al. (2020) and Acharya and Dogra (2022).
Figure 1: Quantitative Tightening in the U.S. and Capital Inflows to Other Countries

Notes: Each line represents a time series of weekly bond fund flows as a ratio of the previous week’s bond fund allocation for 16 emerging market countries. The data on weekly bond fund flows are from the weekly the Emerging Portfolio Fund Research dataset.

assets and away from emerging market investments. We confirm the findings in these previous studies and provide a potential explanation in which the increase in liquidity leads to an overall increase in saving and lending in advanced countries.

Figure 1 illustrates these effects on capital flows to emerging markets. In June 2013, the chairman of the Federal Reserve, Ben Bernanke, announced that the Fed could begin to scale back its purchases of long-term bonds. The so-called “taper tantrum” ensued. In the figure, we plot the capital flows into 16 emerging markets in 2012-2013.\(^5\) It is apparent that there was a sharp decline in flows immediately following Bernanke’s announcement of quantitative tightening (QT). The figure also picks up an acceleration of flows into those countries following the announcement of QE3, the third round of quantitative easing by the Fed, in September 2012.

The framework in which we address this question is one in which some U.S. government bonds are more liquid in the sense that they are traded in deep markets, the creditworthiness of the government is well known, and the bonds can easily be sold on secondary markets. For example, short-term bonds, and reserves held at the Fed are very liquid government liabilities, while longer term bonds may be less liquid. We investigate the distributional consequences of changing the composition of government bond issuance as the U.S. government’s portfolio of debt shifts from longer term, less liquid debt to shorter term, more liquid debt. We are concerned about the effects on loan availability to emerging markets. Because the U.S. is a borrower on global markets, it competes with the emerging market countries (EMs) for loans from the rest of the world (ROW). We want to understand the effect of a shift in the composition of the U.S. debt per se, holding the total debt issuance constant.

\(^5\)See Section 2 for details on the data.
In our model, liquidity is valued because it provides flexibility. The essence of our argument is that when a greater volume of liquid assets become available, agents are willing to save more because they are not as concerned with being locked into illiquid investments. They can hold a larger portfolio of assets - more liquid and more illiquid assets - and be assured that they can sell a sufficient amount of the liquid assets when funds are needed for consumption or investment. The increased supply of liquid assets under QE does not just push down their price and lead investors to switch away from other assets to short-term U.S. government liabilities. It also increases the willingness to increase overall lending, including in the form of less liquid loans to emerging markets.

The U.S. government is a borrower/debtor. Household lenders in the U.S. and ROW decide on a portfolio of loans – short-term (liquid) U.S. bonds, long-term (less liquid) U.S. bonds, and loans to EMs – before the realization of their uncertain income or investment opportunities. When uncertainty is resolved, they can adjust their portfolio by trading with other households that have different outcomes. If a household/lender finds its income is higher than expected, or it has less need for funds for investment in real assets, it can buy liquid bonds from households whose income is lower than expected or who have greater investment opportunities. That is, there is a secondary market in liquid bonds that operates after the resolution of uncertainty, but no such market exists for the other two types of loans. (EMs have no need for the liquid bonds since their income path is known at the time of the initial portfolio choice, and, in fact, we are assuming that the time discount factor of EMs is low such that EMs always want to be borrowers and therefore have no demand for U.S. government bonds of either type.) Hence, we associate “liquidity” with “flexibility” as in the classic works of Hahn et al. (1990) and Jones and Ostroy (1984).

Holding total saving constant, the government purchases illiquid debt and pays with liquid debt under quantitative easing. Holding the size of households’ total debt portfolio constant, the increase supply of liquid debt to markets leads to a drop in their price and induces a shift in portfolios from less liquid assets toward the liquid debt. Why then would lending to emerging markets increase? We posit that lenders do not hold their total savings constant. These lenders want to save/lend to smooth consumption or because they have poor investment opportunities, but they are leery of being burdened with illiquid assets. If they receive a bad income shock in the short-run or a shock that requires new investment, they are forced to lower their consumption. A greater supply of liquid assets allows investors to sell liquid assets if their incomes turn out to be low; so liquid instruments offer flexibility. Investors are more confident they can save/lend a greater amount, and some of that lending is channeled to EMs.

In the data, while all U.S. government debt appears to earn a convenience yield or liquidity yield relative to privately issued debt that is equally riskless or relative to government debt of other countries, the market considers short-term debt to be more liquid than long-term debt. Our model simplifies by allowing some debt to be traded on secondary markets, while other government obligations are not.
We are not interested in, and abstract from, the fiscal implications of the U.S. debt composition. That is, the timing of taxes is not the focus of our study. As the literature has noted, even in a setting with infinitely-lived representative agents, the presence of a liquidity return to government bonds breaks Ricardian equivalence. The particular fiscal policy that is optimal depends on the specifics of how liquidity interacts with private-market decisions, as well as the interdependence of fiscal and monetary policy.\(^6\) The government of our model simply borrows and makes lump-sum transfers, and the driver of U.S. current account deficits is government borrowing.\(^7\)

We have empirically examined the effects of Federal Reserve quantitative easing (and subsequent quantitative tightening) as a way of verifying the predictions of the model. As explained above, T-bills and reserves are effectively substitutes – nearly equally liquid assets (though we have explored the possibility that reserves are more liquid than T-bills, and our empirical results are not affected.) We examine Fed purchases and sales of longer-term Treasury bonds. These are actions that do not materially affect the overall debt of the consolidated government but change the liquidity of U.S. government obligations held by the public.\(^8\) We make use of weekly data on equity and bond flows to emerging markets from the Emerging Portfolio Fund Research (EPFR) dataset. We find that in weeks in which the Fed makes large purchases (sales) of longer-term Treasuries, there is a subsequent increase (decrease) in U.S. funds flowing to EMs. This effect is statistically significant out to a horizon of at least 52 weeks. This effect is robust to controls for U.S. conditions (VIX, returns on 10-year U.S. Treasury bonds, changes in the S&P 500 index) as well as recipient-country specific controls (central bank policy rates, change in industrial production, change in the exchange rate relative to the dollar, year-on-year CPI inflation and reserves/GDP.)\(^9\)

Section 2 presents empirical evidence to support the claim that quantitative easing leads to increased capital flows to emerging markets. Section 3 presents a model that helps explain this phenomenon, while section 4 examines the model quantitatively. Section 5 concludes.

### 2 Empirical Evidence

There is a large literature that has examined the impact of quantitative easing and other unconventional Federal Reserve monetary policies on emerging markets. Some have studied high-frequency responses of financial variables in EMs to announcements of Fed policies. Another strand of research has employed structural vector autoregressions to look at macroeconomic impacts of these policies on EMs. The latter includes a number of papers that have investigated the effects on capital flows to a broad set of emerging markets. A common finding of this work is that quantitative easing

\(^6\)See Azzimonti and Yared (2019), Bigio et al. (2019), Bassetto and Cui (2018), Bayer et al. (2023), Acharya and Dogra (2022), Krishnamurthy et al. (2018), Berentsen and Waller (2018) and Andolfatto and Martin (2018).

\(^7\)Although there are other reasons for the U.S. current account deficit in these models, Kekre and Lenel (2021) and Jiang et al. (2021) are examples of models in which the seigniorage from issuance of safe assets plays an important role in determining the U.S. deficit.

\(^8\)We ignore the fiscal effects that arise because as interest rates on longer term debt differ from shorter term debt.

\(^9\)The analysis is weekly, and U.S. variables are weekly, but most EM variables are measured monthly.
increases flows to EMs. Here we present some additional evidence using high-frequency responses of capital flows to QE.

Lim et al. (2014) finds large effects of QE on quarterly capital inflows to 60 emerging markets and developing countries in the 2000-2013 period. Ahmed and Zlate (2014) use regression analysis for 12 emerging markets from 2002-2013 and find that large-scale asset purchases by the Federal Reserve lead to higher quarterly capital inflows to these countries. Tillmann (2016) estimates a VAR in which surprise QE shocks are identified and finds a strong impact on U.S. quarterly capital flows to emerging markets in Asia and Latin America in the 2007-2013 period. Duca et al. (2016) find that quarterly corporate bond issuance in 18 emerging markets increases following QE in the 2001-2013 period. Anaya et al. (2017) use a large-scale structural global VAR, which identifies QE shocks using Fed balance sheet data, and find a significant impact on U.S. monthly capital flows to 19 emerging markets in the 2004-2018 period. Bhattacharai et al. (2021) estimate a structural VAR for the US, then use the identified QE shocks from that VAR to examine the impact on macro variables for thirteen emerging markets in 2008-2014. They find a large and persistent effect of QE in raising monthly capital inflows into these markets. Chari et al. (2021) use high-frequency identification to obtain a measure of unconventional US monetary policy surprises. Then using monthly data from the U.S Treasury TIC survey, they examine the effects of surprises in the periods of QE and QT, and the pre-crisis period in the 1994-2008 span on fifteen emerging markets, confirming findings in the other studies cited here. Georgiadis and Jarocinski (2023) use high-frequency changes in asset prices to identify the effects of different unconventional US monetary policy shocks, and find that QE triggers quarterly capital inflows to an aggregate of EMs in the 1996-2019 interval.

Kolasa and Wesołowski (2020) build a calibrated structural model to explain how QE in the US might lead to flows into purchases of EM’s sovereign bonds. As returns on US assets decline due to QE, investors find potential returns on EM sovereign bonds more attractive. Their analysis, which centers on default risk, complements our focus on liquidity. Kim (2023) builds a money-search model in which QE in a country such as the US, whose assets serve as collateral, can lead to capital flows to EMs.

The empirical study closest to ours is Fratzscher et al. (2018). Using the high frequency capital flow data from 2008 to 2012 on portfolio flows from EPFR, the study assesses the effects of QE announcements on capital flows. Relative to this work, we look at actual large purchases or sales of Treasury assets by the Federal Reserve, rather than the response to announcements, and find more significant effects. As we detail next, we look at the dynamics of capital flows to EMs, and find that the effects of these operations on capital flows to EMs are quite persistent. Our study uses a longer time span that includes periods of quantitative tightening as well as large scale asset purchases.

Related are a large number of papers that examine spillovers of U.S. monetary policy more generally (that is, not specifically spillovers from quantitative easing or tightening) to emerging markets, including Rey (2016), Bruno and Shin (2015), Passari and Rey (2015), Aizenman et al.
Our empirical approach considers quantitative easing as an alteration of the maturity of government debt held by the investors. When the Federal Reserve buys long-term bonds, the Fed pays with money, increasing reserves of the banking system. QE changes the liquidity composition of government debt - the public holds less long-term Treasury obligations and more short-term debt in the form of reserves. We are interested in how the shift in the liquidity composition of government debt has affected capital inflows to other countries.

2.1 Data Sources

We employ weekly bond flows data from the EPFR dataset. The EPFR dataset collects data on individual fund flows and fund allocations, and aggregates them to flows into and out of specific countries and sectors. The weekly capital flows and fund allocations are reported every Wednesday. Flows cover the total flows occurring over the past week. “Allocations” are their values as of every Wednesday. Both flows and allocations are reported in units of the U.S. dollar.\textsuperscript{10} Weekly capital flows cover mutual fund investors and exchange traded fund investors. We employ weekly bond flows and allocations at the country-level.

With this data, we analyze how the weekly capital bond inflows to countries change following a sharp increase in the Fed’s holding of U.S. Treasury securities. The country sample includes 16 emerging market countries: Brazil, Chile, China, Colombia, Hungary, India, Korea, Mexico, Malaysia, Israel, Peru, Philippines, Poland, Russia, Thailand, and Turkey. The sample period is from 05/01/2003—12/31/2017.\textsuperscript{11}

Our identification of the dynamic effects on capital inflows to emerging markets after a large purchase of government bonds by the Fed relies on high frequency capital flow data. The EPFR is the most suited to our analysis given that the capital flow data are available at high frequency compared to other datasets such as the data from the U.S. Department of Treasury International Capital (TIC) System or each country’s balance of payments statistics. The investor coverage of the dataset is incomplete compared to the balance of payments data; nonetheless, the EPFR dataset is highly representative in capturing the aggregate flow data from the balance of payments data (see, for instance, Jotikasthira et al. (2012).)

2.2 Event Study Analysis

We first flag weeks when the weekly log changes in the Fed’s outright holding of U.S. Treas-

\textsuperscript{10}The results do not change when employing allocations and flows in constant 2015 USD.

\textsuperscript{11}The weekly capital flow data from EPFR are only available from May 2003.
sury securities is larger than 1.302%, which corresponds to the 95th percentile of the empirical distribu-
tion of weekly log changes in the Fed’s outright holding of U.S. Treasury securities in 12/18/2002-12/28/2022.\textsuperscript{12}

We estimate the following weekly panel local linear projections (Jordà (2005)):

\begin{align}
\ln(BondFundAllocation_{i,t+k}) - \ln(BondFundAllocation_{i,t-1}) &= \alpha_{i,k} + \beta_{k} D_{t} + u_{i,k,t} \\
\frac{BondFundFlow_{i,t+k}}{BondFundAllocation_{i,t+k-1}} &= \alpha_{i,k} + \beta_{k} D_{t} + u_{i,k,t}. \tag{2}
\end{align}

Here, $BondFundAllocation_{i,t}$ refers to the total foreign holdings of bonds of country $i$ at weekly date $t$ in the EPFR data, so the change in that variable includes new allocations plus capital gains or losses. $BondFundFlow_{i,t}$ is the flow of funds into country $i$ over the past one week prior to weekly date $t$. $D_{t}$ is a dummy variable that is equal to one, when, on the weekly date $t$, the Fed’s holding of U.S. Treasury securities increases more than 1.302% over one week and zero otherwise. Regression 1 measures how global bond fund allocations to each country $i$ are affected $k$ weeks after, compared to the bond funds allocation one week before, after the Fed’s large purchase of U.S. Treasury securities. In regression 2, we look at how each week’s bond fund flows as a ratio of previous week’s bond fund allocation, $\frac{BondFundFlow_{i,t+k}}{BondFundAllocation_{i,t+k-1}}$, changes $k$ weeks after a large increase in the Fed’s purchase of Treasuries. The coefficients of interest are $\beta_{k}$ for $k = 0, \ldots, 52$. We include country fixed effects in both specifications, and standard errors are clustered at the country-level.

2.3 Results

Figures 2 and 3 show similar patterns. In Figure 2, we see that the bond fund allocation increases by around 25% after a large purchase at the peak and goes down over time. In Figure 3, weekly capital inflows (normalized by the previous period’s fund allocation) increase by one percentage point per week and the fall back to zero around 25 weeks after the quantitative easing.

2.4 Robustness Checks: Additional Exercises

We perform additional robustness exercises. First, we flag only the Fed’s purchases of U.S. Treasury securities that are not U.S. Treasury bills. We create a new dummy that is equal to one, when, on the weekly date $t$, the Fed’s holding of U.S. Treasury securities excluding U.S. Treasury bills increases more than 1.331% over one week and zero otherwise. Second, we define a new shock variable that is 1 when the Fed’s holding of U.S. Treasury securities increases more than 1.302% over one week, -1 when the Fed’s holding of U.S. Treasury securities decreases more than 0.519% over one week, and zero otherwise. 1.302% and -0.519% correspond to 95th and 5th percentile of

\textsuperscript{12}The first observation is on 12/18/2002, and every Wednesday, the Fed reports its holdings of U.S. Treasury securities.
Figure 2: Effect of Large Increase in Fed’s Holding of U.S. Treasuries on Bond Fund Allocations

**Response of Bond Allocations to Fed’s Treasury Holdings Increase**

Notes: The dependent variable is, $\ln(BondFundAllocation_{i,t+k}) - \ln(BondFundAllocation_{i,t-1})$ multiplied by 100. The response shows how the bond funds allocations have changed over 52 weeks after the Fed’s large purchase of the U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.

Figure 3: Effect of Large Increase in Fed’s Holding of U.S. Treasuries on Bond Fund Flows

**Response of Bond Flows to Fed’s Treasury Holdings Increase**

Notes: The dependent variable is, $\frac{BondFundFlow_{i,t+k}}{BondFundAllocation_{i,t+k-1}}$ multiplied by 100. The response shows how week-over-week bond fund inflows have changed over 52 weeks after the Fed’s large purchase of the U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.
the weekly changes in the Fed’s holding of U.S. Treasuries. This measure then allows for both a large increase and decrease in Fed’s holding of U.S. Treasuries.

Third, we add global and country-level control variables to the regressions (1) and (2). Global variables at the weekly frequency include log of the VIX index, the 10-year Treasury bond yield, and changes in the log of the S&P500 index. We also control for the week-over-week changes in the log of bilateral nominal exchange rates against the U.S. dollar to show that the results are not driven by the exchange rate fluctuations. Other country-level controls are mostly at the monthly frequency: central bank policy rates, month-over-month log changes in industrial production index, and year-over-year CPI inflation. Central banks’ reserves as a fraction of GDP is also included but only available at the quarterly frequency.  

Lastly, we examine a more narrow time period from 2008-2015, during which quantitative easing and tightening were actively used as policy tools.

Figure 4 shows that results do not change when we flag only the dates with abnormally higher increase in Fed’s holding of U.S. long-term Treasuries. The results change very little when we use an alternative measure. Figure 5 shows the results are robust to controlling for global and country-level variables.

Figure 6 shows the results where \( D_t \) is equal to one, when, on the weekly date \( t \), the Fed’s holding of U.S. Treasury securities increases more than its 95th percentile value over one week, negative one when its change is less than its 5th percentile value over one week and zero otherwise. Now, the dummy variable captures not only a large purchase of U.S. Treasury securities by the Fed but also its reversal. The results are intact regardless of whether we only capture quantitative easing or both quantitative easing and tightening.

Figure 7 shows that results are similar when we look at a narrower window of time from 2008 to 2015. The results are not driven by the fact that our sample included both non-crisis and crisis times.

In sum, we show that the shift of the liquidity composition of U.S. government towards short-term liquid assets comes with larger capital inflows to emerging markets. At first glance this empirical finding is counterintuitive because quantitative easing supplies the markets with liquid U.S. government assets, which, ceteris paribus, would tend to redirect asset demand toward these U.S. assets and away from investments to other countries. Rationalizing this empirical pattern, we build a simple two-period model that features three different assets with different liquidity profiles and explain how a higher supply of liquid U.S. Treasuries insures investors and leads to more lending to other countries.

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13For example, when data is available only monthly, in the regressions, we give each weekly observation the monthly value of the variable.
Notes: The dependent variable is, \( \ln(BondFundAllocation_{i,t+k}) - \ln(BondFundAllocation_{i,t-1}) \) multiplied by 100. The response shows how the bond funds allocations have changed over 52 weeks after the Fed’s large purchase of the longer-term U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.

Notes: The dependent variable is, \( \frac{BondFundFlow_{i,t+k}}{BondFundAllocation_{i,t-1}} \) multiplied by 100. The response shows how week-over-week bond fund inflows have changed over 52 weeks after the Fed’s large purchase of the longer-term U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.
Figure 5: Controlling for Global and Country-level Variables

Response of Bond Allocations to Fed’s Treasury Holdings Increase with Controls

Notes: The dependent variable is, $\ln(BondFundAllocation_{i,t+k}) - \ln(BondFundAllocation_{i,t-1})$ multiplied by 100. The response shows how the bond funds allocations have changed over 52 weeks after the Fed’s large purchase of the U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.

Response of Bond Flows to Fed’s Treasury Holdings Increase with Controls

Notes: The dependent variable is, $\frac{BondFundFlow_{i,t+k}}{BondFundAllocation_{i,t+k}}$ multiplied by 100. The response shows how week-over-week bond fund inflows have changed over 52 weeks after the Fed’s large purchase of the U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.
Figure 6: Capturing Both Quantitative Easing and Tightening

Response of Bond Allocations to Fed’s Treasury Holdings Change

Notes: The dependent variable is, $\ln(BondFundAllocation_{i,t+k}) - \ln(BondFundAllocation_{i,t-1})$ multiplied by 100. The response shows how the bond funds allocations have changed over 52 weeks after the Fed’s large purchase or sale of the U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.

Response of Bond Flows to Fed’s Treasury Holdings Change

Notes: The dependent variable is, $\frac{BondFundFlow_{i,t+k} - BondFundFlow_{i,t-1}}{BondFundAllocation_{i,t+k-1}}$ multiplied by 100. The response shows how week-over-week bond fund inflows have changed over 52 weeks after the Fed’s large purchase or sale of the U.S. Treasuries. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.
Figure 7: Subsample Period: 2008 – 2015


Notes: The dependent variable is, $\ln(BondFundAllocation_{i,t+k}) - \ln(BondFundAllocation_{i,t-1})$ multiplied by 100. The response shows how the bond funds allocations have changed over 52 weeks after the Fed’s large purchase of the U.S. Treasuries. The sample period is now restricted to 2008 to 2015. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.


Notes: The dependent variable is, $\frac{BondFundFlow_{i,t+k}}{BondFundAllocation_{i,t+k-1}}$ multiplied by 100. The response shows how week-over-week bond fund inflows have changed over 52 weeks after the Fed’s large purchase of the U.S. Treasuries. The sample period is now restricted to 2008 to 2015. The shaded area represents the 95% confidence interval. Standard errors are clustered at the country-level.
3 Model

Our objective in this section is to provide a rationale for why lending to EMs increases with quantitative easing. We build a simple two-period model to illustrate how the changes in the liquidity profile of U.S. Treasuries affects investors’ lending to EMs.

3.1 Environment

There are two periods, 0 and 1. There are three countries, U.S., ROW and EM, populated by mass of households, where \( a = \{\text{US, ROW,EM}\} \). Each country’s households may have different time discount factors, \( \beta^a \), where \( a = \{\text{US, ROW,EM}\} \). Each household exhibits constant relative risk aversion, parametrized by \( \sigma \). We set up a model with three countries to allow for EMs to be borrowers, but to be consistent with the fact that the U.S. is also a net borrower on international markets. In our set-up, the U.S. is a borrower because of the government. Households in the U.S., under our parameter configuration, are savers and are lenders both to the U.S. government and to EMs. We take the amount of U.S. government debt as exogenously given.

**US and ROW Households** In the beginning of period 0, U.S. and ROW households do not know their incomes but know that they may have high income with probability \( p \) or low income with probability \( 1 - p \). \( y_{0}^{a,k} \) denotes the income at time \( t = 0 \) for country \( a = \{\text{US, ROW}\} \) and income type \( k = \{H, L\} \), where \( y_{0}^{a,H} > y_{0}^{a,L} \). In period 1, U.S. and ROW households receive incomes of \( y_{1}^{a} \), where \( a = \{\text{US, ROW}\} \), regardless of their incomes in period 0.\(^{14}\) U.S. and ROW households pay lump-sum taxes in periods 0 and 1, \( T_{0}^{a} \) and \( T_{1}^{a} \), where \( a = \{\text{US, ROW}\} \).

In the beginning of period 0, before U.S. and ROW households learn their incomes, they decide how much to save in three different types of saving vehicles that offer different levels of liquidity: liquid U.S. Treasury bonds \( b^{a} \geq 0 \), less-liquid U.S. Treasuries \( \tilde{b}^{a} \geq 0 \), and illiquid loans \( \ell^{a} \) to EM for country \( a = \{\text{US, ROW}\} \).\(^{15}\) Then, U.S. and ROW households learn their incomes \( y_{0}^{a,k} \) and adjust their liquid U.S. Treasuries holdings. Liquid Treasuries can be bought and sold between households after their income levels are realized. Each household in country \( a \) of income type \( k = \{H, L\} \) buy additional liquid Treasuries \( b^{a,k} \) at the price \( \frac{1}{R_{b,s}} \) (or sell if \( b^{a,k} < 0 \) ), after its income type \( k \) is obtained. The amount that they can sell at this point is restricted by how much the investors hold before the income shock is realized: \( b^{a,k} \geq -b^{a} \).

\(^{14}\)Here, we have allowed for income uncertainty about shocks to exogenous endowments that occur after the saving decisions are made by households. We can reinterpret shocks to endowments as a varying amount of capital required for production of goods in period 1. Households learn about the technology for production in period 1 after their consumption/saving decision is made in the initial period. They may find that additional investment is required for the project to be productive, which would require redirecting resources from consumption to capital investment. A simple framework with investment is summarized in the Appendix.

\(^{15}\)The time discount factors of U.S. and ROW households are much higher than EM households such that EM households want to borrow from the U.S. and ROW households.
Illiquid Treasuries cannot be sold and bought after income realizations. The difference between liquid and less-liquid Treasuries captures the ease of selling assets when needed. The price of illiquid bonds is $\frac{1}{R_b}$.

Illiquid loans to EM households also cannot be sold when the secondary market opens. They also require US and ROW households to pay transaction costs as a function of the size of the amount of loans: $f(\ell^a)$, where $\ell^a$ is the amount of loans extended to EMs, at the price $\frac{1}{R_\ell}$, where $f' > 0$ and $f'' > 0$ for country $a = \{US, ROW\}$. This formulation for the transaction cost is intended as a reduced form that might represent the cost of searching for acceptable projects to invest in. As the amount lent increases, the cost rises, and at an increasing rate to represent the increasing scarcity of acceptable projects larger loans.

Before learning one’s income $y^{a,k}_0$, each household in country $a$ chooses liquid U.S. Treasury bonds $b^a$, less-liquid U.S. Treasuries $\tilde{b}^a$, and illiquid loans $\ell^a$ to EM to maximize the expected utility from consumptions in periods 0 and 1:

$$
\max_{b^a \geq 0, \tilde{b}^a \geq 0, \ell^a} \quad pU^{a,H} + (1-p)U^{a,L} + U(G^a_0, G^a_1),
$$

(3)

where $U^{a,H}$ and $U^{a,L}$ represent the utilities of households in country $a$ with income type H and L, respectively. $U(G^a_0, G^a_1)$ is utility from public goods consumption, provided by each household’s government in periods 0 and 1, and is separable from the utility from consumption for country $a = \{US, ROW\}$. Hence, it does not affect the optimal household saving choices.

Households choose to increase (or reduce if $b^{a,k} < 0$) their liquid Treasury holdings by $b^{a,k}$ after income realizations to maximize $U^{a,k}$, subject to the constraint,

$$
b^{a,k} \geq -b^a,
$$

and $U^{a,k}$ is defined as:

$$
U^{a,k} = \max_{b^{a,k} \geq -b^a} \left( \frac{(c^{a,k}_0)^{1-\sigma}}{1-\sigma} + \beta^a \frac{(c^{a,k}_1)^{1-\sigma}}{1-\sigma} \right),
$$

(4)

where $c^{a,k}_0$ and $c^{a,k}_1$ are the levels of consumption of income type $k$ after learning their incomes in periods 0 and 1, respectively. The budget constraints (for $k = \{H, L\}$) for consumption in periods 0 and 1 are given as:

$$
c^{a,k}_0 = y^{a,k}_0 - T^{a}_0 - \frac{\ell^a}{R_\ell} - \frac{b^a}{R_b} - \frac{\tilde{b}^a}{R_{b,\tilde{b}}} - f(\ell^a), \quad c^{a,k}_1 = y^{a}_1 - T^{a}_1 + \ell^a + b^a + \tilde{b}^a + b^{a,k}. 
$$

(5)

Before learning one’s income, each household in country $a$ saves in liquid U.S. Treasury bonds $b^a$, less-liquid U.S. Treasuries $\tilde{b}^a$, and illiquid loans $\ell^a$ to EMs at the prices $\frac{1}{R_b}$, $\frac{1}{R_{b,\tilde{b}}}$, and $\frac{1}{R_\ell}$, respec-
tively. Then, the household learns its income type \( k \), receiving after-tax income \( y^a_{0,k} - T^a_{0} \), and one rebalancing its liquid Treasuries by buying \( (b^a_{0,k} > 0) \) or selling additionally \( (b^a_{0,k} < 0) \) at the price \( \frac{1}{R_{b,s}} \), and consumes \( c^a_{0,k} \) in period 0. In period 1, each receives its after-tax income \( y^a_{1} - T^a_{1} \) and the payoffs from its lending: liquid U.S. Treasury bonds \( b^a + b^a_{0,k} \), less-liquid U.S. Treasuries \( \tilde{b}^a \), and illiquid loans \( \ell^a \).

**EM Households**

For simplicity, we assume that EM households do not face income uncertainty in period 0 unlike U.S. and ROW households. \( y_{t}^{EM} \) denotes the income at time \( t = 0, 1 \) for EM households. The time discount factor for EM households is much lower than those of ROW and US households, and this low time discount factor renders EM households to be borrowers from ROW and US households. EM households pay the transaction costs as a function of the total value of loans extended to them: \( f(\frac{\ell^{EM}}{R_\ell}) \). This cost is symmetric to the lender’s cost for making the loan, and is meant to be a reduced-form representation of the two-sided problem of matching borrowers and lenders.

Given that both liquid and less-liquid Treasuries offer higher liquidity to savers, in equilibrium, the interest rates on these Treasuries are lower than that on illiquid loans.\(^{16}\) EMs would not optimally choose to save in these lower interest-bearing Treasuries, because they borrow via the most illiquid assets in equilibrium. It would be better to reduce borrowing than to borrow at high interest rates and save at low interest rates, so their saving in both liquid and illiquid U.S. Treasuries is zero.

EM households choose the amount of borrowing via illiquid loans \( \ell^{EM} \) to maximize their utility from consumptions in periods 0 and 1:

\[
\max_{\ell^{EM}} \left( \frac{c^{EM}_{0}}{1 - \sigma} + \beta^{EM} \frac{c^{EM}_{1}}{1 - \sigma} \right),
\]

where \( c^{EM}_{0} \) and \( c^{EM}_{1} \) represent the consumptions in period 0 and 1, respectively. The budget constraints are given as:

\[
c^{EM}_{0} = y^{EM}_{0} + \frac{\ell^{EM}}{R_\ell} - f(\frac{\ell^{EM}}{R_\ell}), \quad c^{EM}_{1} = y^{EM}_{1} - \ell^{EM}.
\]

In period 0, EM households receive endowments of \( y^{EM}_{0} \), choose how much illiquid loans to take, \( \ell^{EM} \), at the price \( \frac{1}{R_\ell} \), and the transaction costs, \( f(\frac{\ell^{EM}}{R_\ell}) \), and they consume \( c^{EM}_{0} \). In period 1, EM households receive the endowment of \( y^{EM}_{1} \), and pay back their loans \( \ell^{EM} \).

**U.S. Government**

The total amount of U.S. government bonds is exogenously given as \( B \). The share of liquid U.S. Treasuries in total U.S. Treasuries is exogenous and set as \( \eta \). We vary \( \eta \) in our experiments to investigate the implication of the liquidity composition for lending to EMs. The U.S. government budget constraints in periods 0 and 1 are given as:

\(^{16}\) We show this hierarchy in the interest rates formally in Proposition 1.
\[ G^{US}_0 = B \left( \frac{\eta}{R_b} + \frac{1-\eta}{R_{\tilde{b}}} \right) + m^{US}T^{US}_0 \] and \[ G^{US}_1 = -B + m^{US}T^{US}_1. \]

U.S. government spending in period 0, \( G^{US}_0 \), is equal to the amount of borrowing in liquid and less-liquid U.S. Treasuries and the lump-sum tax collected from U.S. households, \( m^{US}T^{US}_0 \). We assume that \( G^{US}_0 \) varies with \( \eta \) such that \( T^{US}_0 \) is constant. This assumption allows us to abstract from effects of varying transfers and taxes on government revenues and borrowing as the liquidity composition of the U.S. government bonds. In essence, we assume that if tax collections fall when the government increases the share of bonds that are liquid, government spending in period 0 falls to offset the decline in revenue. U.S. government spending in period 1, \( G^{US}_1 \), is equal to the lump-sum tax collected from U.S. households, \( m^{US}T^{US}_1 \) after paying back their government debt, \( B \). As the total amount of U.S. government bonds \( B \) is fixed as well as the lump-sum tax in period 1, \( T^{US}_1 \), U.S. government spending in period 1 is also a constant.

**ROW Government**

The ROW government runs a balanced budget. The budget constraints of the ROW government in periods 0 and 1 are given as:

\[ G^{ROW}_0 = m^{ROW}T^{ROW}_0 \] and \[ G^{ROW}_1 = m^{ROW}T^{ROW}_1, \]

where ROW government spending \( G^{ROW}_t \) in both periods is equal to the lump-sum tax collected from ROW households, \( m^{ROW}T^{ROW}_t \), for \( t = 0, 1 \).

### 3.2 Market Clearing Conditions

The market clearing conditions for liquid Treasuries, illiquid Treasuries, and illiquid loans are given as below:

\[ m^{ROW}b^{ROW} + m^{US}b^{US} = \eta B, \]

\[ m^{ROW}\tilde{b}^{ROW} + m^{US}\tilde{b}^{US} = (1-\eta)B, \] and

\[ m^{US}l^{US} + m^{ROW}l^{ROW} = m^{EM}l^{EM}. \]

The market clearing condition for rebalancing liquid Treasuries among households after their income realizations is given as:

\[ m^{ROW}(pb^{ROW,H} + (1-p)b^{ROW,L}) + m^{US}(pb^{US,H} + (1-p)b^{US,L}) = 0. \]
3.3 Properties of the Model

In this section, we examine the mechanisms behind the equilibrium determination of interest rates and lending.

Households in country $a$, after they realize that their income type is $k$, adjust their holdings of liquid U.S. Treasuries by choosing $b^{a,k}$ subject to the constraint $b^{a,k} > -b^a$ and the budget constraints 5, to maximize their utility shown in Equation 4. The first order condition (F.O.C) with respect to (w.r.t.) $b^{a,k}$ is given as:

$$-\frac{1}{R_{b,s}}(c_0^{a,k})^{-\sigma} + \beta^a(c_1^{a,k})^{-\sigma} + \mu^{a,k} = 0,$$

(7)

where we denote the Lagrangian multiplier on $b^{a,k} > -b^a$ as $\mu^{a,k}$.

Savers, at the beginning of period 0, before knowing their incomes, optimally choose their savings in liquid Treasuries $b^a \geq 0$, illiquid Treasuries $\tilde{b}^a \geq 0$, loans to EMs, $\ell^a$, subject to the budget constraints 5 of incomes types $k = \{H,L\}$, to maximize their expected utility shown in Equation 3. The first-order conditions w.r.t $b^a$, w.r.t $\tilde{b}^a$, and w.r.t $\ell^a$ are, respectively:

$$p\left(-\frac{1}{R_b}(c_0^{a,H})^{-\sigma} + \beta^a(c_1^{a,H})^{-\sigma} + \mu^{a,H}\right) + (1-p)\left(-\frac{1}{R_b}(c_0^{a,L})^{-\sigma} + \beta^a(c_1^{a,L})^{-\sigma} + \mu^{a,L}\right) = 0,$$

(8)

$$p\left(-\frac{1}{R_b}(c_0^{a,H})^{-\sigma} + \beta^a(c_1^{a,H})^{-\sigma}\right) + (1-p)\left(-\frac{1}{R_b}(c_0^{a,L})^{-\sigma} + \beta^a(c_1^{a,L})^{-\sigma}\right) = 0,$$

and

$$-(\frac{1}{R_\ell} + \frac{1}{R_\ell} f'(\ell^a))(p(c_0^{a,H})^{-\sigma} + (1-p)(c_0^{a,L})^{-\sigma}) + p\beta^a(c_1^{a,H})^{-\sigma} + (1-p)\beta^a(c_1^{a,L})^{-\sigma} = 0.$$  

(10)

Given this environment, we first derive a relationship between the interest rates summarized in Lemma 1. We show that the most liquid assets offer the lowest interest rates, and the interest rate on liquid Treasuries at the issuance is identical to that in the secondary market.

**Lemma 1.** $R_\ell \geq R_{\tilde{b}} \geq R_b = R_{b,s}$.

**Proof.** Using Equations 7 and 8, we can see that $R_{b,s} = R_b$. From Equations 9 and 8,

$$\left(\frac{1}{R_b} - \frac{1}{R_b}\right)\left(p(c_0^{a,H})^{-\sigma} + (1-p)(c_0^{a,L})^{-\sigma}\right) = p\mu^{a,H} + (1-p)\mu^{a,L} \geq 0$$

which implies that $R_b \geq R_b$. Using Equations 9 and 10, we find,

$$\frac{1}{R_b} = \frac{1}{R_\ell}(1 + f'(\ell^a))$$
Therefore, as long as \( \ell > 0 \), \( R_\ell > R_b \).

Lenders get benefits other than interest rates from saving in more liquid assets, so they accept lower interest rates on more liquid assets. Lenders pay no transaction costs for both liquid Treasuries and less-liquid Treasuries, and moreover, they can sell liquid Treasuries when their income levels are realized. These perks make the most liquid assets offer the lowest interest rates.

Also, the U.S. and the ROW households face no unexpected shocks. Therefore, if the interest rates on the primary and the secondary market differ, each household can be better off by arbitraging between the primary and the secondary market. For instance, if the interest rate was higher in the primary market compared to that in the secondary market, a household could make a positive profit by saving more in liquid Treasuries in the primary market and selling them at a higher price in the secondary market. Therefore, both the interest rate at the issuance and in the secondary market are the same.

We then show the amount of loans to EM is the same across lenders due to the presence of transaction costs.

**Lemma 2.** \( \ell_{US} = \ell_{EU} \).

**Proof.** Using Equations 9 and 10, we find,

\[
\frac{1}{R_b} = \frac{1}{R_\ell} (1 + f'(\ell^a))
\]

It can be trivially shown that \( \ell_{US} = \ell_{EU} \).

This result arises because of the increasing cost of making loans. In equilibrium, the marginal cost of loans will be equal for both lenders, which then implies they will lend equal amounts.

Lastly, we show that the low income households sell all their liquid U.S. treasuries in the secondary market when the interest rate on less-liquid Treasuries is higher than that on liquid Treasuries.

**Proposition 1.** When the interest rate on less-liquid Treasuries is higher than that on liquid Treasuries, \( R_{bl} > R_b \), \( b^{a, L} + b^a \geq 0 \) binds while \( b^{a, H} + b^a \geq 0 \) does not bind. Low-income households sell all their liquid assets in equilibrium, while high-income households do not.

**Proof.** When \( R_{bl} > R_b = R_{b,s} \), either \( \mu^{a, H} > 0 \) or \( \mu^{a, L} > 0 \) or both are positive. We show that \( \mu^{a, H} = 0 \) and \( \mu^{a, L} > 0 \) by showing that two other cases are not possible.

Case 1: \( \mu^{a, H} > 0 \) and \( \mu^{a, L} = 0 \).

Household budget constraints can be rearranged as \( c_{a, H}^{a, H} = c_{0}^{a, L} + (y_0^{H} - y_0^{L}) - \frac{b^{a, H} - b^{a, L}}{R_b} \) and \( c_{1}^{a, H} = c_{1}^{a, L} + (b^{a, H} - b^{a, L}) \). And, \( b^{a, H} = -b^a < b^{a, L} \) as the constraint only binds for high income households, and it implies that \( b^{a, H} - b^{a, L} < 0 \). As \( y_0^{H} - y_0^{L} > 0 \) and \( b^{a, H} - b^{a, L} < 0 \), using Equation 4, we can show that
\[
\begin{align*}
\mu^{a,L} &= \frac{1}{R_b} (c_0^{a,L})^{-\sigma} - \beta^a (c_1^{a,L})^{-\sigma} - \sigma = 0 \\
\mu^{a,H} &= \frac{1}{R_b} \left( c_0^{a,H} + \frac{b^{a,H} - b^{a,L}}{R_b} - \frac{b^{a,H} - b^{a,L}}{R_b} \right)^{-\sigma} - \beta^a \left( c_1^{a,L} + (b^{a,H} - b^{a,L}) \right)^{-\sigma} < \mu^{a,L} = 0
\end{align*}
\]

Therefore, we cannot have \( \mu^{a,H} > 0 \).

Case 2: \( \mu^{a,H} > 0 \) and \( \mu^{a,L} > 0 \).

The constraints bind for both high and low incomes so \( b^{a,H} = -b^a \) and \( b^{a,L} = -b^a \). The two equalities cannot hold unless there is a zero supply of liquid Treasuries.

In sum, we conclude the constraint binds for low-income households but does not bind for high-income households.

Proposition 1 shows that households save in lower interest-bearing liquid Treasuries and sell all of them when their income levels turn out to be low. The remainder of their saving/lending is in illiquid Treasuries and loans to EMs. Lenders hold liquid assets, planning to sell them when their income is low in order to smooth consumption across states. These households do not hold more liquid U.S. Treasuries than the amount that they want to sell when their income levels turn out to be low, because they could be better off by lowering their savings in liquid U.S. Treasuries and increasing their savings in less-liquid U.S. Treasuries. When income is realized, high income households buy all the liquid bonds of low income households and earn \( R_b \). The high income households, ex post, wish they had saved more, and are willing to purchase the liquid bonds and earn \( R_b \).

### 3.4 Liquidity Composition and Capital Flows to EM

We next show analytically how capital flows to EM change as the liquidity profile of the outstanding U.S. Treasuries shifts towards more liquid assets. That is, we want to show how \( \ell^{EM} \) changes when the share of liquid Treasuries in total U.S. government bonds \( \eta \) varies.

In doing so, we assume that U.S. and ROW households share the same time discount factors, and the same post-tax income profiles. That is, \( \beta^{US} = \beta^{ROW} \), and \( y_t^{US,k} - T_t^{US} = y_t^{ROW,k} - T_t^{ROW} \) for \( t = 0, 1 \). Without loss of generality, we also assume that \( m^{US} + m^{ROW} = m^{EM} = 1 \). Then, we can collapse our model into two countries, a lender country and a borrower country, with an equal mass of population. Abusing notation, we will re-label those two countries as the U.S. and EM. For simplicity, we further assume that the cost for the U.S. households of lending to EM is zero, \( f(\frac{\mu^{US}}{R_t}) = 0 \).

U.S. households choose \( b^{US,L} \) and \( b^{US,H} \), taking \( b^{US} \), \( b^{US} \), and \( \ell^{US} \) as given. U.S. households choose \( b^{US,k} \) to maximize their utility from consumption shown in Equation 4, given the constraints
\(b^{US,k} + b^{US} \geq 0\) for \(k = \{H, L\}\) and the budget constraints in Equation 5. The F.O.C. w.r.t \(b^{US,k}\) are:

\[
-\frac{1}{R_{b,s}} (c_0^{US,H})^{-\sigma} + \beta^{US} (c_1^{US,H})^{-\sigma} = 0, \quad (11)
\]

\[
-\frac{1}{R_{b,s}} (c_0^{US,L})^{-\sigma} + \beta^{US} (c_1^{US,L})^{-\sigma} + \beta^{US} \mu^{US} = 0. \quad (12)
\]

As we have shown in Proposition 1, the rebalancing constraint does not bind for the high income type: \(b^{US,H} + b^{US} > 0\), i.e., \(\mu^{US} = 0\).

In the beginning of period 0, before the income realizations, U.S. households choose their portfolios of liquid bonds \(b^{US}\), less-liquid bonds \(\tilde{b}^{US}\), and illiquid loans \(\ell^{US}\), subject to the budget constraints in Equation 5 to maximize their expected utility shown in Equation 3, while assuming \(f(\ell^{US}) = 0\). The F.O.C. w.r.t \(b^a\), the F.O.C. w.r.t \(\tilde{b}^a\), and the F.O.C. w.r.t \(\ell^a\) are summarized below, respectively:

\[
p \left( -\frac{1}{R_{\ell}} (c_0^{US,H})^{-\sigma} + \beta^{US} (c_1^{US,H})^{-\sigma} \right) + (1 - p) \left( -\frac{1}{R_{\ell}} (c_0^{US,L})^{-\sigma} + \beta^{US} (c_1^{US,L})^{-\sigma} \right) = 0 \quad (13)
\]

\[
-\frac{1}{R_{b}} (p(c_0^{US,H})^{-\sigma} + (1 - p)(c_0^{US,L})^{-\sigma}) + p\beta^{US} (c_1^{US,H})^{-\sigma} + (1 - p)\beta^{US} (c_1^{US,L})^{-\sigma} + \mu^{US} = 0 \quad (14)
\]

\[
p \left( -\frac{1}{R_{b}} (c_0^{US,H})^{-\sigma} + \beta^{US} (c_1^{US,H})^{-\sigma} \right) + (1 - p) \left( -\frac{1}{R_{b}} (c_0^{US,L})^{-\sigma} + \beta^{US} (c_1^{US,L})^{-\sigma} \right) = 0 \quad (15)
\]

EM households choose how much to borrow from the U.S to maximize utility from consumptions in periods 0 and 1. The F.O.C. w.r.t. \(\ell^{EM}\) is:

\[
\frac{1}{R_{\ell}} (c_0^{EM})^{-\sigma} - \beta^{EM} (c_1^{EM})^{-\sigma} = \frac{1}{R_{\ell}} f'(\ell^{EM}) (c_0^{EM})^{-\sigma} = 0. \quad (16)
\]

The market clearing conditions now become:

\[b^{US} = \eta B, \tilde{b}^{US} = (1 - \eta)B, \text{ and } \ell^{US} = \ell^{EM} = \ell.\]

We show that U.S. households extend more loans to EMs when the total amount of U.S. Treasuries are more tilted towards more liquid short-term Treasury Bills.

**Proposition 2.** Define \(x = \frac{\ell}{R_{\ell}}\). Assuming \(R_{\ell} + x \frac{dR_{\ell}}{dx} > 0\), U.S. households extend more loans to EMs when the liquidity composition of U.S. government bonds shifts to more liquid short-term assets.
That is, \( \frac{dx}{d\eta} > 0 \).

**Proof.** See the Appendix.

Holding total U.S. government debt constant, U.S. (and RoW) households increase their lending to EMs as the liquidity composition of U.S. Treasuries moves toward more liquid assets. The saver households lend more to EM households as the lenders have more “insurance” against the possibility of having low income. They can sell their liquid Treasuries in the secondary market if facing lower income, in contrast to being locked in to illiquid assets. Therefore, lender households have a source of wealth to draw on when their incomes turn out to be low (or investment opportunities arise) since they can adjust their savings in liquid Treasuries.

4 Numerical Analysis

We calibrate the model presented in Section 3.1 to match some key moments from 1980 to 2019, which pertain to the U.S external balance and the interest rates across assets with different liquidity profiles. Section 4.2 presents the numerical results.

4.1 Calibration

One set of parameters is not nailed down in the data, and we vary these to see how liquidity supply influences lending to EMs. The rest are calibrated to match: the average ratio of the U.S. current account balance to the U.S. GDP, the average U.S. saving rate, the average share of foreign investors’ holdings of U.S. Treasuries, the average short-term and long-term interest rates on U.S. Treasuries, and the average interest rate on EM securities. One period in the model is one year.

We assume that high and low income levels in period 0 \((y^a_{0,H} \text{ and } y^a_{0,L})\) obtained by the households of country \(a\) are \(\Delta^a \times 100\%\) higher and lower, respectively, than the average period 0 income of country \(a\), \(y^a_{0}\). That is, \(y^a_{0,H} = y^a_{0}(1 + \Delta^a)\), and \(y^a_{0,L} = y^a_{0}(1 - \Delta^a)\), where \(a = \{\text{US, RoW}\}\). We set the probability of high income to 0.5, and the average incomes in period 0 for the U.S. and the RoW households then are normalized to one, i.e., \(y^a_{0} = 1\). The period 1 income for households in the U.S. and the RoW is also set to one, assuming zero growth rates of incomes. EM households have a constant income across two periods, normalized to one. Assuming the same average incomes for both periods and across households, time discount factors govern how much each household wants to lend or borrow.\(^{17}\) The functional form of the transaction cost of extending illiquid loans

\(^{17}\)We could assume positive growth rates of incomes for all countries with a higher growth rate for EM. In the two period model, the difference in the size of the annual growth rates of GDP is not enough to generate the quantitatively sizable motive to borrow and lend from each other. Therefore, we assume zero growth rates of incomes over one period for all countries but calibrate time discount factors to match the key moments. The alternative specification, imposing the same discount factor for EM households as that of EU households while assuming a positive growth rate of endowments for EM households, produces qualitatively the same results, and the results are available upon request.
is \( f(\ell) = \alpha^{\nu} \nu \), where \( \alpha \) and \( \nu \) are set to one and two, respectively. We compute the ratio of U.S. T-Bills held by public to the total marketable U.S. public debt, both of which are collected from the U.S. Treasury Monthly Statement of the Public Debt (MSPD). We find the ratio to be equal to 0.1955 and set \( \eta \) to 0.1955.

The rest of the parameter values are calibrated by the simulated methods of moments. We target six moments. The first two moments are the average short-term and long-term interest rates of U.S. Treasuries, computed with monthly observations of 3-month and 10-year Treasury annual yields, respectively, in 1980-2018. The third moment is the average interest rate on EM securities, measured by the average of monthly ICE BofA emerging markets corporate plus index effective yields from December 1998 – 2019. The interest rate data are all from the FRED database. The fourth and the fifth moments are the average quarterly U.S. current account balance as a ratio of the U.S. quarterly GDP and the average monthly U.S. household saving rate from 1980 to 2018; both series are collected from the FRED database. Lastly, we compute the average share of foreign holdings of U.S. Treasuries in the total marketable U.S. Treasuries from December 2011 - 2019. The data are fetched from the the U.S. MSPD. In sum, the six targeted moments are: (i) the mean interest rate on the U.S. short-term Treasuries (3.98%), (ii) the mean interest rate on the U.S. long-term Treasuries (6.03%), (iii) the mean interest rate on EM securities (9.89%), (iv) U.S. Current Account/GDP (-3%), (v) U.S. household saving rate (7.3%), and (vi) share of foreign holdings of the U.S. Treasuries (46%). We estimate time discount factors for three countries, the total supply of U.S. Treasuries \( \bar{B} \), and income fluctuations of lender households, \( \Delta (= \Delta^{US} = \Delta^{RoW}) \), and the constant relative risk aversion parameter \( \gamma \) to match moments (i) – (vi).

Although six parameters are jointly determined to match six moments, we can still offer a heuristic description of how each parameter is mostly inferred from an empirical moment. The time discount factor of EM households governs how much they want to front-load their consumption and hence the amount of borrowing and its interest rate, i.e., the interest rate on EM securities. The relative size of the discount factors of the U.S. and the RoW households to that of the EM households helps us to match the U.S. current account. The relative size of the time discount factor of the RoW and to the US captures how much they want to save for the next period, disciplining share of foreign holdings of U.S. Treasuries. The total supply of Treasuries governs both the short-term and long-term interest rates of Treasuries while the size of income fluctuations affects saver households’ desire to hold more liquid U.S. Treasuries, pinning down the difference in the short-term and long-term interest rates on U.S. Treasuries. The relative risk aversion of households regulates the saving rate.

We summarize the predetermined parameter values and internally calibrated parameter values in Table 1, and the targeted moments in Table 2.
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Descriptions</th>
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<tbody>
<tr>
<td>Normalized/Arbitrarily Chosen</td>
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<td></td>
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<tr>
<td>$y_U^0 = y_{RoW}^0 = y_U^1 = y_{RoW}^1$</td>
<td>1</td>
<td>Average Incomes in Periods 0 and 1 for U.S. and RoW</td>
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<tr>
<td>$y_{EM}^0 = y_{EM}^1$</td>
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<td>EM Incomes in Periods 0 and 1</td>
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<td>$p$</td>
<td>0.5</td>
<td>Probability of high-income households</td>
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<td>$\nu$</td>
<td>2</td>
<td>Transaction Costs of Illiquid Loans: $f(x) = \alpha x^\nu$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>Transaction Costs of Illiquid Loans: $f(x) = \alpha x^\nu$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.1955</td>
<td>Share of US T-Bills in Marketable Treasuries</td>
</tr>
</tbody>
</table>

Estimated Parameters from Moment Matching

| $\beta^{US}$ | 1.0000 | Time discount factor of US |
| $\beta^{RoW}$ | 0.9957 | Time discount factor of RoW |
| $B$ | 0.1107 | Supply of US Treasury bonds |
| $\beta^{EM}$ | 0.8435 | Time discount factor of EM |
| $\Delta^{US} = \Delta^{RoW} = \Delta$ | 0.0858 | Income fluctuation of lenders |
| $\sigma$ | 0.4289 | Relative risk aversion |

Notes: The upper panel summarizes the parameters normalized or arbitrarily chosen. The lower panel summarizes the calibrated parameters by the simulated methods of moments.

Table 2: Targeted Moments

<table>
<thead>
<tr>
<th></th>
<th>Sample Period</th>
<th>Data Moments</th>
<th>Model Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate on US short-term Treasuries</td>
<td>1981m1 – 2019m12</td>
<td>3.98%</td>
<td>3.86%</td>
</tr>
<tr>
<td>Interest rate on US long-term Treasuries</td>
<td>1980m1-2019m12</td>
<td>6.03%</td>
<td>7.15%</td>
</tr>
<tr>
<td>Interest rate on EM bonds</td>
<td>1998m12 – 2019m12</td>
<td>9.89%</td>
<td>9.40%</td>
</tr>
<tr>
<td>US Current Account/GDP</td>
<td>1980q1-2019q4</td>
<td>-3%</td>
<td>-3%</td>
</tr>
<tr>
<td>US household saving rate</td>
<td>1980m1 – 2019m12</td>
<td>7.3%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Share of foreign holdings of US Treasuries</td>
<td>2011m12 – 2019m12</td>
<td>46%</td>
<td>47%</td>
</tr>
</tbody>
</table>

Notes: We compute the historical average of variables of our interest from 1980 to 2019 if the data are available. The average short-term and long-term interest rates on U.S. Treasury are computed with 3-month and 10-year Treasury annual yields, respectively. The average of ICE BofA Emerging markets corporate plus index effective yields is used to compute the average interest rate on EM bonds. The share of foreign holdings of U.S. Treasuries is computed as foreign holdings of U.S. Treasuries over the total U.S. marketable Treasuries, where both data are from the U.S. Treasury Monthly Statement of the Public Debt (MSPD). All other data are from the St.Louis Fed’s FRED database.
4.2 Numerical Results

With the calibrated parameters, we numerically solve the model summarized in Section 3.1. Especially, we are interested in how the equilibrium interest rates, loans to EMs and the welfare of households in each country vary with the share of liquid bonds in the U.S. Treasuries. Figures 8 - 11 depict the equilibrium outcomes against $\eta$, where an $\eta$ fraction of total amount of U.S. Treasuries is liquid.

In Figure 8, as $\eta$ increases, the supply of U.S. liquid Treasuries goes up, while that of U.S. illiquid Treasuries goes down. The increase in the liquidity composition of U.S. Treasuries increases the interest rate on liquid U.S. government bonds, while lowering that on illiquid U.S. government bonds. As the volume of liquidity increases, the liquidity “return” declines, narrowing the gap between the yield on liquid bonds and other bonds. At the same time, overall saving increases, lowering the return on less liquid government bonds.

Since liquid bonds offer flexibility if the investor has lower income (or greater investment opportunities), U.S. and ROW households are willing to save more and lend more to EMs, shown in Figure 9. Figure 8 shows the interest rate on these loans falls as their supply increases.

Consequently, EM households benefit more as the liquidity composition of U.S. Treasuries shifts to more liquid assets, and therefore, their welfare increases with higher $\eta$, shown in Figure 10.

Figure 10 also shows the ex ante welfare in the U.S. and ROW as $\eta$ increases. U.S. and ROW households experience a slight fall in their average welfare at a low value of $\eta$. They also experience a much more subdued overall increase in their welfare compared to EM households even when $\eta$ reaches a large value. This welfare result comes from the offsetting effects of an increase in the liquidity share on low income and high income households. Figure 11 shows that as the share of liquid of U.S. Treasuries increases, low income households in the U.S. and ROW benefit from higher liquidity in the market, insuring them from over-saving; however, high income households become worse off. High income households now face lower returns from their savings directly because the composition of Treasuries shifts to more liquid and hence lower interest bearing assets, but also because the interest rates on both less-liquid Treasuries and loans to EMs fall slightly with a higher supply of liquid assets. These changes lead to a reduction in their average welfare initially. Nonetheless, when the share of liquid bonds is high enough, the interest gap between liquid Treasuries and less-liquid Treasuries narrows. And, the positive effect from higher amount of loans to EMs, which yields the highest interests to households, outweighs the former negative effects on the welfare of high income households.

We then explore how the total amount of loans extended to EM changes with the liquidity composition of the U.S. Treasuries, while varying one parameter at a time from its baseline value. Specifically, we experiment with different parameter values of $\alpha, p$, and $\Delta$, 25% lower or higher than the baseline values in Figures 12, 13, and 14, respectively. In all these figures, we see a
Figure 8: Interest Rates Against the Liquidity Composition of Treasuries

Notes: $R_b$, $R_{b,s}$, $R_b^{\ast}$, and $R_{\ell}$ represent the interest rate on liquid U.S. government bonds (dotted pink line), their secondary market rate (black solid line), the interest rate on illiquid U.S. government bonds (blue dotted line) and the interest rate on EM loans (red dashed line). $\eta$ is the share of liquid bonds in the U.S. government bonds.

Figure 9: Loans to EMs Against the Liquidity Composition of Treasuries

Notes: $x^{EM}$ is the total amount of loans to EM households: $\frac{i^{EM}}{\eta}$. $\eta$ is the share of liquid bonds in the U.S. government bonds.
Figure 10: Welfare Across Countries Against the Liquidity Composition of Treasuries

Notes: Welfare gain/loss is computed as a consumption equivalent (%). $\eta$ is the share of liquid bonds in the U.S. government bonds. The welfare of the U.S. and the ROW households does not include the utility from the government spending.

Figure 11: Welfare of High vs. Low Income Households in the U.S. Against the Liquidity Composition of Treasuries

Notes: Welfare gain/loss is computed as a consumption equivalent (%). $\eta$ is the share of liquid bonds in the U.S. government bonds. The welfare of the U.S. and the ROW households does not include the utility from the government spending. The solid line is the U.S. high income households’ welfare against $\eta$, and the dashed line is the U.S. low income households’ welfare against $\eta$. 
positive relationship between the amount of loans extended to EM and the share of liquid bonds in the U.S. Treasuries for the range of parameter values that we have explored. The numerical result is consistent with what we have shown analytically in Proposition 2.

In Figure 12, the amount of illiquid loans to EMs is higher for every $\eta$ when $\alpha$ is lower. With lower transaction costs that households need to pay when lending/borrowing via illiquid loans, they lend and borrow more from each other. A larger amount of illiquid loans for every $\eta$ is observed when $p$ is higher, shown in Figure 13, which is a consequence of having a higher average income in period 0 for the U.S. and the RoW households as the probability of having a high income is higher. As lender households have more average endowments in period 0, they lend more to the emerging markets.

Lastly and most interestingly, in Figure 14, EM households borrow more when $\Delta$ is lower for a given level of $\eta$; however, an increase in loans extended to EMs, as $\eta$ increases, is larger when the period 0’s income dispersion for lender households, $\Delta$, is higher. For a given level of liquid Treasuries supplied, lender households are more willing to lend to EMs as lender households know that their incomes in period 0 would be close to the average income, so they do not have to worry about having too much saving when their income levels turn out to be low. On the other hand, a higher supply of liquid Treasuries – higher $\eta$ – benefits the lender households the most when they obtain their income significantly lower than the average income. Therefore, lender households with higher $\Delta$ increase their lending to EMs by more when the supply of liquid Treasuries increases.
Figure 12: Loans to EMs Against the Liquidity Composition of Treasuries for Different $\alpha$

Notes: $x^{EM}$ is the total amount of loans extended to EM households: $x^{EM} = \frac{EM}{R}$. $\eta$ is the share of liquid bonds in the U.S. government bonds. $\alpha$ governs the size of the transaction costs that households need to pay per one unit of loan lent/borrowed. The total amount of loans extended to EM households is plotted against $\eta$ (i) with our baseline parameter values in a black solid line, (ii) with 25% lower $\alpha$ in a dashed red line, and (iii) with 25% higher $\alpha$ in a dotted blue line.

Figure 13: Loans to EMs Against the Liquidity Composition of Treasuries for Different $p$

Notes: $x^{EM}$ is the total amount of loans extended to EM households: $x^{EM} = \frac{EM}{R}$. $\eta$ is the share of liquid bonds in the U.S. government bonds. $p$ is the probability of having a high income in period 0 for the U.S. and the RoW households. The total amount of loans extended to EM households is plotted against $\eta$ (i) with our baseline parameter values in a black solid line, (ii) with 25% lower $p$ in a dashed red line, and (iii) with 25% higher $p$ in a dotted blue line.
Figure 14: Loans to EMs Against the Liquidity Composition of Treasuries for Different $\Delta$

Notes: $x^{EM}$ is the total amount of loans extended to EM households: $\frac{\ell^{EM}}{R^{0}}$. $\eta$ is the share of liquid bonds in the U.S. government bonds. $\Delta$ governs the size of income fluctuations in period 0 for the U.S. and the RoW households. The total amount of loans extended to EM households is plotted against $\eta$ (i) with our baseline parameter values in a black solid line, (ii) with 25% lower $\Delta$ in a dashed red line, and (iii) with 25% higher $\Delta$ in a dotted blue line.

5 Conclusion

We have seen in the empirical analysis that, holding monetary policy rates constant, a change quantitative easing or tightening in the U.S. appears to influence capital flows to emerging markets. Specifically, the amount of flows increases as the Fed injects liquidity. These flows, then, are not responding to changes in policy rates per se, but rather to the composition of assets offered by the U.S. government/Federal Reserve to the public. Quantitative easing replaces longer term Treasury bonds with reserves in the hands of the public. Following Rogoff (2017), we interpret this as a change in the structure of U.S. government liabilities available to the private sector. Reserves held by the banking sector at the Fed are, in essence, very short-term liquid assets. Hence, we examine the effects of quantitative easing or tightening through the lens of the liquidity of government liabilities offered to the public.

We then offer a possible explanation for why changing the composition of these liabilities leads to greater lending to emerging markets. As liquidity increases, investors have more assurance that their funds will not be tied up in illiquid assets in case of an economic downturn (or, in case attractive investment opportunities arise.) In fact, our quantitative solutions find that this channel is even stronger during times of greater economic uncertainty. Investors have less fear of being “caught short” when they have the ability to sell liquid assets quickly, and therefore are willing to save and invest more, including in illiquid loans to emerging markets.
Emerging market policymakers, in turn, have an interest not only in the restrictiveness of U.S. monetary policy, but also the policy toward liquidity provision. This perspective introduces another dimension to the observation of Rey (2016) concerning the supremacy of U.S. monetary policy decisions in the international transmission mechanism.
References


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Appendix

Proof of Proposition 2

Proof. We multiply Equation 11 by \( p \) and Equation 12 by \( 1 - p \), and compare to Equation 14 and we get \( R_{b,3} = R_b \) From Equation 13 and Equation 15, we get \( R_\ell = R_b \) because illiquid bonds and loans to EM are equivalent for lenders, given the simplifying assumption that U.S. does not bear the cost of lending to EM households. With these equilibrium conditions and \( u'(c) = e^{-\sigma} \), we can write Equations 11–16 as

Then, the first-order conditions give us:

\[
\beta^{US} R_b \left( \frac{\ell}{c_1} \right)^{-\sigma} = (\frac{c_0^{US}}{c_1^{US}})^{-\sigma} \tag{17}
\]

\[
\beta^{US} R_b \left( \frac{\ell}{c_1} \right)^{-\sigma} + \beta^{US} R_b \mu_L = \left( \frac{c_0^{US}}{c_1^{US}} \right)^{-\sigma} \tag{18}
\]

\[
\beta^{US} p \left( \frac{R_\ell - R_b}{R_\ell} \right) \left( \frac{\ell}{c_1} \right)^{-\sigma} + \beta^{US} (1 - p) \left( \frac{R_\ell - R_b}{R_\ell} \right) \left( \frac{c_0^{US}}{c_1^{US}} \right)^{-\sigma} - R_b \mu_L) = 0 \tag{19}
\]

\[
\beta^{EM} R_\ell (\frac{c_0^E}{c_1^E})^{-\sigma} + \sigma \left( \frac{\ell}{R_\ell} \right) \left( \frac{c_0^E}{c_1^E} \right)^{-\sigma} = \left( \frac{c_0^E}{c_1^E} \right)^{-\sigma} \tag{20}
\]

Combining Equations 18 and 19, and we have,

\[
\beta^{US} p \left( \frac{R_\ell - R_b}{R_\ell} \right) \left( \frac{\ell}{c_1} \right)^{-\sigma} + (1 - p) \left( \beta^{US} \left( \frac{c_0^{US}}{c_1^{US}} \right)^{-\sigma} - \frac{1}{R_\ell} \left( \frac{c_0^{US}}{c_1^{US}} \right)^{-\sigma} \right) = 0 \tag{21}
\]

Since we are interested in \( \frac{dx}{d\eta} \), where \( x^{US} = \frac{\ell^{US}}{R_\ell} \), \( x^{EM} = \frac{\ell^{EM}}{R_\ell} \), \( x^{US} = x^{EM} = x \), we replace \( \ell^{US} \) with \( R_\ell x \) and substitute back in the expressions for consumption, we get:

\[
\left( y_0^{US} - \frac{1}{p} \frac{\eta B}{R_b} - x - \left( \frac{1}{R_\ell} \frac{\eta B}{R_b} \right) \right)^{-\sigma} = \beta^{US} R_b \left( y_1^{US} + R_\ell x + B + \frac{1 - p}{p} \frac{\eta B}{R_\ell} \right)^{-\sigma} \tag{22}
\]

\[
\beta^{US} p \left( R_\ell - R_b \right) \left( y_1^{US} + R_\ell x + B + \frac{1 - p}{p} \frac{\eta B}{R_\ell} \right)^{-\sigma} + \beta^{US} (1 - p) R_\ell \left( y_1^{US} + R_\ell x + (1 - \eta) B \right)^{-\sigma}
\]

\[
= (1 - p) \left( y_0^{US} - x - \left( \frac{1}{R_\ell} \frac{\eta B}{R_b} \right) \right)^{-\sigma}
\]

\[
(y_0^{EM} + x - f(x))^{-\sigma} (1 - f'(x)) = \beta^{EM} R_\ell (y_1^{EM} - R_\ell x)^{-\sigma} \tag{24}
\]

Equation 22 becomes:

\[
\left( y_0^{US} - \frac{1}{p} \frac{\eta B}{R_b} - x - \left( \frac{1}{R_\ell} \frac{\eta B}{R_b} \right) \right) = \left( \beta^{US} R_b \right)^{-\frac{1}{\sigma}} \left( y_0^{US} + R_\ell x + B + \frac{1 - p}{p} \frac{\eta B}{R_\ell} \right)^{-\frac{1}{\sigma}} \tag{25}
\]

Now, totally differentiate Equation 23:
\[ \beta^{US} p \left( c_1^{US,H} \right)^{-\sigma} (dR_t - dR_b) - \beta^{US} \sigma p \left( c_1^{US,H} \right)^{-\sigma - 1} (R_t - R_b) \left( xdR_t + R_t dx + \frac{1 - p}{p} B d\eta \right) \\
+ \beta^{US} (1 - p) \left( c_1^{US,L} \right)^{-\sigma} dR_t - \beta^{US} \sigma (1 - p) \left( c_1^{US,L} \right)^{-\sigma - 1} R_t (xdR_t + R_t dx - B d\eta) \]
\[ = -\sigma (1 - p) \left( c_0^{EM} \right)^{-\sigma - 1} \left( -dx + \frac{B}{R_t} d\eta + \frac{(1 - \eta) B}{(R_t)^2} dR_t \right) \]
\]

Then, from Equation 24,
\[ -\sigma (c_0^{EM})^{-\sigma - 1} (1 - f'(x))^2 dx - (c_0^{EM})^{-\sigma} f''(x) dx = \beta^{EM} \left( c_1^{EM} \right)^{-\sigma} dR_t + \beta^{EM} \sigma (c_1^{EM})^{-\sigma - 1} R_t (xdR_t + R_t dx) \]

Solving for \( dR_t \), we find:
\[ dR_t = \left( \frac{\sigma}{\sigma - 1} \right) \left( c_0^{EM} \right)^{-\sigma} \left( 1 - f'(x)^2 \right) dx \]
\[ \equiv \mathcal{A} dx, \text{ where } \mathcal{A} < 0. \]

Then, totally differentiating Equation 25:
\[ -\frac{B}{pR_b} d\eta + \frac{\eta B}{p(R_b)^2} dR_b - dx + \frac{B}{R_t} d\eta + \frac{(1 - \eta) B}{(R_t)^2} dR_t \]
\[ = -\frac{1}{\sigma} \beta^{US} \left( c_0 \right)^{-\sigma} (R_b) \left( c_1^H \right) dR_b + \left( \beta^{US} R_b \right)^{-\frac{1}{\sigma}} \left( R_t dx + x dR_t + \frac{1 - p}{p} B d\eta \right) \]
\]

This can be simplified by noting that \( \left( \beta^{US} R_b \right)^{-\frac{1}{\sigma}} = \frac{c_0^{US,H}}{c_1^{US}} \). We get:
\[ -\frac{B}{p} d\eta + \frac{\eta B}{pR_b} dR_b - dx + \frac{B R_b}{R_t} d\eta + \frac{(1 - \eta) B R_b}{(R_t)^2} dR_t \]
\[ = -\frac{1}{\sigma} c_0^{US,H} c_0^{US,H} R_t + \frac{c_0^{US,H}}{c_1^{US}} R_t \left( R_t dx + x dR_t + \frac{1 - p}{p} B d\eta \right) \]
\]

Now plug in for \( dR_t \) in Equations 26 and 27 and rearrange the terms,
\[ \left[ -\frac{1}{\sigma} \beta^{US} \left( c_0^{US,H} \right)^{-\sigma} \mathcal{A} - \beta^{US} \sigma p \left( c_1^{US,H} \right)^{-\sigma - 1} (R_t - R_b) (x \mathcal{A} + R_t) + \beta^{US} (1 - p) \left( c_1^{US,L} \right)^{-\sigma} \mathcal{A} \right] d\eta \]
\[ = -\left[ \frac{1}{\sigma} c_0^{US,H} + \frac{\eta B}{pR_b} \right] dR_b \]
\]

\[ \left[ \beta^{US} p \left( c_1^{US,H} \right)^{-\sigma} \mathcal{A} - \beta^{US} \sigma p \left( c_1^{US,H} \right)^{-\sigma - 1} (R_t - R_b) (x \mathcal{A} + R_t) + \beta^{US} (1 - p) \left( c_1^{US,L} \right)^{-\sigma} \mathcal{A} \right] d\eta \]
\[ - \beta^{US} \sigma (1 - p) \left( c_0^{US,L} \right)^{-\sigma - 1} R_t (x \mathcal{A} + R_t) - \beta^{US} \sigma (1 - p) \left( c_1^{US,L} \right)^{-\sigma - 1} \left( 1 - \frac{(1 - \eta) B}{(R_t)^2} \right) d\eta \]
\[ + \left[ \beta^{US} p \left( c_1^{US,H} \right)^{-\sigma - 1} R_t B - \beta^{US} \sigma p \left( c_1^{US,H} \right)^{-\sigma - 1} (R_t - R_b) \left( 1 - \frac{(1 - \eta) B}{R_t} \right) B + \beta^{US} (1 - p) \left( c_0^{US,L} \right)^{-\sigma - 1} B \right] d\eta \]
\[ = \beta^{US} p \left( c_1^{US,H} \right)^{-\sigma} dR_b \]
\]

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And, we can define Equations 28 and 29 as $A_1 d\eta + A_2 dx = A_3 dR_b$ and $B_1 dx + B_2 d\eta = B_3 dR_b$, where

$$A_1 d\eta + A_2 dx = A_3 dR_b$$

$$A_1 = \left( -\frac{1}{p} + \frac{R_b}{R_\ell} - \frac{c_0^{\text{US.H}}}{c_1^{\text{US.H}}} \frac{1-p}{p} \right) B < 0$$

$$A_2 = -R_b + \frac{(1-\eta)BR_b}{(R_\ell)^2} A_3 c_0^{\text{US.H}} \frac{R_b}{R_\ell} (x+\eta)$$

$$A_3 = -\frac{1}{\sigma} c_0^{\text{US.H}} \frac{B}{R_\ell} < 0$$

$$B_1 = \beta^{\text{US}} (c_1^{\text{US.H}})^{-\sigma \eta} - \beta^{\text{US}} \sigma p (c_1^{\text{US.H}})^{-\sigma - 1} (R_\ell - R_b) (x+\eta) + \beta^{\text{US}} (1-p) (c_1^{\text{US.L}})^{-\sigma \eta}$$

$$B_2 = -\beta^{\text{US}} \sigma (1-p) (c_1^{\text{US.L}})^{-\sigma - 1} R_\ell (x+\eta) + \sigma (1-p) (c_0^{\text{US.L}})^{-\sigma - 1} \left( \frac{1-(1-\eta)B}{(R_\ell)^2} \right)$$

$$B_3 = \beta^{\text{US}} (c_1^{\text{US.H}})^{-\sigma - 1} R_b (1-p) + \sigma (1-p) (c_0^{\text{US.L}})^{-\sigma - 1} \frac{1}{R_\ell} B > 0$$

$A_2$ and $B_1$ are negative as long as $(R_\ell + x+\eta)$ is positive.

$$\frac{dx}{d\eta} = -\frac{B_2 + B_3 A_1}{A_3}$$

$$B_1 - B_3 \frac{A_2}{A_3} < 0$$

$$-B_2 < 0 \text{ but } B_3 \frac{A_1}{A_3} > 0$$

We show that $-B_2 + B_3 \frac{A_1}{A_3} < 0$. As $B_2$ and $B_3 \frac{A_1}{A_3}$ are positive, we just need to show:

$$B_2 > B_3 \frac{A_1}{A_3}$$
\[
\left( \beta^{US} (1 - p) \right) \left( c_{1}^{USL} \right)^{-\sigma - 1} R_t - \beta^{US} \sigma \left( c_{1}^{USU} \right)^{-\sigma - 1} \left( R_t - R_b \right) \left( \frac{1 - p}{p} \right) + \sigma (1 - p) \left( c_{0}^{USL} \right)^{-\sigma - 1} \frac{1}{R_t} B
\]

\[
> \beta^{US} \left( c_{1}^{USH} \right)^{-\sigma} \left( -\frac{1}{p} + \frac{R_b}{c_{0}^{USH} R_b - \frac{1 - p}{p}} \right) B
\]

\[
\iff \frac{1}{R_t} \left( c_{0}^{USH} \right)^{-\sigma - 1} - \frac{1}{c_{0}^{USH} - \frac{\eta B}{p R_b}} \left( \beta^{US} \sigma (1 - p) \left( c_{1}^{USH} \right)^{-\sigma - 1} - \left( c_{1}^{USH} \right)^{-\sigma - 1} \right) + \beta^{US} (1 - p) \left( c_{0}^{USH} \right)^{-\sigma - 1} \frac{R_b}{R_t - \frac{1 - p}{p}}
\]

\[
\iff \beta^{US} \left( c_{0}^{USH} \right)^{-\sigma} \eta B \sigma (1 - p) R_t \left( \left( c_{0}^{USH} \right)^{-\sigma - 1} - \left( c_{1}^{USH} \right)^{-\sigma - 1} \right) + \beta^{US} (1 - p) \left( c_{0}^{USH} \right)^{-\sigma - 1} \frac{1}{R_t}
\]

\[
\iff \beta^{US} \left( c_{1}^{USH} \right)^{-\sigma} > \beta^{US} \left( c_{0}^{USH} \right)^{-\sigma} \frac{R_b}{R_t}
\]

Using Equation 21,

\[
p \left( \beta^{US} \left( R_t - R_b \right) \left( c_{1}^{USH} \right)^{-\sigma} \right) + (1 - p) \beta^{US} \left( c_{1}^{USH} \right)^{-\sigma} - \frac{1}{R_t} \left( c_{0}^{USL} \right)^{-\sigma} = 0,
\]
\[
\beta^{US} \left( c_0^{USH} (1 - p) R_L + \frac{\eta B \sigma (1 - p) R_L}{p R_b} \right) \left( \left( c_1^{USL} \right)^{-\sigma - 1} - \left( c_1^{USH} \right)^{-\sigma - 1} \right) + c_0^{USH} (1 - p) \left( c_0^{USL} \right)^{-\sigma - 1} \frac{1}{R_L} \\
+ \frac{\beta^{US} \eta B \sigma (1 - p) (c_1^{USH})^{-\sigma - 1}}{p R_b R_L} + \frac{\eta B \sigma (1 - p) (c_0^{USL})^{-\sigma - 1}}{p R_b} - \beta^{US} (1 - p) \left( c_0^{USH} \right)^{-\sigma} + \beta^{US} \left( c_0^{USL} \right)^{-\sigma - 1} \frac{1}{R_L} > 0
\]

\[
\iff \beta^{US} \left( c_0^{USH} (1 - p) R_L + \frac{\eta B \sigma (1 - p) R_L}{p R_b} \right) \left( \left( c_1^{USL} \right)^{-\sigma - 1} - \left( c_1^{USH} \right)^{-\sigma - 1} \right) + c_0^{USH} (1 - p) \left( c_0^{USL} \right)^{-\sigma - 1} \frac{1}{R_L} \\
+ \frac{\beta^{US} \eta B \sigma (1 - p) (c_1^{USH})^{-\sigma - 1}}{p R_b R_L} + \frac{\eta B \sigma (1 - p) (c_0^{USL})^{-\sigma - 1}}{p R_b} - \beta^{US} (1 - p) \left( c_0^{USH} \right)^{-\sigma} > 0
\]

\[
\iff \beta^{US} \left( c_0^{USH} (1 - p) R_L + \frac{\eta B \sigma (1 - p) R_L}{p R_b} \right) \left( \left( c_1^{USL} \right)^{-\sigma - 1} - \left( c_1^{USH} \right)^{-\sigma - 1} \right) + c_0^{USH} (1 - p) \left( c_0^{USL} \right)^{-\sigma - 1} \frac{1}{R_L} \\
+ \frac{\beta^{US} \eta B \sigma (1 - p) (c_1^{USH})^{-\sigma - 1}}{p R_b R_L} + \frac{\eta B \sigma (1 - p) (c_0^{USL})^{-\sigma - 1}}{p R_b} - \beta^{US} (1 - p) \left( c_0^{USH} \right)^{-\sigma} > 0
\]

Since lower income consumption in each period is no greater than high income consumption, all the elements are positive. We show that \( \frac{dx}{d\eta} > 0 \).
A Model with Investment

In this section, we show that the endowment shock in period 0 can map to a shock to investment needed for the production of outputs in period 1. The U.S. and the ROW households are endowed with $K_0^a$ amount of capital and $y_0^a$ amount of goods. These resources can be consumed or invested to produce capital $K$, which is used as production in period 1. At the beginning of period 0, households are not yet informed of the production technology of period 1, but they are aware that with probability $p$, production technology will be high-type and with probability $1 - p$, it will be low-type. The production technology $F^{a,k}(K)$ in period 1 is:

$$
F^{a,H}(K) = \begin{cases} 
y_1^a & \text{if } K \geq K_0^a - \frac{1}{p}I_0^a \\
0 & \text{if } K < K_0^a - \frac{1}{p}I_0^a
\end{cases}
$$

$$
F^{a,L}(K) = \begin{cases} 
y_1^a & \text{if } K \geq K_0^a + \frac{1}{1-p}I_0^a \\
0 & \text{if } K < K_0^a + \frac{1}{1-p}I_0^a
\end{cases}
$$

Once households learn their production technology, the high-type households will sell capital to the low-type households and lower type will give goods in return. With this technology, the high type households have higher “endowments” of goods, $y_0^a + \frac{1}{1-p}I_0^a$, while the low type households have lower “endowments”, $y_0^a - \frac{1}{1-p}I_0^a$. The value of liquidity then arises from the flexibility to adjust saving in U.S. liquid government bonds as needed for investment.