Open Economy Macroeconomics/International Finance

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This is a compilation of material for my PhD course in international macro and finance for spring, 2023. I had two goals in arranging the material the way I did. First, I wanted to do a broad (and therefore necessarily somewhat shallow) survey of models and topics in international finance. If students have the necessary background, they can proceed to state-of-the-art research on their own more readily than starting with current research and forcing them to fill in the blanks. Second, I wanted to teach the course assuming the students had read the material, and then during lecture I could elaborate on some points and answer their questions. I assigned the readings in advance, with detailed guidelines for reading the papers, and with questions that they needed to answer. The questions are not designed as problem sets. They are questions that test the students' understanding of the material.

This packet has (1) the notes for the readings, which include the questions; (2) each set of notes has a list of "ten important papers" on the topic, but that were not assigned or necessarily discussed; (3) my answers to the assigned questions. I did not have lecture slides for most of the class, except in a few cases where they were useful to elaborate on some points, and those are included. As we were running out of time at the end of the semester, I did use lecture slides at the end, and those are attached also.

Reading the literature is important in this class more than probably any other field. There are two problems with the standard lecture format. First, I find that students are tempted to absorb the lectures and not motivated to go to the readings. Second, I get bogged down in details when I lecture, and we don't end up covering enough ground. When you read, you begin to see the details, but we can keep the class moving forward. If you get interested in a particular topic, then that is great – the readings will help you see where the main story line may need to be modified or may be drastically wrong.

Why are the readings so important? One can do research in the field without really giving serious consideration to general equilibrium. There are many important areas of research – like search markets in labor or understanding of financial institutions – that one can study in detail, and then even embed in a simplified general equilibrium model. You can think of that style of research as concentrating on the "building blocks" of macroeconomics. My own interest in international macro has been the foreign exchange rate – for example, the dollar price of foreign currency. But there is no sense in which the foreign exchange market can be studied in isolation from the macroeconomy, as a building block. The exchange rate is really the only national asset price – the only asset price that reflects all the different elements driving the macroeconomy, from monetary and fiscal policy to productivity to financial market changes. To understand the exchange rate, it is unavoidable that you need to understand the general equilibrium of the economy. To do that, you need to start by reading on a broad range of topics – even if at first, your understanding of each area will be superficial. Indeed, even if you end up specializing in one aspect of open-economy macro, you still need to have some broad idea of how everything fits together (at least, that's my opinion!)

In fact, there really isn't something called the dollar price of foreign currency. There are specific exchange rates like the dollar price of the euro. That leads me to mention an important point about this class: my expertise has been more in advanced, high-income economies. When I think of the dollar/euro exchange rate, I want a model that has both the U.S. and Europe in it. A typical way to model a "global" general equilibrium is to build a model in which there are only two countries in the world (U.S. and Europe, say) and work out the equilibrium. A full model like this requires understanding what determines trade flows and capital flows between the two countries, what determines prices and exchange rates, and what determines financial market variables in each country. That is the scope of the models we need, and we will try to cover at least the basics. We will also talk about emerging economies. On the one hand, these economies are easier to model in that we use "small, open-economy" models, which are like two-country models but take the behavior of the "rest of the world" as exogenous. On the other hand, emerging economies are subject to many dramatic shocks, and there is more an emphasis on non-linear solutions to these models, as we will see.

Here is the plan for the semester:

1. Simplified 2-country New Keynesian model

Our goal in this section is to build a super-simple three-equation two-country dynamic New Keynesian model. As I will explain below, this model is intended as a baseline. There are many things it cannot explain, but it serves as a baseline for understanding how various shocks to the economy affect exchange rates, real exchange rates, interest rates (nominal and real), and inflation. Once we have this in hand, the rest of the semester will be aimed at enriching the model, or at least seeing why it needs enriching and how it might be enriched.

A. But before we get to the 3-equation model, we will study two topics. Each of these realistically could take a whole semester to study. The first is goods pricing. In the end, we will put some focus on something called "local currency pricing" or LCP, because that is the pricing model that makes our simple 3-equation model work best. I also think there is substantial merit in this model, though it certainly needs to be augmented in many important ways to get a realistic model of pricing.

We actually will start by looking at a recent paper of mine in the *AER*, co-authored with Berka and Devereux. It is primarily an empirical paper. But one thing we can get from it is a model of real exchange rates that encompass two prominent, simple, neoclassical models in which prices are perfectly flexible and there is perfect competition.

We will study a fairly broad range of pricing models but will keep an eye on how they might end up getting incorporated into general equilibrium models. One important point is that predictions about pricing from models of a single firm often change dramatically when we embed that firm in a larger economy. For example, a canonical way to express a first-order approximation to a firm's pricing decision is that it is weighted average of unit cost and the prices of its competitors. But once we then embed this firm into a model where other firms are similar – and jointly they determine the price of the competitors – the implications for how prices respond to costs can be quite different than what we learn from a single firm model.

In any case, though, slow price adjustment is also an important feature of the data, and so we will need to consider that. Actually, usually even though the lessons from optimal price setting point to important ways to modify standard models of goods prices, those lessons are not incorporated into models with slow price adjustment, usually because of algebraic complexity.

B. The second big topic we cover is "uncovered interest parity" or UIP. UIP is a theory that says that investors want the same return on a short-term domestic deposit as on a short-term foreign deposit. The domestic deposit is riskless, but the foreign deposit has foreign exchange risk, because the return on the foreign deposit depends not only on the interest rate, but also on how the exchange rate changes during the holding period. UIP is a theory that says investors ignore that risk.

There is a famous empirical result, often called the UIP puzzle or Fama puzzle, which says that the when the interest rate in one country (relative to the other) rises, the expected return on deposits in that country increase relative to the other. That is not a trivial statement, because when you compare expected returns, you are comparing not just the interest rates, but also incorporating the expected change in the exchange rate. So, one way to say this is that when a country's interest rate increases, its deposits become riskier. If relative expected returns are not equal, UIP does not hold (since UIP is a theory that says investors accept equal expected returns.)

That is a traditional way to interpret the finding – that a foreign exchange risk premium must account for what we see. We will study models like this briefly, though we could spend weeks on them.

But there are other possible explanations for expected returns on one asset to rise relative to another. There can be financial market imperfections, and we will look at a model that simply assumes investors are not constantly monitoring their portfolios. Later in the semester, we will think about this again in the context of models where investors are constrained in how much they can adjust their portfolios.

Another possible explanation is that some assets have an unobserved, or at least hard-to-measure, component to their return that is not monetary. Specifically, an asset may be valued for its liquidity, and therefore pay a lower expected monetary return than a less liquid asset. We will also address this later in the semester.

In the field of finance, most focus is put on how a shock at time *t* affects $E_t r_{t+1}$, the expected return on the asset between *t* and $t+1$. The expected return, relative to the return on the riskless asset, may be predictable, which is to say the risk premium on the asset may be predictable. In economics, we are often more concerned with how a shock at time *t* affects r_t, the return on the asset between $t-1$ and *t*. For example, in a model with no risk premium, so UIP holds, shocks don't affect the risk premium (because the risk premium is zero.) But they do affect the exchange rate at time *t*, which has an influence on the *ex post* return on assets between $t-1$ and *t*. We care about how the exchange rate changes because it may have effects on international prices, and therefore imports and exports, etc. We will look at my 2016 *AER* paper that raises the concern that models that can explain the foreign exchange risk premium often have very bad predictions about the effects of shocks on *ex post* returns and the exchange rate.

We'll also look at a paper I wrote recently with some of our PhD students that indicates the Fama puzzle has completely disappeared once nominal interest rates have fallen very low and close to zero.

C. Then we will move on to the simple 3-equation model. One thing that makes it possible to express a model of two economies in just three equations is that we assume the economies are exactly symmetric.

But we also make other dramatic simplifying assumptions. For one, despite all our readings about the failure of UIP, we will assume that it holds (or, more accurately, that deviations from UIP come from an exogenous random variable.)

Another is that we ignore most of what we learned about goods pricing. We will use a model in which the only reason for real exchange rate fluctuations is nominal price stickiness and LCP.

The third equation describes monetary policy, and it will be very simple. We will assume in each country, the central bank sets the short-term interest rate to target inflation.

This is an obviously simplified model, but I believe it helps us understand how various shocks, such as monetary policy shocks, cost-push shocks, and *exoegenous* shocks to the UIP condition affect nominal and real exchange rates, nominal and real interest rates, expected excess returns, and inflation.

But one thing that is odd about the model is that it can be written to solve three variables as a closed system: the real exchange rate, relative inflation, and relative nominal interest rates. What is odd is that there is no feedback from the rest of the economy. The current account balance, consumption, output, etc., do not influence these variables. This is a block-recursive system. The three equations solve for those three variables. They affect the rest of the economy (that is, the current account, consumption, output, etc.) but there is no feedback going the other way. This is obviously unsatisfactory as a general model, and you can think of much of the rest of the semester as enriching the model and allowing for this mutual feedback.

One nice feature of the 3-equation model is that we can see not only how current shocks affect the economy, but also how news (or, signals) about future shocks affect the variables now. The exchange rate in particular, is an asset price whose current value is very much affected by expectations of the future. When the economy gets news about the future, that affects exchange rates now, even if there are no current economic shocks.

2. <u>Filling in the real economy</u>

In this section, we are going to read from two textbooks: Obstfeld and Rogoff's 1996 text, and the more recent text from Schmitt-Grohe and Uribe.

The Obstfeld-Rogoff book is a great source for understanding some of the basic forces that might determine real exchange rates in the long run, current account imbalances, and many other important things that anybody in the field should know about. It is a very long book, and our readings from it will be very selective, but you might find many chapters interesting that I don't assign.

The SGU book builds up a stochastic New Keynesian DSGE model of a small open economy, so it is very much useful for thinking about modern models that are used in research. Again, we will only look at a few chapters. It goes less into detail on economic mechanisms relative to Obstfeld-Rogoff, but we will be able to understand it better having read some of OR first.

3. <u>Optimal monetary policy</u>

In some ways, this is an arcane subfield, but I think an important one. First, we will read the last chapter of OR. They are the first to build a New Keynesian two-country model, though their 1996 model is now pretty old-fashioned. But the chapter is great in motivating why the models have the elements they have.

Then we will look at my 2011 *AER* paper. This paper is useful because it both explains why optimal monetary policy models are formulated the way they are, and the appendix contains all the very tedious algebra worked out step by step. My particular contribution is to find optimal cooperative monetary policy when there is LCP.

4. <u>Sudden stops</u>

This section follows more directly from the models we studied in SGU. Here, we will study Bianchi's classic 2011 *AER* paper on "overborrowing" by a small open economy. The purpose of the paper is to understand "sudden stops" – a phenomenon by which these economies suddenly move from a position of international borrowers to one where they can no longer borrow and must pay back their loans. This is often accompanied by a rapid slowdown of the economy.

A key aspect of these models is non-linear, global solution methods. The reason is that these large changes that come with a sudden stop are not usually caused by a large shock. Small shocks eventually can lead to very large changes – something that does not happen in a linear model.

5. <u>International Portfolios</u>

We would like to understand how countries choose their portfolios of assets, but we don't understand it well. We know there is a lot of "home bias" in equity holdings – that is, countries hold a disproportionate share of their own equities. We also know that in terms of foreign asset holdings, the U.S. holds disproportionately more risky assets (equities and foreign direct investment) compared to other countries whose portfolios are more in safe bonds.

We'll see that a standard, straightforward model of intertemporal optimization under uncertainty is both very hard to solve, but also doesn't tend to yield very satisfactory solutions relative to the data. We'll then look at some recent papers that look at other assumptions, which some might consider *ad hoc*, that are useful in explaining portfolios.

6. <u>Financially-constrained intermediaries</u>

An important development in the field has been the recognition that most asset trade is done not so much by individuals, but by financial intermediaries. But balance sheet constraints may limit the amount of financial intermediation that is achieved.

We'll start with the important paper by Gabaix and Maggiori, in the *QJE* in 2015. This one has many insights into the role of constrained financial intermediaries. We will also look at Itskhoki and Mukhin's 2021 *JPE* paper on "exchange rate disconnect", where intermediaries that are quite risk averse play a key role.

We'll move on to my new paper with Devereux and Wu. This paper is useful because, unlike the previous two, it is a completely standard New Keynesian DSGE model, with financial intermediaries that are constrained just as in the papers by Gertler and Karadi. We make one innovation – we assume U.S. government bonds are better collateral for lenders to banks than other assets – but the main reason we'll look at this paper is because the modeling is so standard. We can also see how many of the things we learned about in Obstfeld and Rogoff, and SGU, come into play, though the channels through which shocks affect endogenous variables is not always easy to understand in a very rich model.

7. Sovereign default

We won't cover this at all. This literature is fascinating, and there are deep connections between it and important areas of economic theory. We just don't have time to cover it, and I'm no expert here. An absolutely great source is the recent book by Aguiar and Amador, "The Economics of Sovereign Debt and Default".

The Plan:

For each section on the reading list, I will give you specific papers to read. Importantly, I will also direct you to the parts of the paper that I want you to pay closest attention to.

I will ask a series of questions on the papers, and you will need to write out the answers. Mostly these are questions that make sure you comprehend what the paper is doing. They won't be problems (except maybe I'll ask you to do a 2-good example of some formula that has n goods, for example.) Answering these questions is the only requirement for the course.

The time frame for reading and answering the questions on each section depends on how much reading there is, and how difficult the reading is. The first section on pricing might take up two weeks. There is a lot to cover, so we can't get too bogged down.

Econ 872
Spring 2023
C. Engel

International Finance and Macroeconomics

Course Outline

I. International Goods Pricing
   A. Real exchange rates (Balassa-Samuelson)
   B. Terms of trade
   C. Pricing to Market
   D. Nominal price stickiness and open-economy Phillips curve

II. International Asset Pricing
   A. Uncovered interest rate parity
   B. Stochastic discount factors in international setting
   C. Foreign exchange risk premium (long-run risks model)
   D. Delayed portfolio adjustment

III. Baseline Open-Economy NK DSGE model
   A. 3-equation "canonical" NK DSGE model
   B. Review of solving linear rational expectations models
   C. Shocks – to monetary policy, productivity, risk premium
   D. News and exchange rates
   E. Forecasting exchange rate changes

IV. Enriching the models
   A. Obstfeld-Rogoff, chapters 1, 2, 4, 5, 9, 10
      i. Intertemporal models, current account
      ii. Non-traded goods, terms of trade
      iii. Complete markets, asset pricing, portfolio choice
      iv. Dornbusch model
      v. New Keynesian DSGE model
   B. Uribe and Schmitt-Grohe, chapters 2, 3, 4, 7, 8
      i. Small open-economy dynamic model
      ii. Small open-economy RBC model
      iii. Real exchange rates and terms of trade

V. Optimal Monetary Policy in Open Economies

VI. International Portfolio Choice

VII. Emerging Markets and “Sudden Stops”

VIII. The Role of Financial Intermediaries and Balance-Sheet Constraints

IX. Sovereign Default (not covered)

Assignment 1: International pricing

First, I would like you to read these two papers:

Engel, Charles. 1993. "Real Exchange Rates and Relative Prices: An Empirical Investigation," **Journal of Monetary Economics** 32, 35-50.

Engel, Charles. 1999. "Accounting for U.S. Real Exchange Rate Changes," **Journal of Political Economy** 107, 507-538.

You don't need to read these in great detail. I would spend no more than 45 minutes on each, and maybe less. I have no questions to ask you about these.

Before we go on, let me give you my view of how the consumer price real exchange rate is determined. I think for most goods (not including goods such as gasoline or fresh foods whose prices are not sticky), consumer prices are set in the currency of the consumer and are sticky in that currency. (Reminder: we are focusing on low-inflation, advanced economies.) Let p_i be the log of the price of good *i* in the "home"country denominated in the home currency and p_i^* be the log of the price of the same good in the "foreign" country denominated in the foreign currency. Let *s* be the log of the nominal exchange rate, expressed as the home currency price of foreign currency.

Now consider $p_i - s - p_i^*$. If the law of one price held, this would always equal zero. But instead, I think p_i is sticky in home currency and p_i^* is sticky in foreign currency. I think *s* is very volatile and persistent, and driven by things like monetary policy, financial market shocks, etc. In this case, $p_i - s - p_i^*$ can be very different from zero for a long time as the exchange rate fluctuates. Our expectation of this variable far into the future may be zero – that is, it maybe stationary with a mean of zero – but until prices fully adjust, there will be deviations from the law of one price due to price stickiness.

Implicit in this view is that consumers find it very costly to arbitrage price differences across borders. A typical macro model assumes firms can ship the good costlessly (though some models do include per unit shipping costs), but the costs for consumers is very, very high.

Moreover, I think price adjustment could be very prolonged. In the Calvo price-setting framework, when a firm does reset prices, it resets the price of its good both at home and in the foreign country equal to unit cost, so when prices are reset for that good, the law of one price holds. But this result follows because the Calvo model typically assumes constant elasticity of demand for the good. As we will see in the readings, more generally the optimal price (if prices are flexible, but also the optimal "reset" price) can be approximately expressed as a weighted average of unit cost and the price of competitors:

$$p_i = \alpha c_i + (1-\alpha) p$$

Here, c_i is the cost per unit in home currency and *p* is the home currency price of competitors in the home country. For the foreign country, the price is set as:

$$p_i^* = \alpha(c_i - s) + (1-\alpha)\, p^*$$

$c_i - s$ is the unit cost in foreign currency, and p^* is the foreign-currency price charged by competitors in the foreign country. Now suppose firms put a lot of weight on prices of competitors. In the extreme, suppose $\alpha = 0$. Then the optimal reset price at home is *p* and the optimal reset price in the foreign country is p^*, and so when prices are reset, we find $p_i - s - p_i^* = p - s - p^*$. If all goods are priced in a similar way, so there is this "local-currency pricing" (LCP), then $p - s - p^*$ is also determined as a deviation from the law of one price for all the competitors. The reset price does not set $p_i - s - p_i^*$ to zero but instead to the already existing deviation from the law of one price for other goods.

In a staggered price setting framework such as Calvo's, if there are price-setting complementarities, the price adjustment process can be very prolonged. When each firm decides on its price for the future, it is concerned not only with its current and expected future costs, but also with the prices of other firms, particularly those that are not currently adjusting their prices. For example, suppose there has been a one-time, permanent 5% monetary expansion that in the long run will have all firms raising their prices 5% (and nominal costs rising 5%). In the short run, even if the firm's nominal costs rose immediately by 5%, it would not want to raise its price immediately by 5%, because it must consider the prices of other firms. If many other firms have not yet adjusted their prices, this firm will raise its price by less than 5%. Subsequently, when these other firms change their prices, they will note that our original firm has not fully increased its price, so they will not raise their price all the way by 5%. And so on. This could lead to prolonged nominal price adjustment. Note that it is misleading to measure how frequently firms change prices and make a conclusion about how sticky prices are – the speed of adjustment depends very much on the degree of pricing complementarities.

As a broad approximation, I think this describes the pricing behavior for almost all consumer goods, whether they are traded or not. That is, the most important determinant of the real exchange rate is that goods prices are sticky in local currency and there are fluctuations in the exchange rate. This is not true for all goods. Goods that are more like commodities, such as gasoline and food do not behave this way.

Moreover, there are of course relative price changes of goods within countries, so it is not literally true that the deviation from the law of one price is the same for all goods. But as the 1993 paper shows, these "internal" relative price changes are much smaller than fluctuations in the deviation from the law of one price.

One objection to this line of thinking is that consumer prices are in fact the price of a composite good. They price both the actual good, and the distribution service that brings the good to the consumer. That is of course true, and distribution services can be a large part of the price of the good. Let p_i^d be the price of the distribution service and p_i^g be the price of the actual good. Then maybe the consumer price is a weighted average of those two prices:

$$p_i = \lambda p_i^g + (1-\lambda)\, p_i^d$$

We can write, assuming the same weights in the foreign country:

$$p_i - s - p_i^* = \lambda\left(p_i^g - s - p_i^{g*}\right) + (1-\lambda)\left(p_i^d - s - p_i^{d*}\right)$$

This is consistent with my view that there are deviations from the law of one price for almost all goods. There are deviations for non-traded distribution services, and for the actual good. And if the distribution service has a high weight, it accounts for a lot of the movement of $p_i - s - p_i^*$. But that is not what the critics have in mind. Instead, they think the law of one price holds for the actual good: $s = p_i^g - p_i^{g*}$. Substituting that back into the expression above, we get:

$$p_i - s - p_i^* = (1-\lambda)\left(p_i^d - p_i^g - \left(p_i^{d*} - p_i^{g*}\right)\right)$$

In this view, the nominal exchange rate doesn't matter at all. Fluctuations in the real exchange rate are driven by changes in the price of distribution services relative to the price of the good itself in one country relative to another. This is very implausible. First, note that because $1-\lambda < 1$, this relative relative price must be more volatile than the deviation from the law of one price measured using the consumer price, but that goes against what we see in the 1996 paper. The 1999 paper also notes that if the price of distribution services is correlated with other non-traded goods, it would imply a strong correlation between real exchange rates and the relative price of non-traded goods, which is not in the data. Moreover, the 1999 paper uses actual data from Japan on distribution services – the only paper I know of that does so – and shows there is no relation between those prices and real exchange rates.

There is a more insidious critique. Let me explain in this way. In France, the most popular toothpaste is called Signal. In the U.S., the most popular toothpaste is Crest. These two toothpastes are almost identical. If I were modeling preferences for them, I would say the elasticity of substitution is very high. Why don't we see Signal sold very much in the U.S., and Crest in France? I think there are costs to trade, especially the fixed cost of establishing a brand. They cannot overcome the small profit margin available for goods that have close substitutes.

What is the right way to model this? The right way is probably exactly as I expressed in the previous paragraph – close substitutes plus trade costs. But in practice our macro models don't have this level of detail, because different potentially tradable goods have different elasticities of substitution. The models are simplified. So, what simplified model describes this best?

One common tack is to say that Americans don't like Signal and French don't like Crest. The model doesn't set these as very close substitutes, but instead they are just as substitutable as a pair of pants are for an armchair. But the fact that the U.S. doesn't import much Signal means we have a strong "home bias" in preferences – Americans really like Crest and French really like

Signal. In this approach, the small amount of trade means the foreign goods are not important for each country's overall price level (even if spending on toothpaste itself was quite large.) I think the truth is that the relative price of Crest to Signal, $p^C - s - p^{S*}$, fluctuates a lot because the toothpastes are priced in local currency and there is price stickiness, and the exchange rate fluctuates a lot. But taking the view that these are different goods, and prices are flexible, these large fluctuations are then supposedly driven by changes in relative demand and supply for Crest versus Signal. It is highly unlikely that we could see such large price fluctuations as occur in the data driven by conditions in the toothpaste market. Moreover, these fluctuations are correlated strongly with all the other law-of-one price deviations, which seems unlikely if this were a flexible-price world and relative demand and supply for toothpaste was the force behind these volatile price changes.

It seems far more sensible to model $p^C - s - p^{S*}$ as a deviation from the law of one price. By the way, in empirical work, sometimes a related issue arises. Some scholars think the best way to detect deviations from the law of one price is to look at the prices of identical goods across borders: Crest toothpaste in the U.S. and France, or Signal toothpaste in the U.S. and France. I think this is a mistake. If Crest is sold at all in France, it is probably only sold at luxury hotels catering to U.S. tourists and will have a very high price. It strikes me that the right comparison is between the price of Crest in the U.S. and Signal in France. In practical terms, it is better to look at the prices of narrow categories of goods (as the next paper does) rather than trying to match the exact product.

While I think the right way to model consumer prices is sticky-price LCP, there are other prices to consider. "Border prices", as Burstein and Gopinath call them – the price of imports and exports – are much less likely to be sticky in the currency of the importer. They do seem to be sticky in the currency of invoicing, but that is not always the currency of the importer. Indeed, recent work has shown that much trade is invoiced in U.S. dollars even if neither of the trading partners are the U.S.!

Actually, most imports are not final consumer goods. They are intermediate products that go into the production of locally produced goods. Suppose the trade price is denominated in the exporter's currency, so the import price fluctuates one-for-one with the exchange rate. We still know very little about how much of that fluctuation is passed along to the producer that uses the intermediate good. That is, the importer may not be the same as the domestic producer using the imported intermediate good. There may be an import-export firm that decides how much of the exchange-rate change to pass along to the producer. It is probably not 100%, but probably more than 0%. At this point, we don't have conclusive answers.

So, there are many fine points in the theory and empirics of pricing, and the exact way in which goods are priced has implications for how exchange rate changes affect the macroeconomy. But I do think a fairly safe general assumption is that consumer prices are priced in local currency, and those prices are slow to adjust.

Next, please read:

Berka, Martin; Michael B. Devereux; and, Charles Engel. 2018. "Real Exchange Rates and Sectoral Productivity in the Eurozone" **American Economic Review** 108, 1543-1581.

Here is how I want you to read the paper. Read section 1, part A casually. It is set up as a fairly standard New Keynesian model but with fixed exchange rates. Even though the "labor wedge" plays a big role in this paper, don't pay special attention to it. I mainly want you to understand how productivity shocks to the traded sector affect real exchange rates, and the flexible-price version of this model demonstrates two different important channels. But answer this question:

1. In light of my discussion above, why does this paper argue that we are more likely to see traditional channels of real exchange rate determination like the Balassa-Samuelson effect in economies that share a common currency?

   Read section 1, parts B, C, and D more carefully. Then answer these questions:

2. Suppose the home and foreign traded goods are perfect substitutes, so λ is infinity. Referring to equation (11), this means $p_F^* - p_H = 0$. How does an increase in productivity in the home traded sector affect the real exchange rate? (A drop in *q* is a home real appreciation, meaning consumer prices in the home country rise relative to the foreign country.) Importantly, explain the economic mechanism. This is the classic Balassa-Samuelson theory.

3. Now assume there are no nontraded goods or nontraded distribution services. Also assume that there are no deviations from the law of one price. Referring to the equation at the bottom of page 1552, only the middle term remains. First, how would an increase in productivity in the home country affect $p_F^* - p_H$? How does that affect the real exchange rate, assuming home bias in preferences ($\omega > 0.5$)?

The last two questions show that depending on the model set-up, under flexible prices an increase in productivity in the traded sector could cause a real appreciation or depreciation. Sometimes the literature is confusing on this point.

Read the rest of the paper casually, both to see the empirical results and to see a typical way in which macroeconomists verify a model.

The next paper is quite long, and very rich. It is both too long and too compact to mine every detail for this class, but it is a valuable resource if you want to continue to study international goods pricing. I do want you to read the whole paper, and I give some specific instructions for reading each section below.

Burstein, Ariel, and Gita Gopinath. "International prices and exchange rates." *Handbook of international economics*. Vol. 4. Elsevier, 2014. 391-451.

Read sections 1 and 2.

4. In your own words, simply restate empirical findings 1-5.

Read the first part of section 3 lightly. The notation gets confusing. The main point is that all 5 empirical findings can be explained in a flexible-price world if distribution services are an important part of consumer prices, and if there is a lot of home bias in preferences. I have already explained above why I don't believe these explanations will work well for accounting for real exchange rate movements.

Section 3.1 is difficult. Here I am going to ask you to do some exercises that will get across the main points:

Demand for firm *i*'s product is given by $D(P_i / P)$. Demand depends on the price the firm charges, P_i, relative to the prices of competitors of firm *i*, *P*. Assume the (nominal) cost per unit of producing the good is C_i. The firm chooses its price to maximize profits:

$$P_i D(P_i / P) - C_i D(P_i / P)$$

Remember that the elasticity of demand can be expressed as:

$$\varepsilon(P_i / P) \equiv -\frac{(P_i / P) D'(P_i / P)}{D(P_i / P)}.$$

We are not assuming that the elasticity of demand is some constant. In general, the elasticity of demand depends on the price, so we write $\varepsilon(P_i / P)$.

5. With this definition in hand, show that we can rewrite the first-order condition for profit maximization as:

$$P_i = \frac{\varepsilon(P_i / P)}{\varepsilon(P_i / P) - 1} C_i.$$

6. Take a first-order log approximation to $P_i = \frac{\varepsilon(P_i / P)}{\varepsilon(P_i / P) - 1} C_i$ to show we can write

$$p_i = \frac{\eta}{\varepsilon - 1 + \eta} p + \frac{\varepsilon - 1}{\varepsilon - 1 + \eta} c.$$

where η is the elasticity of ε: $\eta = \frac{P_i}{P}\frac{\varepsilon'}{\varepsilon}$

The log of the price of good *i* is a weighted average of the cost and the price of its competitors, as long as $\eta \neq 0$. Pass-through of costs to prices is less than one-for-one if $\eta > 0$, that is, if the elasticity of demand increases with the price.

The expression above doesn't seem to have anything to do with international economics. However, suppose we are talking about a good sold in the U.S., but some or all of the cost is incurred in a foreign country (that is the U.S. imports the good.) For example, a fraction α is incurred abroad, and cost in foreign currency is w^*, so in home currency is $s + w^*$. The rest of the cost is in the U.S. – maybe the good is finished in the U.S., or maybe these are distribution costs. Then we have:

$$p_i = \frac{\eta}{\varepsilon - 1 + \eta} p + \frac{\varepsilon - 1}{\varepsilon - 1 + \eta}\left((1-\alpha) w + \alpha\left(s + w^*\right)\right).$$

Then holding *p*, *w*, and w^* constant, the "pass-through" of a change in *s* to p_i is $\frac{\alpha(\varepsilon - 1)}{\varepsilon - 1 + \eta}$.

Suppose one wanted to estimate a pass-through regression of the exchange rate into the price of good *i*. There really are two points to be made here. First, the regression needs to control for *p*, *w*, and w^*. Second, there is an error term in the regression. What is driving it? If it is some macro shock, then the error term might be correlated with some or all of the right-hand-side variables, which makes the estimation of the parameters inconsistent.

But then section 3.1 goes on to make the point that even if pass-through is incomplete for a firm, that does not mean the firm charges a different price in the U.S. then it does in its own country. That is, there is not necessarily "pricing to market".

From above, we have, assuming that the degree of strategic complementarities is the same in both the home and foreign markets (that is, α is the same in both markets):

$$p_{i,t} = (1-\alpha)c_t + \alpha p_t \text{ and } p_{i,t}^* = (1-\alpha)(c_t - s_t) + \alpha p_t^*.$$

Then we have

$$s_t + p_{i,t}^* - p_{i,t} = \alpha\left(s_t + p_t^* - p_t\right)$$

That is, the amount of pricing to market here depends only on aggregate prices, over which the firm has no control. This point exemplifies why we need to be careful about micro studies of pricing to market that "assume" that the macroeconomy is "exogenous" for the firm and therefore can be ignored in empirical work using micro data.

Section 3.1 discusses what is necessary to get pricing to market. Section 3.2 discusses sticky prices. It makes the point that I made above – which is that when there are pricing complementarities, price adjustment may be much slower, and pass-through of exchange rates to prices lower. This section shows that when there are pricing complementarities, the prices of competitors at time *t* is determined not just by firms that reset prices at time *t*, but by firms that reset prices in the past. This section also works through my 2006 paper that shows that the optimal currency of price-setting for exports under sticky prices depends on how sensitive the export price would be to exchange rates if the firm could flexibly choose its optimal price.

I don't have specific questions to ask from section 3.2. I only ask you to read it carefully. Then read section 4. Here, one thing that I want you to try to derive on your own is that, from the perspective of the individual firm, each of the three models presented end up being models that give us the key necessary ingredient for incomplete pass-through that we derived above: that the elasticity of demand facing a given firm increases as the firm increases its price. The Atkeson-Burstein model is particularly important. In that model, the elasticity of demand for a firm's product is the weighted average of two elasticities – the elasticity of substitution for goods within a narrow category (say, men's clothing), and the elasticity of substitution of goods across categories (men's clothing for furniture, for example.) When a firm has a large share of its market, then its competitors are more from other categories of goods, but when it has a small share, its main competitors are within its own market. The within elasticity is obviously higher than the across categories elasticity. When a firm increases its price, it loses market share, and so it no longer cares as much about competition from other categories but instead cares more about competition from within its own category of goods, for which the elasticity of substitution is higher. As a result, as it raises its price, its elasticity of demand increases.

Read section 5 for your education. I don't think it is necessary to work through each equation. Here, I just want you to see some of the scope of models of international pricing.

The next question you need to answer just asks you to demonstrate a result shown in section 6.

7. Above, we saw $s_t + p^*_{i,t} - p_{i,t} = \alpha\left(s_t + p^*_t - p_t\right)$.Suppose the strategic complementarities are the same for all goods (that is, α does not depend on the good., so $s_t + p^*_{j,t} - p_{j,t} = \alpha\left(s_t + p^*_t - p_t\right)$ for any good *j*.) Also assume that the price indexes that give us the aggregate prices *p* and p^* have the same weights on each good, so $p_t = \sum_{j=1}^{N} \gamma_j p_{j,t}$ and $p^*_t = \sum_{j=1}^{N} \gamma_j p^*_{j,t}$. Then show $s_t + p^*_{j,t} - p_{j,t} = 0$ for all *j*.

But there is an even stronger result, which is that for the aggregate price, pass-through does not depend on the degree of price complementarities. The last question I ask is for you to complete the proof of that. In this case, to simplify, let's assume there are only two goods: a home good (produced entirely at home) and a foreign good (produced entirely abroad.)

8. Assume

$$p_{h,t} = (1-\alpha) c_t + \alpha p_t$$

$$p_{f,t} = (1-\alpha)\left(s_t + c_t^*\right) + \alpha p_t$$

Let $p_t = \gamma p_{h,t} + (1-\gamma) p_{f,t}$. Then, calculate p_t and show the pass-through of the exchange rate is $1-\gamma$. That is, it depends only on the share of foreign goods in the price index and not on the degree of strategic complementarities.

As the reading notes, this is a special case, in that it requires the strategic complementarities to be the same for all goods. It also depends on price flexibility, which the reading also discusses.

Section 7 presents a general equilibrium model with LCP. It notes that there is some funny behavior of wages, if nominal wages are flexible, so it extends the model to have sticky nominal wages. Read this section for fun – that is, again, do not work through it super carefully.

This chapter points to a lot of richness in international pricing. Prices at the dock (border prices) are priced differently than consumer prices. Producer prices are different than either border prices or consumer prices. Transportation costs – both fixed and variable – matter for pricing traded goods. Distribution costs matter for pricing consumer goods. Price behavior will be different for goods that have close substitutes produced abroad versus those that do not.

It is tempting to say that a good model should incorporate all these features of pricing. Many quantitative macro models are very rich. Parameters are calibrated or estimated to try to "match" various moments in the data. So, it is feasible to construct open-economy models with all this richness. However, the more detail in the model, the more difficult things are to interpret. There is an old analogy that you have probably heard: a map that is on a 1:1 scale (that is, one meter on the map represents one meter in the area being mapped) is not a useful map. It has all the detail but doesn't help us navigate. Simpler models may, in many cases, be better than more complicated models. Inevitably, that means the simpler model will get some predictions wrong – it will fail to "match" some moments. The model is useful if it helps us understand an important mechanism in the global economy, but we need to be mindful of whether the simplifications in the model undermine the usefulness of the insight from the model. Modeling the open economy is an art as well as a science.

Ten Important Papers

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. "Importers, exporters, and exchange rate disconnect." *American Economic Review* 104.7 (2014): 1942-78.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. "International shocks, variable markups, and domestic prices." *The Review of Economic Studies* 86.6 (2019): 2356-2402.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. "Dominant currencies: How firms choose currency invoicing and why it matters." *The Quarterly Journal of Economics* 137.3 (2022): 1435-1493.

Atkeson, Andrew, and Ariel Burstein. "Pricing-to-market, trade costs, and international relative prices." *American Economic Review* 98.5 (2008): 1998-2031.

Auer, Raphael, Ariel Burstein, and Sarah M. Lein. "Exchange rates and prices: evidence from the 2015 Swiss franc appreciation." *American Economic Review* 111.2 (2021): 652-86.

Blanco, Andrés, and Javier Cravino. "Price rigidities and the relative PPP." *Journal of Monetary Economics* 116 (2020): 104-116.

Fitzgerald, Doireann, and Stefanie Haller. "Pricing-to-market: evidence from plant-level prices." *Review of Economic Studies* 81.2 (2014): 761-786.

Gopinath, Gita, et al. "Dominant currency paradigm." *American Economic Review* 110.3 (2020): 677-719.

Gopinath, Gita, and Oleg Itskhoki. "Dominant Currency Paradigm: A Review." *NBER Working Papers* 29556 (2021).

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon. "Currency choice and exchange rate pass-through." *American Economic Review* 100.1 (2010): 304-36.

Addendum

In class, I briefly talked about this paper:

de Walque, Gregory; Thomas Lejeune; Ansgar Rannenberg; and, Raf Wouters. 2023. “Low Pass-through and International Synchronization in General Equilibrium: Reassessing Vertical Integration.” Journal of International Economics 140, 103710.

The paper builds a New Keynesian model that is aimed at better accounting for two facts: that consumer prices adjust more slowly than trade prices and helps explain the high synchronization of business cycles across countries in response to some shocks. It also produces a fair amount of “disconnect.”

I don’t want to go into the details of the paper, but roughly speaking it has goods being LCP at the dock and LCP for final goods, but the imported goods serve as intermediate goods into the production of final goods in the home goods sector. There is also a distribution sector, which uses home final goods to produce both final goods. But I don't think the distribution sector is the key thing here. (This description is not entirely accurate because what I am calling final goods are in fact themselves “producer” goods that are combined to make a final homogeneous consumption good that has flexible prices and is sold in competitive markets, but we could just think of these producer goods as being final consumer goods and the household has utility over those goods.)

The main thing for thinking about speed of price adjustment is that there is a double layer of stickiness for the home final good. Its price is sticky, but also the price of the imported intermediate input has a sticky price, which makes the price of final goods stickier than trade prices. That is, there is not only slow adjustment of the price of the final good, but there is slow adjustment of the cost of one of the inputs.

This paper is related to the papers by Amiti, Itskhoki and Konings on the reading list of “important papers.”

Answers to questions on goods price setting

1. In light of my discussion above, why does this paper argue that we are more likely to see traditional channels of real exchange rate determination like the Balassa-Samuelson effect in economies that share a common currency?

Answer:

When nominal goods prices are sticky, particularly when they are sticky in the consumers' currencies, most of the movement in real exchange rates over short- and medium-horizons will come from changes in the nominal exchange rate, which may be driven by monetary or financial shocks. In other words, if $Q = \frac{SP^*}{P}$, but the nominal prices react slowly, then movements in *S* will mostly determine movements in *Q*. But when *S* is rigidly fixed, as in a currency zone, that does not happen. Then even when prices are sticky, if we look at horizons of a year or more, we will see channels like the Balassa-Samuelson effect play a larger role.

2. Suppose the home and foreign traded goods are perfect substitutes, so λ is infinity. Referring to equation (11), this means $p_F^* - p_H = 0$. How does an increase in productivity in the home traded sector affect the real exchange rate? (A drop in *q* is a home real appreciation, meaning consumer prices in the home country rise relative to the foreign country.) Importantly, explain the economic mechanism. This is the classic Balassa-Samuelson theory.

Answer:

In this case, the equation for the log of the real exchange rate reduces to:

$$q = a_F^* - a_H - \left(a_N^* - a_N\right)$$

Holding productivity in other sectors constant, an increase in a_H leads to a home real appreciation. The logic is this: Since the traded goods are perfect substitutes, their prices will be the same. So, the only way that the home consumer price level can rise relative to the foreign level is if home nontraded prices rise relative to foreign nontraded prices. When productivity in the home traded sector rises, it increases the marginal product of labor, thus increasing the wage that producers of traded goods are willing to pay. Labor is mobile between sectors, so wages are equalized between sectors. This means nontraded producers must pay a higher wage, but they've experienced no increase in productivity, so the cost and price of their product rises.

3. Now assume there are no nontraded goods or nontraded distribution services. Also assume that there are no deviations from the law of one price. Referring to the equation at the bottom of page 1552, only the middle term remains. First, how would an increase in

productivity in the home country affect $p_F^* - p_H$? How does that affect the real exchange rate, assuming home bias in preferences ($\omega > 0.5$)?

Answer:

The equation for the real exchange rate reduces to $q = (2\omega - 1)\tau$, where $\tau = p_F^* - p_H$. Now the foreign and home goods are imperfect substitutes. The increase in productivity of the traded good in the home country lowers its costs, and so lowers the price relative to the price of the foreign traded good. When $\omega > 0.5$, this leads to a home real appreciation. The reason is that the home consumers put a higher weight on the home traded good in their consumption basket, so the decline in the relative price of the home traded good makes the home consumer price level fall relative to the foreign consumer price level.

4. In your own words, simply restate empirical findings 1-5.

Answer: (In Burstein and Gopinath's words):

Empirical Finding 1. Real exchange rates for consumer prices co-move closely with nominal exchange rates at short and medium horizons. The persistence of these RERs is large with long half-lives.

Empirical Finding 2. Movements in RERs for tradeable goods are roughly as large as those in overall CPI-based RERs when tradeable goods prices are measured using consumer prices or producer prices, but significantly smaller when measured using border prices.

Empirical Finding 3. ERPT into consumer prices is lower than into border prices. ERPT into border prices is typically incomplete in the long run, displays dynamics, and varies considerably across countries.

Empirical Finding 4. Border prices, in whatever currency they are set in, respond partially to exchange rate shocks at most empirically estimated horizons.

Empirical Finding 5. There are large deviations from relative PPP for traded goods produced in a common location and sold in multiple locations. On average, these deviations co-move with exchange rates across locations.

5. With this definition in hand, show that we can rewrite the first-order condition for profit maximization as:

$$P_i = \frac{\varepsilon(P_i / P)}{\varepsilon(P_i / P) - 1} C_i .$$

Answer:

The home firm chooses P_i to maximize $P_i D(P_i / P) - C_i D(P_i / P)$. The 1st-order condition is:

$$D(P_i / P) + \left(P_i / P\right) D'(P_i / P) - \left(C_i / P\right) D'(P_i / P) = 0$$

Rearrange this expression:

$$PD(P_i / P) + P_i D'(P_i / P) = C_i D'(P_i / P)$$

$$\frac{PD(P_i / P)}{D'(P_i / P)} + P_i = C_i$$

$$P_i\left(1 + \frac{(P_i / P) D(P_i / P)}{D'(P_i / P)}\right) \equiv P_i\left(1 - \frac{1}{\varepsilon(P_i / P)}\right) = C_i \text{, so } P_i = \frac{\varepsilon(P_i / P)}{\varepsilon(P_i / P) - 1} C_i$$

6. Take a first-order log approximation to $P_i = \dfrac{\varepsilon(P_i / P)}{\varepsilon(P_i / P) - 1} C_i$ to show we can write

$$p_i = \frac{\eta}{\varepsilon - 1 + \eta} p + \frac{\varepsilon - 1}{\varepsilon - 1 + \eta} c.$$

where η is the elasticity of ε: $\eta = \dfrac{P_i}{P}\dfrac{\varepsilon'}{\varepsilon}$.

<u>Answer</u>

Let lower case letters represent the logs of upper-case letters, and so rewrite the condition as:

$$e^{p_i} = \frac{\varepsilon(e^{p_i - p})}{\varepsilon(e^{p_i - p}) - 1} e^{c_i}$$

Now do a 1st-order Taylor-Series expansion, where the subscript 0 represents the value of the variable at the point of approximation, and for simplicity let the lower case letters be the deviation of the log from its value at the point of approximation.

$$P_{i0} p_i = \frac{\left(P_{i0} / P\right) \varepsilon_0{}' \left(\varepsilon_0 - 1\right)\left(p_i - p\right) - \left(P_{i0} / P\right) \varepsilon_0{}' \varepsilon_0 \left(p_i - p\right)}{\left(\varepsilon_0 - 1\right)^2} C_{i0} + \frac{\varepsilon_0}{\varepsilon_0 - 1} C_{i0} c_i \text{, which}$$

simplifies to:

$$P_{i0}p_i = \frac{\eta\varepsilon_0(\varepsilon_0-1)(p_i-p)-\eta(\varepsilon_0)^2(p_i-p)}{(\varepsilon_0-1)^2}C_{i0}+P_{i0}c_i$$

$$P_{i0}p_i = \frac{\eta(\varepsilon_0-1)(p_i-p)-\eta(\varepsilon_0)(p_i-p)}{\varepsilon_0-1}P_{i0}+P_{i0}c_i \;\Rightarrow\; (\varepsilon_0+\eta-1)p_i = \eta p+(\varepsilon_0-1)c_i$$

7. Above, we saw $s_t + p^*_{i,t} - p_{i,t} = \alpha\left(s_t + p^*_t - p_t\right)$.Suppose the strategic complementarities are the same for all goods (that is, α does not depend on the good., so $s_t + p^*_{j,t} - p_{j,t} = \alpha\left(s_t + p^*_t - p_t\right)$ for any good *j*.) Also assume that the price indexes that give us the aggregate prices *p* and p^* have the same weights on each good, so $p_t = \sum_{j=1}^{N}\gamma_j p_{j,t}$ and $p^*_t = \sum_{j=1}^{N}\gamma_j p^*_{j,t}$. Then show $s_t + p^*_{j,t} - p_{j,t} = 0$ for all *j*.

Answer:

From the definitions of the price indexes, we have:

$$s_t + p^*_t - p_t = s_t + \sum_{j=1}^{N}\gamma_j\left(p^*_{j,t}\right) - \sum_{j=1}^{N}\gamma_j p_{j,t} = \sum_{j=1}^{N}\gamma_j\left(s_t + p^*_{j,t} - p_{j,t}\right).$$

where the second equality follows because the index weights sum to one. But then, because $s_t + p^*_{i,t} - p_{i,t} = \alpha\left(s_t + p^*_t - p_t\right)$ for all *i*, we get from the last line of the equation above: $s_t + p^*_t - p_t = \alpha\left(s_t + p^*_t - p_t\right)$, which gives us $s_t + p^*_t - p_t = 0$ if $\alpha \neq 0$. It then follows that because $s_t + p^*_{i,t} - p_{i,t} = \alpha\left(s_t + p^*_t - p_t\right)$, we must have $s_t + p^*_{j,t} - p_{j,t} = 0$ for all *j*.

8. Assume

$$p_{h,t} = (1-\alpha)c_t + \alpha p_t$$
$$p_{f,t} = (1-\alpha)\left(s_t + c^*_t\right) + \alpha p_t$$

Let $p_t = \gamma p_{h,t} + (1-\gamma)p_{f,t}$. Then, calculate p_t and show the pass-through of the exchange rate is $1-\gamma$. That is, it depends only on the share of foreign goods in the price index and not on the degree of strategic complementarities.

Answer:

$$\begin{aligned} p_t &= \gamma\left[(1-\alpha)c_t + \alpha p_t\right] + (1-\gamma)\left[(1-\alpha)\left(s_t + c_t^*\right) + \alpha p_t\right] \\ &= \gamma(1-\alpha)c_t + (1-\gamma)(1-\alpha)\left(s_t + c_t^*\right) + \alpha(\gamma + 1 - \gamma)\, p_t \end{aligned}.$$

We get:

$(1-\alpha)\, p_t = \gamma(1-\alpha)c_t + (1-\gamma)(1-\alpha)\left(s_t + c_t^*\right)$, which gives us $p_t = \gamma c_t + (1-\gamma)\left(s_t + c_t^*\right)$

Assignment 2: Uncovered interest rate parity (UIP) and Risk Premiums

The first reading for this is

Engel, Charles. 2015. "Exchange Rates and Interest Parity", **Handbook of International Economics** 4, 494-515.

Note that I am asking you only to read pages 494-515 at this point.

It is necessary to provide a fair amount of background before starting. UIP says that a riskless asset denominated in the home currency, which earns the interest rate i_t between period *t* and $t+1$ earns the same expected return as a riskless foreign asset that earns i_t^* with certainty, which means the expected return in home currency on the foreign asset is $i_t^* + E_t s_{t+1} - s_t$, where s_t is the log of the home currency price of foreign currency. $E_t s_{t+1} - s_t$ is the expected depreciation of the home currency, or the expected appreciation of the foreign currency. The actual depreciation will be $s_{t+1} - s_t$, so there is foreign exchange risk since s_{t+1} is not known at the time of the investment, *t*. UIP says that for some reason, investors do not care about this risk, and markets drive the two expected returns into equality.

A well-known test of UIP is to regress the ex-post change in the exchange rate on the interest rate differential. Under UIP, the only thing that should be able to systematically predict $s_{t+1} - s_t$ is the interest rate differential, since UIP can be stated as:

$$E_t s_{t+1} - s_t = i_t - i_t^*$$

Another way of saying this is that the ex-post change should equal the interest differential plus an unpredictable expectation error: $s_{t+1} - s_t = i_t - i_t^* + u_{t+1}$, where $E_t u_{t+1} = 0$. In principle, no other variable known at time *t* should help predict $s_{t+1} - s_t$ except $i_t - i_t^*$. In practice, we find UIP is usually rejected, using only $i_t - i_t^*$ to predict $s_{t+1} - s_t$, because $E_t s_{t+1} - s_t$ does not change one for one with $i_t - i_t^*$. Specifically, a common test is to estimate:

$$s_{t+1} - s_t = \alpha + \beta\left(i_t - i_t^*\right) + u_{t+1}.$$

Under the null of UIP, we should find $\alpha = 0$, $\beta = 1$. While generally we do not reject the null of $\alpha = 0$, surprisingly, for many currency pairs and over many time periods (particularly from 1970-2000), we find $\hat{\beta} < 1$ and usually $\hat{\beta} < 0$. We can reject $\beta = 1$.

This regression can be rewritten as:

$$i_t^* + s_{t+1} - s_t - i_t = \alpha + (1-\beta)\left(i_t^* - i_t\right) + u_{t+1},$$

When $\hat{\beta} < 1$, this regression says the excess return on the foreign asset ("excess" over the riskless home asset) tends to rise as $i_t^* - i_t$ rises. In other words, an increase in $i_t^* - i_t$ helps predict an expected excess return on the foreign asset: $i_t^* + E_t s_{t+1} - s_t - i_t = \alpha + (1-\beta)\left(i_t^* - i_t\right)$. Often in the literature, the "expected excess return" is called a "risk premium", though several weeks of the class are devoted toward other explanations for why there may be an expected excess return.

The log of the real exchange rate is given by $q_t = s_t + p_t^* - p_t$, which is the price level in the foreign country relative to the home country, when expressed in a common currency. The ex-ante) real interest rate in each country are given by $r_t \equiv i_t - E_t\left(p_{t+1} - p_t\right)$ and $r_t^* = i_t^* - E_t\left(p_{t+1}^* - p_t^*\right)$. By subtracting relative expected inflation differentials from both sides, UIP can be written as:

$$E_t q_{t+1} - q_t = r_t - r_t^* .$$

Theories are usually written in terms of real interest rates and real exchange rates, even though much of the empirical work uses nominal exchange rates and interest rates. This is justifiable because goods prices are so stable relative to nominal exchange rates (for low-inflation, advanced economies.) For much of the discussion, we will pretend that the standard test is the regression:

$$q_{t+1} - q_t = \alpha + \beta\left(r_t - r_t^*\right) + u_{t+1} .$$

Note that to implement this exactly, we would need measures of the ex-ante real interest rates, which would require measures of the rationally expected inflation rates.

A point worth making is that these log approximations are not exact, and when we get to the theory, for some purposes it is important to point out the difference between the approximation and the exact formulation. In empirical work, the approximation is indistinguishable from the exact representation (that is, the tests give the same conclusions) if the exact representation is even testable. This may be puzzling considering the theory. For the most part, I will ignore these issues, but I will point them out now. Investors care about real returns, so the exact specification of UIP for a home investor is:

$$(1) \qquad \left(1+i_t\right) E_t\left(\frac{1}{P_{t+1}/P_t}\right) = \left(1+i_t^*\right) E_t\left(\frac{S_{t+1}/S_t}{P_{t+1}/P_t}\right)$$

The upper-case letters for the exchange rates and prices are their actual values, rather than the logs of their values. Suppose the variables that are random at time *t* (which are $P_{t+1}, S_{t+1}, P_{t+1}^*$) are lognormally distributed. If *X* and *Y* are lognormally distributed, we have:

$$E\left(XY\right) = E\left(e^{x+y}\right) = e^{Ex+Ey+.5\operatorname{var}(x)+.5\operatorname{var}(y)+\operatorname{cov}(x,y)},$$

where lower case letters are the logs of upper-case letters. Then it follows that

$$\ln\left[E\left(XY\right)\right] = Ex + Ey + .5\operatorname{var}(x) + .5\operatorname{var}(y) + \operatorname{cov}(x,y),$$

so, we do not have simply $\ln\left[E\left(XY\right)\right] = Ex + Ey$. Let's express UIP in real terms from the perspective of the home investor and assume r_t and r_t^* (the real returns in units of home consumption and foreign consumption, respectively) are known at time *t*:

$$1+r_t=\left(1+r_t^*\right)E_t\left(\frac{Q_{t+1}}{Q_t}\right).$$

where Q is the level (not the log) of the real exchange rate. Then under log normality, and assuming $\ln(1+x)\approx x$ when x is not random, we have:

$$r_t=r_t^*+E_tq_{t+1}-q_t+.5\operatorname{var}_t\left(q_{t+1}\right)$$

That is, even when expected returns are equal, we have exactly (rather than approximately)

(2) $\quad r_t^*+E_tq_{t+1}-q_t-r_t=-.5\operatorname{var}_t\left(q_{t+1}\right).$

In practice, the term on the right-hand side in the data is so small that in empirical work we can ignore it and say that UIP implies $r_t^*+E_tq_{t+1}-q_t-r_t=0$.

To confuse and complicate things, let me digress and talk about the UIP condition if the investor evaluates returns in units of the foreign good (for example, a foreign investor that consumes the foreign basket):

$$\left(1+r_t^*\right)=\left(1+r_t\right)E_t\left(\frac{1}{Q_{t+1}/Q_t}\right)$$

Then UIP gives us:

(3) $\quad r_t^*+\left(E_tq_{t+1}-q_t\right)-r_t=.5\operatorname{var}_t\left(q_{t+1}\right).$

Note that the right-hand-sides of (2) and (3) are different. Both arise from risk-neutral investors with rational expectations equating excess returns. The important point is that home investors and foreign investors evaluate returns in different units – home in terms of the home consumption basket and foreign in terms of the foreign consumption basket. This is different from "standard" finance that assumes everyone agrees on what the real return on an asset is. If two people have different consumption baskets, they may disagree on what the real return is. The not-so-important digression is that the incompatibility of (2) and (3) means there may be no equilibrium in the market with two risk-neutral investors that have different consumption baskets. It's as if there are two bettors on a horse race that are risk neutral but disagree on where the finish line is – one will bet all the money he can on the sprinter and the other all the money he can on the long-distance horse. If there is no limit to how much they can bet (and borrow to do so), there will be no equilibrium. But empirically, the r.h.s. term, $.5\operatorname{var}_t\left(q_{t+1}\right)$ is small, so we would test the null of $r_t^*+\left(E_tq_{t+1}-q_t\right)-r_t=0$.

Now let's move beyond the risk neutral case. We know that when we have time-separable expected utility with discount factor β, that the Euler equation for choosing any asset is:

$$E_t\left(\frac{\beta U'\left(C_{t+1}\right)\left(1+r_{j,t+1}\right)}{U'\left(C_t\right)}\right)=1, \text{ for all assets } j.$$

Let's use the short-hand notation $M_{t+1}=\dfrac{\beta U'\left(C_{t+1}\right)}{U'\left(C_t\right)}$. We can write:

$$E_t\left(M_{t+1}\left(1+r_{j,t+1}\right)\right)=E_t\left(M_{t+1}\right)E_t\left(\left(1+r_{j,t+1}\right)\right)+\operatorname{cov}_t\left(M_{t+1},r_{j,t+1}\right)=1,$$

so

$$E_t\left(\left(1+r_{j,t+1}\right)\right)=\frac{1-\operatorname{cov}_t\left(M_{t+1},r_{j,t+1}\right)}{E_t\left(M_{t+1}\right)}$$

The convention is to write the return on the asset whose real return is known at time *t* and therefore riskless without the *j* subscript, and with a time *t* subscript (even though it refers to the return between *t* and $t+1$.) We have

$$1+r_t=\frac{1}{E_t\left(M_{t+1}\right)}$$

Hence, we can write:

$$E_t\left(\left(r_{j,t+1}-r_t\right)\right)=-\frac{\operatorname{cov}_t\left(M_{t+1},r_{j,t+1}\right)}{E_t\left(M_{t+1}\right)}$$

The "risk premium" on asset *j* is positive if $r_{j,t+1}$ is negatively correlated with $U'\left(C_{t+1}\right)$, which happens when $r_{j,t+1}$ tends to be high when C_{t+1} is high. If asset *j* is more like insurance, the "risk premium" could even be negative, because the asset tends to have high returns when C_{t+1} is low.

Perhaps confusingly, a well-known result in finance (though this is really a linear algebra result) is the following. Let's write $1+r_{j,t+1}=\frac{Z_{j,t+1}+D_{j,t+1}}{Z_{j,t}}$, where $Z_{j,t}$ is the price of the asset, and $D_{j,t+1}$ is the dividend. The condition we wrote above can be written as:

$$E_t\left(M_{t+1}\left(Z_{j,t+1}+D_{j,t+1}\right)\right)=Z_{j,t}.$$

Here is the "linear algebra" result. Under certain conditions (my recollection is that the payoff in every state must be weakly positive (that is, limited liability), if the payoff in any state is strictly positive then the price of the asset at time *t* is strictly positive, and if any two assets have the same payoff in every state then their price is the same), there exists an M_{t+1}, called a *stochastic discount factor* or sdf that satisfies

$E_t\left(M_{t+1}\left(Z_{j,t+1}+D_{j,t+1}\right)\right)=Z_{j,t}$, or equivalently, $E_t\left(M_{t+1}\left(1+r_{j,t+1}\right)\right)=1$ for all assets *j*.

Because this is a linear algebra result, it does not depend on any assumptions about investor behavior. Investors don't have to act rationally or maximize expected utility. They do not have to have rational expectations (but to be clear, the expectation symbol above refers to the expectation of the true probability distribution). Indeed, there really don't even have to be investors. This is just a mathematical result.

If there are complete markets, or if a more limited set of assets can replicate the payoffs of complete markets, there is an even stronger result, which is that M_{t+1} is unique. A lot of finance papers implicitly assume complete markets. But if markets are not complete, there are an infinite number of sdfs that satisfy the above relationship.

What I am about to say is a useful though not entirely fair characterization of one branch of empirical finance. It is devoted to finding empirical representations of M_{t+1} for which the null hypothesis of $E_t\left(M_{t+1}\left(1+r_{j,t+1}\right)\right)=1$ cannot be rejected for some set of assets. For example, maybe it finds M_{t+1} as a function of some statistical decomposition into factors. More interestingly, some studies find M_{t+1} as a function of macroeconomic variables. These papers then tell a story of why $\text{cov}_t\left(M_{t+1}, r_{j,t+1}\right)$ might be positive or negative for certain assets.

We won't read papers from this literature. I find these quite interesting and insightful. But they are a little hard for my mind to wrap around, because I always want a model – even a very simple one – to help me understand things, and these papers usually tell a story, rather than using a model, about the macroeconomy, albeit a plausible sounding story.

Here is one thing to note in the empirical context: Suppose $1+r_{j,t+1}$ is the real return in units of the home consumption basket. Then we can always do the following regrouping:

$$E_t\left(M_{t+1}\left(1+r_{j,t+1}\right)\right)=E_t\left(\left(\frac{M_{t+1}Q_{t+1}}{Q_t}\right)\left(\frac{1+r_{j,t+1}}{Q_{t+1}/Q_t}\right)\right)=E_t\left(M^*_{t+1}\left(1+r^*_{j,t+1}\right)\right)=1$$

That is, if M_{t+1} is an sdf for returns in units of home consumption, then M^*_{t+1} is an sdf for returns on those same assets when their returns are expressed in units of the foreign consumption basket, where,

$$M^*_{t+1}=\frac{M_{t+1}Q_{t+1}}{Q_t}..$$

If markets are complete, M_{t+1} and M^*_{t+1} are unique. Note, this statement does not require that there be home investors and foreign investors. The theorem itself is a linear algebra result, and as noted above, there really don't even have to be investors for it to be true.

Instead, we will stick to models where M_{t+1} comes from somebody's utility function. But you probably know that there are problems when we use the standard expected utility model, especially when we assume constant relative risk aversion, so $U\left(C_t\right)=\frac{1}{1-\gamma}C_t^{1-\gamma}$, where γ is the coefficient of relative risk aversion. The Euler equation implies:

$$E_t\left(\beta\frac{C_{t+1}^{-\gamma}\left(1+r_{j,t+1}\right)}{C_t^{-\gamma}}\right)$$

Assuming log-normality, this can be written as:

$$\ln(\beta)+\gamma c_t-\gamma E_t c_{t+1}+E_t(r_{j,t+1})+.5\gamma^2 \operatorname{var}_t(c_{t+1})+.5\operatorname{var}_t(r_{t+1})-\gamma \operatorname{cov}_t(c_{t+1},r_{t+1})=0$$

For the riskless asset, we get:

$$\ln(\beta)+\gamma c_t-\gamma E_t c_{t+1}+r_t+.5\gamma^2 \operatorname{var}_t(c_{t+1})=0$$

Subtracting the second from the first, we get:

$$E_t(r_{j,t+1}-r_t)=\gamma \operatorname{cov}_t(c_{t+1},r_{j,t+1})$$

This equation says the risk premium on asset *j* depends on the covariance of returns with consumption. One problem is that empirically, this equation does not hold up. But also, to explain large risk premiums, such as seem to hold for equity returns, γ would need to be very large. While psychological experiments suggest γ may be in the range of two to four, to account for the size of risk premiums on equities, it needs to be more like 100. But if it were that large, then the implications for the behavior of the riskless rate of return are weird. From above, we have that

$$E_t c_{t+1}-c_t=\frac{1}{\gamma}\left(r_t+.5\gamma^2 \operatorname{var}_t(c_{t+1})+\ln(\beta)\right)$$

A large γ means expected consumption growth is very insensitive to the riskless real rate. For this type of utility function, we have that γ is the coefficient of relative risk aversion, but $1/\gamma$ is the intertemporal elasticity of substitution. This does not work well empirically.

One popular assumption in the finance literature is to assume people have a different type of utility function. Not only does it get away from time-separable, constant discount factor, constant relative risk aversion expected utility – it does not even assume people maximize *expected* utility! These are Epstein-Zin-Weil (or, often, simply Epstein-Zin) preferences.

With standard expected utility, we can write $V_t=(1-\delta)E_t\sum_{j=0}^{\infty}\delta^j C_{t+j}^{1-\gamma}$, which can be written recursively as:

$$V_t=(1-\delta)C_t^{1-\gamma}+\delta E_t V_{t+1}$$

Now, define $U_t=V_t^{\frac{1}{1-\gamma}}$, then we could write $U_t=\left\{(1-\delta)C_t^{1-\gamma}+\delta E_t U_{t+1}^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}$

Epstein-Zin preferences generalize this to:

$$U_t=\left\{(1-\delta)C_t^{\frac{1-\gamma}{\theta}}+\delta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}.$$

Epstein-Zin preferences violate the axioms of expected utility. We will briefly discuss some of those. One advantage of these preferences is that they allow for separate parameters for relative risk aversion and the intertemporal elasticity of substitution. Under standard preferences ($\theta=1$), γ is the coefficient of relative risk aversion and $1/\gamma$ is the intertemporal elasticity of substitution.

With EZ preferences, γ is still the coefficient of relative risk aversion, but $1/\psi$ is the IES, where $\psi = \frac{\theta}{\theta+\gamma-1}$.

We will assume $\gamma > 1$ and $\psi > 1$. This implies $\gamma > \frac{1}{\psi}$. This latter condition means people prefer an "early resolution of uncertainty." What does this mean?

Consider a lottery in which you receive C in the first two periods. Then a coin is flipped at the end of the second period. If it comes up heads, you receive C in every period from the third period onward, if tails, you receive C* in every period from the third period onward.

Compare that with a lottery that is the same, but the coin is flipped at the end of the first period. That is, you still receive C in each of the first two periods, and then C or C* for every period from the third onward. But the coin flip occurs earlier. There is early resolution of uncertainty.

In an expected utility framework, the two lotteries would yield the same utility at the beginning of period 1. But under EZ preferences, if $\gamma > \frac{1}{\psi}$, you prefer the second lottery.

Here is another interpretation of EZ preferences:

Write $U_t = \left\{ (1-\delta) C_t^{\frac{\psi-1}{\psi}} + \delta \left(E_t U_{t+1}^{\frac{\theta(\psi-1)}{\psi}} \right)^{\frac{1}{\theta}} \right\}^{\frac{\psi}{\psi-1}}$. Let $V = U^{\frac{\psi-1}{\psi}}$, so

$$\begin{aligned} V_t &= (1-\delta) C_t^{\frac{\psi-1}{\psi}} + \delta \left(E_t V_{t+1}^{\theta} \right)^{\frac{1}{\theta}} = (1-\delta) C_t^{\frac{\psi-1}{\psi}} + \delta e^{E_t v_{t+1} + \frac{1}{2}\theta \operatorname{var}_t(v_{t+1})} \\ &= (1-\delta) C_t^{\frac{\psi-1}{\psi}} + \delta E_t V_{t+1} e^{\frac{1}{2}(\theta-1)\operatorname{var}_t(v_{t+1})} \end{aligned}.$$

If $\theta < 1$, an increase in $\operatorname{var}_t(v_{t+1})$ reduces utility. This is equivalent to the condition $\gamma > \frac{1}{\psi}$. So, when an increase in $\operatorname{var}_t(v_{t+1})$ reduces utility, there is a preference for early resolution of risk.

(Note this expression when $\theta = 1$ is our standard utility recursion.)

Let's be clear what this means. Under standard expected utility, you can dislike variance of consumption. But you don't care if your period-by-period utility varies. You just care about expected utility. For a given level of expected consumption, higher consumption variance lowers expected utility. But you are just as happy with a given expected utility whether it is achieved through high expected consumption and low volatility of consumption, or vice-versa. Under EZ preferences with a preference for early resolution of uncertainty, you prefer your period-by-period utility of consumption be relatively constant. That is, you would prefer period-by-period utility not fluctuate, so you want more than just high expected utility – you prefer constant expected utility in the future. Intuitively, that lines up with the idea that you want early resolution of uncertainty, because you don't want any surprises about your period utility.

Now, in the international context, we can think of M_{t+1} as being the s.d.f. for home investors that discount returns in home units of consumption and M^*_{t+1} as the s.d.f. for foreign investors that discount returns in units of the foreign consumption basket. We still have

$$M^*_{t+1} = \frac{M_{t+1}Q_{t+1}}{Q_t} ..$$

If markets are complete, M_{t+1} and M^*_{t+1} are unique, and we will use this result later in the semester.

Now you are ready to read the first reading!

1. Usually, we think of the risk premium as depending on the covariance of returns with the s.d.f. But equation (67) tells us $r^*_t + E_t q_{t+1} - q_t - r_t = .5\left(\text{var}_t\left(m_{t+1}\right) - \text{var}_t\left(m^*_{t+1}\right)\right)$. Why are there no covariance terms here?

Read section 4.2 carefully. Most of the derivations are straightforward. The derivation of the s.d.f. for EZ preferences is not so easy. Just take equations (79) and (80) as given. We will return to this model when we read Engel (2016) below, so make sure you follow things here.

2. The article gives the conditions on parameters for $\text{cov}\left(E_t d_{t+1}, r_t - r^*_t\right) < 0$. A weaker finding is that the covariance of expected excess returns covaries with the interest rate differential. That is, $\lambda_t = r^*_t - r_t + E_t d_{t+1}$. What condition on parameters gives us $\text{cov}\left(\lambda_t, r_t - r^*_t\right) < 0$?

By the way, one thing I want to point out about the model of Bansal and Shaliastovich is that it is essentially a partial-equilibrium model. It takes as exogenously given how consumption evolves in the home and foreign country but doesn't delve into what drives consumption. A useful contrast is to the closed-economy versions of this model. Those are usually specified by saying the economy is an endowment economy that exogenously receives an endowment that follows some stochastic process like the ones for consumption in this paper. In a closed economy endowment model with no government and no storable goods, consumption equals output. We could interpret those as a full general equilibrium (albeit very simple) model. But in the open economy, we definitely do not have that consumption always equals output. Even if there were exogenous stochastic processes for output in each country, the equilibrium outcome for consumption in each country would depend on how the full economy is modeled. So, we must interpret this model as a partial-equilibrium model. However, there are papers that use EZ preferences in general equilibrium macro models to explain some of the international asset-pricing models, specifically Colacito and Croce (Journal of Finance, 2013), Colacito, Croce, Ho, and Howard (AER, 2018), Colacito, Riddiough, and Sarno (Journal of Financial Economics, 2020.)

An odd thing about the models with exogenous consumption is that they are able to solve for the dynamics of the real exchange rate without specifying anything about how goods markets work or how prices are set. The real exchange rate is the relative price of the foreign consumption basket to the home consumption basket. In economics, we want to know how firms are setting the price and how shocks to demand for the good and cost of the good affect the price. In the exogenous consumption models, somehow all of this is behind the scenes. There may be shocks to productivity, or to income of consumers, and maybe there are sticky prices. Somehow all of that works out to give us the stochastic processes for consumption that are given as a starting point in these models. It is worthwhile to perform this mental exercise: suppose we lived in a world where there were no barriers to trade, all goods are traded, and the weights in the consumption baskets are the same in home and foreign countries. Then the real exchange rate is constant (relative purchasing power parity holds.) Under complete markets, consumption growth rates in the two countries must be equal. What are the exogenous consumption models implicitly assuming about the stochastic processes for consumption that rules out this case?

You can read most of section 4.3 just for your enjoyment. It’s kind of neat, but only work through it equation by equation if you have time. We will however return to the model of slow portfolio adjustment that starts toward the bottom of page 509.

In the first model of expectations that are not rational:

3. Use your own words to explain why this model could explain the UIP puzzle (even though the article does explain it.). Why do we say expectations are not rational in this model?

Beyond that first model, you should read the rest of section 4.3 and then all of 4.4, but I have no specific questions.

The next paper we will read is:

Engel, Charles. 2016. “Exchange Rates, Interest Rates, and the Risk Premium.” **American Economic Review** 106, 436-474.

Some people find this paper hard to read because it goes through both a relatively elaborate empirical section and also then works through a few models. But with the background you already have, it should not be that hard.

Here is the basic idea. The Fama regression, which can be written as either
$q_{t+1} - q_t = \alpha + \beta\left(r_t - r_t^*\right) + u_{t+1}$, or

$$r_t^* + q_{t+1} - q_t - r_t = \alpha + (1-\beta)\left(r_t^* - r_t\right) + u_{t+1}$$

If we find $\beta < 1$, we infer that as $r_t^* - r_t$ rises, $r_t^* + q_{t+1} - q_t - r_t$ also rises. That is, $\text{cov}\left(r_t^* + q_{t+1} - q_t - r_t, r_t^* - r_t\right) > 0$. If we attribute the deviation from UIP to a risk premium, then as the foreign interest rate rises relative to the home interest rate, the foreign bond becomes riskier.

If the foreign bond becomes riskier, that ought to work toward making the foreign currency weaker – there is less demand for the foreign bond. So, holding other things constant, that ought to make $\text{cov}\left(q_t, r_t^* - r_t\right)$ smaller (either less positive or more negative.) However, suppose there were no risk premium at all, so UIP held. The paper provides a way to measure what the real exchange rate would be if UIP held. Let's call that measure q_t^{IP}. If there is no risk premium, only earnings on the bonds matter, and a higher foreign interest rate would increase demand for the foreign bond and tend to make its currency stronger. In equation form, that means $0 < \text{cov}\left(q_t^{IP}, r_t^* - r_t\right)$. If the actual real exchange rate does incorporate a risk premium, then the logic of this paragraph tells us that $\text{cov}\left(q_t, r_t^* - r_t\right) < \text{cov}\left(q_t^{IP}, r_t^* - r_t\right)$.

Part of the work of this paper was establishing an empirical measure of q_t^{IP}. In retrospect (and after having written the last paper that is assigned in this section of readings), I realize that measuring q_t^{IP} is tricky. If I were writing the paper now, I would emphasize not the fact that the risk premium models are built to explain $\beta < 1$, but indeed are aimed at explaining $\beta < 0$ in the Fama regression. This means that the risk premium effect outweighs the effect that higher interest rates mean more earnings. Think of it like this. We can write the Fama regression as:

$$q_{t+1} - q_t = \alpha - \beta\left(r_t^* - r_t\right) + u_{t+1}.$$

But we can "decompose" the left-hand side:

$$\left(r_t^* + q_{t+1} - q_t - r_t\right) - \left(r_t^* - r_t\right) = \alpha - \beta\left(r_t^* - r_t\right) + u_{t+1}.$$

If there were no risk premium effect, the first term in parentheses on the left-hand-side would be zero, and so we would have $\beta = 1$. But if we find $\beta < 0$, then when $r_t^* - r$ rises, the increase in the risk premium, $\left(r_t^* + q_{t+1} - q_t - r_t\right)$, must be greater than the direct effect on the interest rate differential, $\left(r_t^* - r_t\right)$. In that case, when $r_t^* - r$ rises, we should see that investors are so averse to buying the foreign bond because of exchange-rate risk, that even though the foreign interest rate is higher, they flee from foreign bonds, and the foreign currency depreciates. That is, we should see $\text{cov}\left(q_t, r_t^* - r_t\right) < 0$. But the data strongly show the opposite, $0 < \text{cov}\left(q_t, r_t^* - r_t\right)$. [I do discuss this point, starting with the second paragraph of page 462.]

This strikes me as a fundamental failing of the risk premium models. To explain the Fama puzzle, the models imply that as a country's interest rate is higher, its currency is weaker, but the data clearly show the reverse is true. Since I wrote my 2016, researchers in the finance field have come up with different representative agent models of the risk premium that can reconcile my findings, though generally they do not focus on the main problem I just mentioned, but instead focus on the "reversal" of expected excess returns, (pages 453-456.)

I've been skeptical of risk premium models based on representative consumers for a long time, for a different reason. It can be illustrated here. The Epstein-Zin preferences allow the inverse of the intertemporal elasticity of substitution and the coefficient of relative risk aversion to be different. But the models we look at here still require a very large degree of risk aversion to account for the empirical findings of the Fama regression. Large values of risk aversion do not cause the problem for the volatility of the riskless real interest rate that I mentioned above if preferences are EZ, since that relationship depends on the intertemporal elasticity of substitution. But measures of relative risk aversion based on experimental studies, for example, still suggest that risk aversion is not so large. It seems unlikely the coefficient of relative risk aversion is greater than 10, and it is probably less. But the marginal utility of average consumption per capita needs to be volatile to generate large risk premiums. That in turn requires very large degrees of relative risk aversion. If people were really that risk averse, they would be very afraid of taking risks – they would drive to work in tanks, they would live in bomb shelters, they would always boil their water before drinking it, they would never do something like going skiing, etc. Typically, the way the literature has gotten around this is to say that risk aversion is not so high, but that "long-run risks" are high. That is, there really is a lot of riskiness in consumption. A shock today could have very large permanent effects on consumption. There is some tricky logic here, because the problem is that we cannot measure long-run riskiness of consumption without very very long time series. So, we cannot rule out long-run risks. Generally, the evidence in favor of the long-run risks model comes from asset prices, rather than actual consumption data. That is, the behavior of many different asset prices is consistent with long-run risks and EZ preferences.

This is not to say that foreign exchange risk premia are unimportant. My view is that they are not explained well in any model with a representative, unconstrained, risk-averse consumer in each country. I think there is promise in examining the foreign exchange risk premiums in models with financial intermediaries, such as the ones we will examine later in the semester. In those models, households put their money in financial intermediaries (call them banks), and the banks are the ones choosing portfolios. In those models, the main focus has been how balance-sheet constraints on the banks prevent them from exploiting expected profit opportunities, and hence lead to deviations from UIP. But it is also the case that if the owners are risk averse, there can be risk premiums, and those potentially can be quite large (much larger than ones generated in models of representative consumers.) Most of the models with financial intermediaries use solution methods that preclude examination of the risk premiums (for example, because the models are linearized to solve them), but I have seen some closed-economy models and one instance of an open economy model that are solved allowing for risk premiums, and those models can account for quite large deviations from UIP.

With that background, first answer these questions about the empirical part of the paper:

4. Show that the real exchange rate in the empirical model on page 445 is stationary under the assumptions mentioned on page 446.

5. How is $q_t - q_t^{IP}$ calculated? (Note that I don't actually use the notation q_t^{IP}, though I do use s_t^{IP}, so you have to think about this for a minute.)
6. The paper assumes we can calculate expectations of future variables using the VECM that is estimated. What major assumption is needed for that to be true? (This is related to the following point. Suppose we estimate the Fama regression: $q_{t+1} - q_t = \alpha + \beta\left(r_t - r_t^*\right) + u_{t+1}$. If the null hypothesis, $E_t q_{t+1} - q_t = r_t - r_t^*$, we should not be able to reject $\alpha = 0$, $\beta = 1$. But suppose we find with almost near certainty (maybe because we have a huge sample) that $\alpha = 0$, $\beta = 1$. While we could not reject $E_t q_{t+1} - q_t = r_t - r_t^*$, why can we not conclude $E_t q_{t+1} - q_t = r_t - r_t^*$ is true? That is, why might there be some other explanation for finding $\alpha = 0$, $\beta = 1$?

   The only question I will ask about the risk premium model is:
7. Explain intuitively the difference between the model I attribute to Bansal and Shaliastovich, and the one I attribute to Lustig, Roussanov and Verdelhan, starting on page 460.

Next, read the section on delayed overshooting briefly, but I now disavow what I say in it. In that section, I build a "toy" model that I thought captured the essence of Bacchetta and van Wincoop's model of delayed overshooting. You can read about why delayed overshooting can explain the Fama puzzle by reading this section. But my thought, which is incorrect, is that since delayed overshooting requires an underreaction of exchange rates to interest rate changes in the short run, it must violate the finding that the actual real exchange rate has a bigger reaction to changes in the interest rate differential than the UIP real exchange rate does. That is, in the data, $0 < \operatorname{cov}\left(q_t^{IP}, r_t^* - r_t\right) < \operatorname{cov}\left(q_t, r_t^* - r_t\right)$, which can be written as: $0 < \operatorname{cov}\left(q_t - q_t^{IP}, r_t^* - r_t\right)$.

In reality, "underreaction" means that the actual unexpected change in the real exchange rate in response to an unexpected change in the interest rate differential is smaller than it would be under UIP:

$$\operatorname{cov}\left(q_t - q_t^{IP} - E_{t-1}\left(q_t - q_t^{IP}\right), r_t^* - r_t - E_{t-1}\left(r_t^* - r\right)\right) < 0 .$$

This is quite different than saying the Bacchetta and van Wincoop model implies $\operatorname{cov}\left(q_t - q_t^{IP}, r_t^* - r_t\right) < 0$. In my toy model of delayed overshooting, we do have $\operatorname{cov}\left(q_t - q_t^{IP} - E_{t-1}\left(q_t - q_t^{IP}\right), r_t^* - r_t - E_{t-1}\left(r_t^* - r\right)\right) < 0$, but I set it up so that we also have the wrong relationship (compared to the data) for the unconditional covariance: $\operatorname{cov}\left(q_t - q_t^{IP}, r_t^* - r_t\right) < 0$. The Bacchetta and van Wincoop model is capable of delivering the correct signs (ones consistent with the data) for both covariances.

One way to see the issue is to write out the Wold representation for $q_t - q_t^{IP}$ and $r_t^* - r_t$. We can write those (in the case of a finite MA representation) as, for example:

$$q_t - q_t^{IP} = a_0 u_t + a_1 u_{t-1} + \ldots + a_k u_{t-k} ,$$

$$r_t - r_t^* = b_0\varepsilon_t + b_1\varepsilon_{t-1} + \ldots + b_k\varepsilon_{t-k},$$

where u_t and ε_t are mean-zero, serially uncorrelated random variables. The condition for underreaction, $\operatorname{cov}\left(q_t - q_t^{IP} - E_{t-1}\left(q_t - q_t^{IP}\right), r_t^* - r_t - E_{t-1}\left(r_t^* - r\right)\right) < 0$ is $a_0 b_0 \operatorname{cov}(u,\varepsilon) < 0$. The condition for $\operatorname{cov}\left(q_t - q_t^{IP}, r_t^* - r_t\right) > 0$ (which is what I find in the data) is $\sum_{j=0}^{k} a_j b_j \cdot \operatorname{cov}(u,\varepsilon) > 0$. Clearly, we could have both conditions being true, depending on the model. We will see in the next reading that in the actual Bacchetta and van Wincoop model, both could be true under certain parameter restrictions.

Finally, read the section on Liquidity Return, but don't toil through it. We will spend plenty of time on models such as this (and with better micro-foundations) during the semester. Just note that if $\alpha = 0$ in equation (27), we have essentially the simple model of deviations from UIP that introduce a small deviation from rational expectations, discussed in section 4.4 of the reading from the previous Handbook of International Economics. It is the assumption of $\alpha > 0$ that gives the excessive reaction of the real exchange rate – the model assumes that when the home interest rate goes up, it's liquidity return also rises, so the desirability of the home bond increases both because it pays a higher rate of return, and because it is more valued for its liquidity.

Next, read:

Bacchetta, Philippe, and Eric Van Wincoop. "Puzzling exchange rate dynamics and delayed portfolio adjustment." *Journal of International Economics* 131 (2021): 103460.

I have no questions to ask about this. I just want you to read this paper to see how the delayed portfolio adjustment model works. Louphou Coulibaly and I are working on a model that provides "microfoundations" for slow portfolio adjustment.

Let me make this comment on the Bacchetta and van Wincoop paper. In a sense, that paper and my AER paper have explanations for the puzzle that are not so different. BvW assume that bond returns are taxed, and taxes follow a stochastic process. Those taxes are unobserved by the "econometrician" – the people performing Fama regressions, etc. In that sense, these taxes are a lot like the unobserved liquidity return that the bonds earn in the model in my AER paper. The big difference is what drives the Fama puzzle. In their model, it is slow portfolio adjustment. In my model, it is something akin to the error in expectations described in my Handbook paper (which equation 3 refers to), though the connection here is subtle. I think what must be true is that if I took the "toy" model of BvW that I wrote in the AER paper, but then added in the unobserved liquidity returns, it would give me another way in which we can find both the Fama puzzle and the excess volatility of exchange rates.

Finally, read the mostly empirical paper I wrote recently with three graduate students:

Engel, Charles, et al. "A reconsideration of the failure of uncovered interest parity for the US dollar." *Journal of International Economics* 136 (2022): 103602.

This should be an easy paper to read. Mostly it is empirical results. The main takeaway is that estimates of the slope coefficient in the Fama regression are very unstable over sub-samples of data. In the 2000s, when nominal interest rates are very low, the UIP puzzle seems to disappear. The main reason for this finding is that the slope coefficient is estimated very imprecisely. This follows because the variance of the right-hand-side variable is very small (since interest rates are all close to zero during this period), but the left-hand-side variable remains quite volatile (though perhaps less volatile than in the 1980s and 1990s.)

But also pay attention to the story we tell of delayed overshooting in the Conclusions section. We find that year-on-year inflation is a good measure of expected monetary policy in the near future. When y.o.y. inflation is relatively high, we can predict, at least since the mid-1980s, that monetary policy will tighten. But we say there is a delayed response of exchange rates. Note that if the nominal interest rates are zero, the expected excess return on the foreign bond is simply the expected rate of depreciation of the home currency: $E_t s_{t+1} - s_t$. If the home monetary policy tightens, we find the expected return on home bonds rises relative to foreign bonds, which means $E_t s_{t+1} - s_t$ falls.

8. In your own words, explain how delayed portfolio adjustment can account for this finding.

Also note in the conclusions section that while there is also a risk premium story we could tell, we are skeptical of that story, for the reason I explained above: the riskier currency will be weaker, but higher expected future interest rates are associated with a stronger currency.

<u>Ten Important Papers</u>

Bacchetta, Philippe, Eric van Wincoop, and Eric R. Young. *Infrequent Random Portfolio Decisions in an Open Economy Model*. No. 22-10. Swiss Finance Institute, 2022.

Bansal, Ravi, and Ivan Shaliastovich. "A long-run risks explanation of predictability puzzles in bond and currency markets." *The Review of Financial Studies* 26.1 (2013): 1-33.

Colacito, Riccardo, and Mariano M. Croce. "International asset pricing with recursive preferences." *The Journal of Finance* 68.6 (2013): 2651-2686.

Colacito, Riccardo, Steven J. Riddiough, and Lucio Sarno. "Business cycles and currency returns." *Journal of Financial Economics* 137.3 (2020): 659-678.

Hassan, Tarek A., and Rui C. Mano. "Forward and spot exchange rates in a multi-currency world." *The Quarterly Journal of Economics* 134.1 (2019): 397-450.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan. "Common risk factors in currency markets." *The Review of Financial Studies* 24.11 (2011): 3731-3777.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan. "Countercyclical currency risk premia." *Journal of Financial Economics* 111.3 (2014): 527-553.

Lustig, Hanno, Andreas Stathopoulos, and Adrien Verdelhan. "The term structure of currency carry trade risk premia." *American Economic Review* 109.12 (2019): 4142-77.

Lustig, Hanno, and Adrien Verdelhan. "The cross section of foreign currency risk premia and consumption growth risk." *American Economic Review* 97.1 (2007): 89-117.

Verdelhan, Adrien. "The share of systematic variation in bilateral exchange rates." *The Journal of Finance* 73.1 (2018): 375-418.

Addendum 1:

I briefly talked about this paper in class:

Jiang, Zhenyang. 2023. "Market Incompleteness and Exchange-Rate Spillover." NBER working paper no. 30856.

Although the two-country case is not the main focus of the paper, this is a good venue to look at the "exogenous consumption" view of the real exchange rate, in which the real exchange rate is determined entirely in financial markets. The paper does not explicitly state that it is taking consumption as exogenous, but the discussion is in those terms: it intimates that the stochastic discount factor is related to the path of intertemporal consumption, and it treats the s.d.f.s as fundamentals (that is, not endogenous.)

The last equation on page 12 breaks down the growth in the real exchange rate into two components: the difference in the logs of the s.d.f.s, and the residual. If markets were complete, the growth rate of real exchange rate would equal the first component, so the second component is, so the paper says, what is attributable to market incompleteness.

As in my discussion above, suppose the conditions held in goods markets and preferences such that purchasing power parity holds (the real exchange rate is constant): there were no barriers to trade, all goods are traded, and the weights in the consumption baskets are the same in home and foreign countries. Then the l.h.s. of the equation on page 12 would be zero. How do we interpret the r.h.s.?

Imagine we were living in this world. The s.d.f.s in the case of constant relative risk aversion are proportional to the growth rates of consumption. What does the first term on the r.h.s. mean – the difference in the s.d.f.s? The paper wants us to think that the real exchange rate should equal that difference if markets are complete. The residual term on the r.h.s. is just minus the first term since the l.h.s. is zero. In other words, the first term is supposed to be what the real exchange rate should be if markets are complete. But since the l.h.s. is zero, the fact that real exchange rates are not changing supposedly is due to market incompleteness.

That logic makes sense if we insist that consumption is exogenous. Then behind the scenes, goods markets must be adjusting to give us the path we see for consumption. In this example, if we could just complete the markets, purchasing power parity would no longer hold, and the real exchange rate change would equal the first term, the difference in the s.d.f.s.

My point is that, generally, to understand the real exchange rate we need at minimum to specify how goods markets set prices as well as an asset market specification. My example of purchasing power parity is at one extreme: in this case, only the goods market matters. The exogenous consumption literature is at the other extreme: only the asset market matters. I like to think of model economies as representations of a simple world – I imagine households, firms, governments acting in some country just like in the model. But I have a hard time understanding exogenous consumption. Is a dictator making people consume at some specified level? And then, if that is the story, the asset market tells us how the real exchange rate must behave, and somehow goods markets must adjust in equilibrium to be compatible with asset market equilibrium?

Addendum 2

We had a brief discussion of whether the u.i.p. puzzle held in emerging markets. I made reference to these two papers:

Bansal, Ravi, and Magnus Dahlquist. 2000. "The Forward Premium Puzzle: Different Tales from Developed and Emerging Economies." Journal of International Economics 51, 115-144.

Frankel, Jeffrey, and Jumana Poonawala. 2010. "The Forward Market in Emerging Currencies: Less Biased than in Major Currencies." Journal of International Money and Finance 29, 585-598.

But then in an email, I wrote that I had meant to mention the important recent papers by Sebnem Kalemli-Ozcan. Before I mention her main conclusion, I pointed out that she is using data on the expectations of future exchange rates from surveys of foreign exchange traders. So rather than look to see if $s_{t+1} - s_t + i_t^* - i_t$ is predictable, this work looks at the behavior of $s_{t,t+1}^e - s_t + i_t^* - i_t$, where $s_{t,t+1}^e$ is the expectation of the traders. We can then look to see whether these expectations are rational (unbiased, for example), or whether there is an expected excess return, or both. I mentioned that there are papers by Frankel and Froot and Chinn and Frankel (and many others) that have previously used survey data.

Froot, Kenneth A., and Jeffrey A. Frankel. 1989. "Forward Discount Bias: Is It an Exchange Risk Premium?" Quarterly Journal of Economics 104, 139-161.

Frankel, Jeffrey A., and Kenneth A. Froot. 1990. "Chartists, Fundamentalists, and Trading in the Foreign Exchange Market." American Economic Review 80, 181-185.

Chinn, Menzie, and Jeffrey Frankel. 2020. "A Third of a Century of Currency Expectations Data: The Carry Trade and the Risk Premium." University of Wisconsin-Madison, working paper.

The main conclusion from Kalemli-Ozcan is that while the failure of u.i.p. in advanced countries is mostly a deviation from rational expectations, in emerging markets it is a "risk premium." Here are the papers:

Kalemli-Ozcan, Sebnem. 2019. "U.S. Monetary Policy and International Risk Spillovers." Proceedings of the Jackson Hole Symposium (Federal Reserve Bank of Kansas City), 95-191.

Kalemli-Ozcan, Sebnem and Liliana Varela. 2022. "Five Facts about the UIP Premium." University of Maryland, working paper.

These papers attribute, empirically, the deviation from UIP in emerging markets to a risk premium that is generated from policy uncertainty. But how does policy uncertainty lead to a deviation from UIP that we can call a "risk premium"? The authors do not mean uncertainty about the interest rate payoff – that is, they are not really thinking about default or expropriation. I believe this paper answers that question:

Akinci, Ozge; Sebnem Kalemli-Ozcan; and, Albert Queralto. 2022. "Uncertainty Shocks, Capital Flows, and International Risk Spillovers." University of Maryland, working paper.

In the end, policy uncertainty affects the balance sheet constraints of financial intermediaries. In turn, that affects the s.d.f. of the intermediaries – their s.d.f. is not just the s.d.f. of the households that own the intermediary because the intermediary faces constraints on its asset holdings. We will examine this type of model in detail later in the semester. Footnote 13 suggests the UIP premium arises from the covariance of the intermediary's s.d.f. and the exchange rate.

Answers to questions from Assignment #2 on UIP and Risk Premiums

1. Usually, we think of the risk premium as depending on the covariance of returns with the s.d.f. But equation (67) tells us $r_t^* + E_t q_{t+1} - q_t - r_t = .5\left(\operatorname{var}_t(m_{t+1}) - \operatorname{var}_t(m_{t+1}^*)\right)$. Why are there no covariance terms here?

Answer:

As the article explains, the risk premium for foreign bonds is different for home households and foreign households. For home households, the expected excess return on the foreign bond is given by:

$$\lambda_t^H = r_t^* - r_t + E_t d_{t+1} + \tfrac{1}{2}\operatorname{var}_t d_{t+1}, \text{ where } d_{t+1} \equiv q_{t+1} - q_t$$

Then , since equation (67) says $r_t^* + E_t d_{t+1} - r_t = \frac{1}{2}\left(\operatorname{var}_t(m_{t+1}) - \operatorname{var}_t(m_{t+1}^*)\right)$, it follows that $\lambda_t^H = \frac{1}{2}\left(\operatorname{var}_t(m_{t+1}) - \operatorname{var}_t(m_{t+1}^*)\right) + \frac{1}{2}\operatorname{var}_t d_{t+1}$. But under complete markets, $d_{t+1} = m_{t+1}^* - m_{t+1}$. So, $\operatorname{var}_t(d_{t+1}) = \operatorname{var}_t(m_{t+1}^*) + \operatorname{var}_t(m_{t+1}) - 2\operatorname{cov}_t(m_{t+1}^*, m_{t+1})$.

We have then that

$$\begin{aligned}\lambda_t^H &= \tfrac{1}{2}\left(\operatorname{var}_t(m_{t+1}) - \operatorname{var}_t(m_{t+1}^*)\right) + \tfrac{1}{2}\operatorname{var}_t(m_{t+1}^*) + \tfrac{1}{2}\operatorname{var}_t(m_{t+1}) - \operatorname{cov}_t(m_{t+1}^*, m_{t+1}) \\ &= \operatorname{var}_t(m_{t+1}) - \operatorname{cov}_t(m_{t+1}^*, m_{t+1}) = \operatorname{cov}_t(m_{t+1} - m_{t+1}^*, m_{t+1}) = -\operatorname{cov}_t(m_{t+1}, d_{t+1})\end{aligned}$$

That is, when we properly express the risk premium on the foreign bond for the home consumer, it is indeed related to the covariance of the risky part of the return and the s.d.f. for the home household, $-\operatorname{cov}_t(m_{t+1}, d_{t+1})$. Likewise, we can show for the foreign household, the risk premium on the foreign bond equals $-\operatorname{cov}_t(m_{t+1}^*, -d_{t+1})$.

The reason covariance terms do not show up on the right hand side of the equation $r_t^* + E_t q_{t+1} - q_t - r_t = .5\left(\operatorname{var}_t(m_{t+1}) - \operatorname{var}_t(m_{t+1}^*)\right)$ is that $r_t^* + E_t q_{t+1} - q_t - r_t$ is not the risk premium for the home nor the foreign investor, though it is the average of those, as the article shows.

2. The article gives the conditions on parameters for $\operatorname{cov}\left(E_t d_{t+1}, r_t - r_t^*\right) < 0$. A weaker finding is that the covariance of expected excess returns covaries with the interest rate differential. That is, $\lambda_t = r_t^* - r_t + E_t d_{t+1}$. What condition on parameters gives us $\operatorname{cov}\left(\lambda_t, r_t - r_t^*\right) < 0$?

Answer:

Equations (81) and (82) of the paper show that in this model

$$Ed_{t+1} = \gamma_u^r \left(u_t^h - u_t^f\right)$$
$$\lambda_t = \tfrac{1}{2}\left(\lambda_x^r\right)^2 \left(u_t^h - u_t^f\right),$$

Where $\gamma_u^r \equiv \alpha\left(\alpha - \rho\right)/2$ and $\lambda_x^r \equiv 1 - \alpha$.

From above, $r_t - r_t^* = E_t d_{t+1} - \lambda_t$. It follows that

$$\operatorname{cov}\left(\lambda_t, r_t - r_t^*\right) = \operatorname{cov}\left(\lambda_t, E_t d_{t+1} - \lambda_t\right) = \operatorname{cov}\left(\lambda_t, E_t d_{t+1}\right) - \operatorname{var}\left(\lambda_t\right)$$

Then using equations (81) and (82), we get

$$\operatorname{cov}\left(\lambda_t, E_t d_{t+1}\right) - \operatorname{var}\left(\lambda_t\right) = \tfrac{1}{2}\left[\gamma_u^r \left(\lambda_x^r\right)^2 - \tfrac{1}{2}\left(\lambda_x^r\right)^4\right] \operatorname{var}\left(u_t^h - u_t^f\right) = \tfrac{1}{2}\left(\lambda_x^r\right)^2 \left[\gamma_u^r - \tfrac{1}{2}\left(\lambda_x^r\right)^2\right] \operatorname{var}\left(u_t^h - u_t^f\right)$$

This covariance is negative when

$$\gamma_u^r - \tfrac{1}{2}\left(\lambda_x^r\right)^2 = \frac{\alpha\left(\alpha - \rho\right)}{2} - \tfrac{1}{2}\left(1 - \alpha\right)^2 = \frac{\alpha^2 - \alpha\rho - 1 - \alpha^2 + 2\alpha}{2} = \frac{2\alpha - \alpha\rho - 1}{2} < 0,$$

which means we need $\alpha\left(2 - \rho\right) < 1$.

3. Use your own words to explain why this model could explain the UIP puzzle (even though the article does explain it.). Why do we say expectations are not rational in this model?

Answer:

Suppose the only shock was the expectational error. Holding interest rates constant, a positive value of u_t means there is an expectation of appreciation ($E_t s_{t+1} - s_t < 0$), which is accomplished by a home depreciation (s_t rises.) The depreciation will generate inflation in the home country, leading the home monetary policy maker to raise the home interest rate, so $i_t - i_t^*$ rises. We have then an increase in $i_t - i_t^*$, and a decrease in $E_t s_{t+1} - s_t$, which is in line with the Fama puzzle. If the expectational error has a large enough variance, this endogenous response of monetary policy to the exchange rate is large enough to generate the negative correlation between $i_t - i_t^*$ and $E_t s_{t+1} - s_t$.

4. Show that the real exchange rate in the empirical model on page 445 is stationary under the assumptions mentioned on page 446.

We can write out in full detail the first two equations in the VECM, using the notation $\Delta x_t \equiv x_t - x_{t-1}$, $p_t^r \equiv p_t - p_t^*$, and $i_t^r \equiv i_t - i_t^*$:

$$s_t - s_{t-1} = c_1^0 + g_{11}s_{t-1} - g_{11}p_{t-1}^r + g_{13}i_{t-1}^r + c_{11}^1\Delta s_{t-1} + c_{12}^1\Delta p_{t-1}^r + c_{13}^1\Delta i_{t-1}^r$$
$$+c_{11}^2\Delta s_{t-2} + c_{12}^2\Delta p_{t-2}^r + c_{13}^2\Delta i_{t-2}^r + c_{11}^3\Delta s_{t-3} + c_{12}^3\Delta p_{t-3}^r + c_{13}^3\Delta i_{t-3}^r + u_{1t}$$

$$p_t^r - p_{t-1}^r = c_2^0 + g_{21}s_{t-1} - g_{21}p_{t-1}^r + g_{23}i_{t-1}^r + c_{21}^1\Delta s_{t-1} + c_{22}^1\Delta p_{t-1}^r + c_{23}^1\Delta i_{t-1}^r$$
$$+c_{21}^2\Delta s_{t-2} + c_{22}^2\Delta p_{t-2}^r + c_{23}^2\Delta i_{t-2}^r + c_{21}^3\Delta s_{t-3} + c_{22}^3\Delta p_{t-3}^r + c_{23}^3\Delta i_{t-3}^r + u_{1t}$$

Now subtract the second equation from the first equation, and use the notation $q_t = s_t - p_t^r$:

$$q_t = c_1^0 + \left(1 + g_{11} - g_{21}\right)q_{t-1} + \left(g_{13} - g_{23}\right)i_{t-1}^r + \left(c_{11}^1 - c_{21}^1\right)\Delta s_{t-1} + \left(c_{12}^1 - c_{22}^1\right)\Delta p_{t-1}^r + \left(c_{13}^1 - c_{23}^1\right)\Delta i_{t-1}^r$$
$$+\left(c_{11}^2 - c_{21}^2\right)\Delta s_{t-2} + \left(c_{12}^2 - c_{22}^2\right)\Delta p_{t-2}^r + \left(c_{13}^2 - c_{23}^2\right)\Delta i_{t-2}^r + \left(c_{11}^3 - c_{21}^3\right)\Delta s_{t-3} + \left(c_{12}^3 - c_{22}^3\right)\Delta p_{t-3}^r + \left(c_{13}^3 - c_{23}^3\right)\Delta i_{t-3}^r + u_{1t}$$

Examining this equation, we see that since Δs_t, Δp_{t-1}^r, and i_t^r are all stationary, then q_t will be stationary as long as $1 + g_{11} - g_{21} < 1$, that is, $g_{11} < g_{21}$.

5. How is $q_t - q_t^{IP}$ calculated? (Note that I don't actually use the notation q_t^{IP}, though I do use s_t^{IP}, so you have to think about this for a minute.)

<u>Answer</u>:

q_t^{IP} is the value of the real exchange rate if UIP holds: $E_t q_{t+1}^{IP} - q_t^{IP} = r_t - r_t^*$. Iterating this forward, $q_t^{IP} = -\sum_{j=0}^{\infty} E_t\left(r_{t+j} - r_{t+j}^*\right) + \bar{q}$, where $\bar{q}$ is the unconditional mean of q, and so $q_t - q_t^{IP}$ is just the difference between q_t and this measure of q_t^{IP}. The expected infinite sum in practice is calculated with forecasts from the VECM.

6. The paper assumes we can calculate expectations of future variables using the VECM that is estimated. What major assumption is needed for that to be true? (This is related to the following point. Suppose we estimate the Fama regression: $q_{t+1} - q_t = \alpha + \beta\left(r_t - r_t^*\right) + u_{t+1}$. If the null hypothesis, $E_t q_{t+1} - q_t = r_t - r_t^*$, we should not be able to reject $\alpha = 0$, $\beta = 1$. But suppose we find with almost near certainty (maybe

because we have a huge sample) that $\alpha = 0$, $\beta = 1$. While we could not reject $E_t q_{t+1} - q_t = r_t - r_t^*$, why can we not conclude $E_t q_{t+1} - q_t = r_t - r_t^*$ is true? That is, why might there be some other explanation for finding $\alpha = 0$, $\beta = 1$?

Answer:

As noted in the answer to the previous question the expected infinite sum is calculated with forecasts from the VECM. But we want the true expectation of the variables in this sum, and the VECM is only giving us the true expectation if the data is actually generated by the process in the VECM in the real world.

In answer to the question, suppose we estimate the Fama regression and find $\alpha = 0$, $\beta = 1$. That means that a projection of $q_{t+1} - q_t$ on $r_t - r_t^*$ gives us an intercept of zero and a slope of one, but that is different than saying $E_t q_{t+1} - q_t = r_t - r_t^*$. In other words, there could be a complex process that drives $q_{t+1} - q_t$ that involves many driving variables and perhaps is nonlinear, but if we just regress $q_{t+1} - q_t$ on $r_t - r_t^*$ we find a slope of one and intercept of zero.

7. Explain intuitively the difference between the model I attribute to Bansal and Shaliastovich, and the one I attribute to Lustig, Roussanov and Verdelhan, starting on page 460.

Answer:

In the Bansal and Shaliastovich setting, the home and foreign bonds have different risk characteristics. That is, when u_t^h rises relative to u_t^f, the "pseudo" risk premium on foreign bonds increases. (I'm calling it a "pseudo" risk premium because of the discussion in section 1. We can think of it as the average of the risk premium perceived by home and foreign investors.) at the same time $r_t^* - r_t$ rises. Investors in both countries have identical preferences – the same degree of risk aversion and the same intertemporal elasticity of substitution. The movements in the risk premium and interest rates are determined by the properties of idiosyncratic shocks to the variance of the growth rates of consumption in the two countries.

In the LRV model, the focus is on the common component (common between the two countries) of shocks to the variance of growth rates of consumption. The risk premium and the interest rate differential are determined by differences in how home and foreign investors react to those shocks, which is determined by differences in their preferences. It is differences in home and foreign risk aversion and intertemporal elasticity of substitution that determine the comovements.

8. In your own words, explain how delayed portfolio adjustment can account for this finding.

Answer:

First, take the case where portfolios could adjust instantly. Suppose the two economies are at the ZLB but central banks can use an unconventional monetary policy tool. When $\pi_t - \pi_t^*$ is below average at time *t*, the Fed might credibly announce that at time $t+k$, the nominal interest rate will be lower than markets previously had anticipated. If uncovered interest parity held, the dollar would depreciate immediately at time *t*, and would be expected to remain constant until time $t+k$. That is, $E_t s_{t+k} - s_t$ would equal zero, and the constancy of $E_t s_{t+k} - s_t$ would match the constancy of short-term interest rates between period *t* and period $t+k-1$. Hence, under UIP, the realization of $\pi_t - \pi_t^*$ would not help forecast excess returns because ex ante excess returns would be zero.

However, suppose portfolio adjustments were costly or were constrained from full adjustment. Then, when $\pi_t - \pi_t^*$ has a low realization, markets believe that U.S. interest rates will fall in the future. Some agents are able to shift some of their resources from U.S. deposits to foreign deposits, but the rebalancing of portfolios is not sufficient to eliminate the low returns on U.S. assets relative to foreign assets. At time *t*, the dollar is expected to depreciate because there has not been full adjustment and it is expected that in period $t+1$, there will be further adjustment. Then, $E_t s_{t+1} - s_t > 0$. Given the constancy of interest rates (at, or very near, the ZLB), we have $i_t < i_t^* + E_t s_{t+1} - s_t$, and the foreign deposits will continue to have ex ante excess returns until portfolios fully adjust. We conclude that low values of $\pi_t - \pi_t^*$ predict high values of $s_{t+1} - s_t + i_t^* - i_t$.

Simple New Keynesian model

The main reading for this section is:

Engel, Charles. "Exchange rates and interest parity." *Handbook of international economics* 4 (2014): 453-487.

For this section, you are only being asked to read up to page 487.

The purpose of this section of the course is to build a very simple New Keynesian model that will give us a baseline to think about more difficult problems.

Section 1:

Section 1 seems fairly straightforward, but I want to make an important comment. Equation (4) is not a full model of the exchange rate. It is derived only from the uncovered interest parity condition and makes no other assumption. Equation (5) is even more general since it does not even require UIP to hold, with λ_t being defined as the deviation from UIP.

To have a full model, though, we need something that tells us how interest rates behave over time. Our simple model will have a monetary policy rule which posits that nominal interest rates respond to inflation, possibly with interest rate smoothing. That means the nominal interest rate is endogenous in the model, and we can't just "plug in" interest rates into equation (4) or (5) assuming they follow some exogenous process. Likewise, inflation will be endogenous in our model, and will depend on the real exchange rate (which in turn depends on the nominal exchange rate and inflation.) And we will introduce a very simple model for the deviation from UIP that is not exogenous but depends on interest rates.

In other words, it may be tempting to treat (4) or (5) as a full model and then just posit real interest rates and deviations from UIP move exogenously, but that would not make for a coherent macro model.

One interesting way to see that equation (4), for example, is incomplete is to think about the case of flexible nominal prices, where purchasing power parity always holds so the real exchange rate is always constant, and money is neutral so does not affect any real variable. If we look at (4), the final term is zero, since the log of the real exchange rate is zero ($q_t = 0$). The summation term involving real interest rates would be zero, because the real representation of UIP would imply the real interest rate differential was zero: $r_t - r_t^* = E_t q_{t+1} - q_t = 0$. The equation then simply says $s_t = p_t - p_t^*$, which is just a restatement of $q_t = 0$, and so equation (4) really tells us nothing in this case about how exchange rates are determined.

In that introductory section, the argument is that we can look at this equation to get some intuition about how things might work in a sticky-price New Keynesian model. Let's intuitively consider three shocks – a monetary shock that "tightens" monetary policy in the home country; a shock to relative productivity that changes the long-run real exchange rate as in, perhaps, the Balassa-Samuelson model; and an exogenous shock to the liquidity of the home bond, so that it pays a higher non-monetary liquidity return.

Let's say that nominal prices are quite sticky so none of these shocks has a very large effect on the overall nominal price level in either country initially, so that the effect on $p_t - p_t^*$ is very close to zero. First consider a monetary shock. If the monetary tightening raises real interest rates – that is, the nominal interest rate at home increases more than expected inflation – then, holding the foreign country constant, $r_t - r_t^*$ rises, and probably the expected future real interest rate differential rises as well if the effects are persistent. Equation (5) says that the direct effect of that shock is a home currency appreciation, meaning s_t falls. Intuitively, investors want to shift their funds from foreign bonds to home bonds because of the higher home interest rate and will continue to do so until s_t has fallen enough that bond market equilibrium (equation (2)) is restored.

In the simple model we will look at, the increase in interest rates also influences the liquidity return, λ_t, but let's set that aside for now. In our simple model, the monetary shock has no effect on the expected long run real exchange rate (money is neutral in the long run), so the last term, $\lim_{j\to\infty} E_t q_{t+j+1}$ is not affected by the monetary shock.

Instead, let's just look directly at the effect of a change in the liquidity return on home bonds, as if there were an exogenous shock that raised the liquidity of those bonds. If the current or future liquidity of home bonds rises (relative to foreign bonds), λ_t rises, and the home currency appreciates. This works like an increase in the home interest rate. The liquidity return is like a non-monetary return to holding the home bond. In the simple model we look at, there are feedback effects, as the change in the exchange rate leads to a change in inflation, which affects interest rates, etc. On the other hand, in the simple model, liquidity shocks are also neutral in the long run, so $\lim_{j\to\infty} E_t q_{t+j+1}$ is not affected.

Suppose there is some shock that causes a long-run real appreciation, such as a shock to the productivity in the home country as in the Balassa-Samuelson model. Then $\lim_{j\to\infty} E_t q_{t+j+1}$ is affected. A long-run real appreciation leads to an immediate nominal appreciation, *ceteris paribus*. But in the simple model, there will be feedback to interest rates and the liquidity return.

My point is that equations (4) or (5) are useful to some degree for intuition of how shocks affect the nominal exchange rate, but since there are endogenous responses, we need a full model. Section 2 presents a very simple full model.

Section 2.1:

I think the derivation of the model, up to the matrix representation at the top of page 460, is pretty straightforward. Notice that the elements of the simple model are: a) UIP holds. There is no deviation at all (not even a liquidity yield as in section 1.) b) The price adjustment equation assumes no home bias in preferences and LCP pricing. c) Monetary policy in each country targets only inflation, but with interest rate smoothing. d) The two countries are exactly symmetric.

These elements make huge simplifying assumptions relative to the literature we have already read. The point, again, is to provide some baseline to think about things. We can start with this stripped-down model and then start adding elements.

1. Set $\alpha = 0$ in the model at the top of page 460. Rewrite the matrix version of the model in a "block recursive" form: the first block is in terms of two variables, $\pi_t - \pi_t^*$ and q_t, the second block simply determines $i_t - i_t^*$ as a function of $\pi_t - \pi_t^*$.

Now I want you to review how to solve a matrix first-order expectational difference equation. (Note, the next question (question 2) is not asking you to solve the specific model from question 1, which would involve actually calculating eigenvalues of the matrix *A* in the following equation. Instead, it is a review of solving a simple system of expectational difference equations.) Suppose in general we have:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ y_t \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

x_t and y_t are the endogenous variables we want to solve for. A is a 2x2 matrix of parameters. u_{1t} and u_{2t} are exogenous random variables that are not necessarily i.i.d.

Follow the Wikipedia article on "Eigendecomposition of a matrix":

https://en.wikipedia.org/wiki/Eigendecomposition_of_a_matrix

We can write $A = Q\Lambda Q^{-1}$, where Q and Λ are defined in the article. It is important to note that Λ is diagonal. Then note by multiplying through by Q^{-1}, we can write the above system as:

$$Q^{-1} \begin{bmatrix} E_t x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \Lambda Q^{-1} \begin{bmatrix} x_t \\ y_t \end{bmatrix} + Q^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Let $Q^{-1} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$, then we can write this system as:

$$\begin{bmatrix} E_t z_{1t+1} \\ E_t z_{2t+1} \end{bmatrix} = \Lambda \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} + \begin{bmatrix} q_{11}u_{1t} + q_{12}u_{2t} \\ q_{21}u_{1t} + q_{12}u_{2t} \end{bmatrix}$$

2. Explain why this converts the system into two single equation difference equations, each a function of only one variable.

When we have a single equation difference equation, such as $E_t z_{t+1} = \lambda_1 z_t + v_t$, the "stable" solution depends on whether $\lambda_1 < 1$ or $\lambda_1 > 1$. If $\lambda_1 < 1$, the stable solution to the difference equation is "backward looking" and can be written as simply $z_{t+1} = \lambda_1 z_t + v_t$, or equivalently, $z_t = \lambda_1 z_{t-1} + v_{t-1}$. This is a solution because it solves for the endogenous variable, z_t, in terms of predetermined and exogenous variables. We could express the solution for z_t as a function of the whole past sequence of v's by iterating backward: $z_t = v_{t-1} + \lambda_1 v_{t-1} + \lambda_1^2 v_{t-1} + \ldots$, But this solution is only "stable" when $\lambda_1 < 1$. If $\lambda_1 > 1$, the terms in the infinite sum explode.

When $\lambda_1 > 1$, we rewrite the equation as $z_t = \frac{1}{\lambda_1} E_t z_{t+1} - \frac{1}{\lambda_1} v_t$. We can then iterate forward:

$z_t = -\frac{1}{\lambda_1}\left[v_t + \frac{1}{\lambda_1} E_t v_{t+2} \right] + \left(\frac{1}{\lambda_1} \right)^2 E_t z_{t+2}$, and continue to find: $z_t = -\frac{1}{\lambda_1} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_1} \right)^j E_t v_{t+j}$. If we know the stochastic process for v_t, we can simplify the discounted expected sum by substituting in expressions for $E_t v_{t+j}$. (We are not going to worry about the special case of $\lambda = 1$ for now.)

3. With this reminder of how to solve dynamic models, show that if one of the diagonal elements of Λ above is <1 and one is >1, then the solution for x_t and y_t will in general have a "backward" and "forward" element. (Hint: After solving for the two variables in question 2, use the fact that $\begin{bmatrix} x_t \\ y_t \end{bmatrix} = Q \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix}$ to recover solutions for x_t and y_t.) What is the parameter that determines the speed of adjustment of the backward-looking component? What is the parameter that determines the "discount factor" of the forward-looking component?

Questions 2-3 are a simple example of the solution method presented in Blanchard and Kahn (Econometrica, 1980.) In the article, if I make the arbitrary assumption that $\sigma\beta = 1$, I can actually solve the 3-equation model as presented, because solving for the eigenvalues involves only solving a quadratic equation under this assumption (and recognizing that the third eigenvalue must be $1/\beta$.) That is how I derived (20), which you can try to derive on your own.

In this case, we see that the real exchange rate follows a first-order autoregression, if the monetary shock is i.i.d. (The only exogenous variable here is the monetary shock.) With a lot of work, one can show a contractionary home monetary shock (an increase in $\varepsilon_t - \varepsilon_t^*$) leads to a real appreciation. The persistence of the real exchange rate depends on both the degree of interest rate smoothing in the monetary policy rule, and the sluggishness of prices. The article states that the persistence can be no greater than the greater of the persistence of the interest-rate smoothing or the probability of leaving prices unchanged in the Calvo price setting algorithm. This claim is proven in my 2019 Journal of Monetary Economics paper on the persistence of real exchange rates.

It is striking that even if nominal prices are sticky, if there is no interest-rate smoothing in the monetary policy rule, the real exchange rate has no persistence (other than any persistence that comes from the monetary policy shock itself.) This was shown by Benigno (2004) and is not very intuitive. My 2019 JME paper explains the intuition. It also shows that if there is a closely-related price-setting rule (that I call a Dornbusch rule in that paper), then interest-rate smoothing is no longer needed to get real exchange rate persistence, and in fact, under one simple form of the rule, the speed of adjustment of the real exchange rate is entirely determined by the speed of adjustment of nominal prices. The relationship between these two price-adjustment mechanisms is also explained in that article.

For most of the rest of the way, we will assume no interest-rate smoothing in the Taylor rule but still use the Calvo price-setting mechanism, which means our solutions will only give us persistence in the real exchange rate if the exogenous driving variables are persistent. That is not realistic, but let's not worry about it. I will hypothesize that even with Calvo price setting, if there were pricing complementarities (so the "reset" price depends not only on costs but on the prices of competing firms), the real exchange rate would be persistent even if there were no interest-rate smoothing, but that is not something I've tried to prove.

If we assume no interest-rate smoothing, then in (20), we find $\mu_2 = 0$. Equation (21) shows the full forward-looking solution in this case – that is, the solution even when $\varepsilon_t - \varepsilon_t^*$ is not i.i.d. The rest of section 2.1 then emphasizes how "news" about future monetary policy shocks can influence today's exchange rate, and in fact, may be a major factor in determining the exchange rate today. Since it is difficult for an empirical researcher to measure "the news", the researcher may only see how current monetary shocks affect the exchange rate and not news about the future. The article emphasizes though, that when there is news, we should not expect to find a strong connection between current exogenous driving variables and the current exchange rate.

The conclusion that I mention toward the end that the presence of news cannot explain volatility in the *change* in the real exchange rate is a special result that only holds when the real exchange rate solution has no backward-looking element - that is, no persistence.

Section 2.2 foreshadows what we will look at in the next section of readings (from Obstfeld and Rogoff, and Schmitt-Grohe and Uribe) – how the trade balance or current account will come into a model of exchange-rate determination. We can ask, though, why is it that the three-equation model here can completely determine the real exchange rate, real interest rates, the nominal exchange rate, inflation? Why is there no feedback from output, consumption, investment, etc., to these variables The answer is that we can think of the global economy as being "block recursive". These three equations can be solved autonomously, but then the solutions for the variables in this block do affect the rest of the economy. The key reasons why this system is block-recursive are exactly the set of assumptions that I listed at the beginning of the discussion of section 2.1. That is why we made these assumptions – to give us a simple autonomous system that can solve for some of the variables, making the whole system for the global economy block-recursive. Richer and more realistic models will not have this property.

Before we move on from section 2.1, let's consider a slightly richer version of this 3-equation model. It will not have interest-rate smoothing, but it will have deviations from UIP, and it will have exogenous movements in the long-run real exchange rate.

The model and its solution are laid out on the assigned pages of this reading (don't read all the statistical tables that impinge on the part we want to read – we just want to look at the section with the model and its solution.):

Engel, Charles, et al. "The uncovered interest parity puzzle, exchange rate forecasting, and Taylor rules." *Journal of International Money and Finance* 95 (2019): section 4, 325-329.

I want you to focus on the solutions for the model, equations (17)-(20). You can try deriving these yourself. In deriving these, I did not use the method described above, but instead used the "method of undetermined coefficients" (there's a Wikipedia article on that too!), sometimes called "guess and verify", but either way would work. Note that under the assumptions of the model, $D_i > 0$, $i = 1, 2, 3$.

The questions are aimed at testing your understanding of the intuition of the solutions:

4. Equation (20) has a solution for the real exchange rate. From that, we can see how a shock that implies a monetary contraction, a shock to the long-run real exchange rate, and a shock to the expected excess return on foreign bonds affect q_t. Say in words how each of these affect the real exchange rate, but say it clearly. In other words, don't write "a monetary shock leads to a real appreciation." You need to say something like "a monetary contraction in the home country leads to a home real appreciation." That is, you need to say clearly which direction of shock you are talking about and which direction of change of the real exchange rate. Then, compare your answers to the descriptions of the effects of these shocks from the forward-looking, single-equation "model" that I gave on page 2 of these notes.

5. Note from equation (17) that the effect of a monetary shock on the nominal interest differential is ambiguous. Explain why that is the case.

Be sure to examine these solutions and understand how each shock affects each variable, because this model is giving us the baseline intuition on which we will build.

6. δ can be considered a measure of how much inflation responds to deviations of the real exchange rate from its long-run value. A small value of δ implies price adjustment is slower. How does the size of δ influence the response of the real exchange rate to a monetary shock?

Section 2.2

As I noted above, section 2.2 very briefly describes two-way feedback between trade balances and the exchange rate that will be examined in more detailed models in our next set of readings. Here is the basic point that is being made: Suppose the home currency depreciates, perhaps because of a foreign monetary contraction. In a lot of international models, where consumer demand does respond to exchange rates, the home depreciation makes foreign goods more expensive and reduces home demand for those goods. Initially, the home country will run a trade balance surplus as it imports fewer goods (and exports more, since the same forces make home goods cheaper abroad.) But the trade surplus does not last forever. As the home country runs trade surpluses, it must be the case that it is lending abroad and so its wealth is rising over time. As its wealth rises, consumption rises, which drives higher inflation. As home inflation rises, home monetary policy contracts, which works to appreciate the currency. This appreciation counteracts the initial depreciation and works as a self-equilibrating mechanism.

We will spend some time discussing the Gourinchas-Rey paper mentioned at the end of section 2.2 and its implications later in the semester.

Section 3.1

Here I have two questions for you to answer from this section:

7. What is delayed overshooting, and how could it account for the uncovered interest parity puzzle?
8. Why does news that home inflation is higher than expected lead to an appreciation of the home currency? In other words, it seems like if markets expect home inflation to be higher, the value of the currency should be lower and it should depreciate. So why isn't that true?

Section 3.2

Here, the part of the reading that requires some thought is the Engel-West theorem. Let me help by explaining what my intuition was that led me to propose this theorem (which West proved!)

Suppose we have a model for the exchange rate that looks like this:

(1) $$s_t = f_t + bE_t s_{t+1}$$

f_t is some economic "fundamental" that drives the exchange rate. The equation says the log of the exchange rate is determined by today's fundamental, but also by expectations of next period's exchange rate. Those expectations are "discounted" by the constant b, where $0 < b < 1$.

A crucial assumption is that the fundamentals have a unit root. Here is just a quick review of what that means. First, it means that the first-difference of the fundamentals, $f_t - f_{t-1}$, is stationary. What does stationary mean? Here is a useful property of stationary random variables:

$$\lim_{j\to\infty} E_t x_{t+j} = Ex_t \,.$$

The left-hand-side of this equation is the forecast, conditional on time *t*, of x_t very far into the future. The right-hand-side is the unconditional mean of x_t. The equation says that the forecast of *x* far into the future converges to the unconditional mean of *x*.

Another useful thing to know is the Beveridge-Nelson decomposition. Any variable f_t that has a unit root can be written as the sum of a stationary random variable, and a variable whose change is unforecastable:

$$f_t = x_t + p_t \,.$$

Here, x_t is the stationary random variable. We can write for p_t (the "permanent component" of f_t):

$$p_t = p_{t-1} + \varepsilon_t$$

where ε_t is a mean-zero, serially uncorrelated random variable. The forecast of p_t at any period into the future is just equal to p_t:

$$E_t p_{t+k} = p_t \,.$$

From what we said above, we can conclude that the unconditional mean of p_t does not exist, and so neither does the unconditional mean of f_t. Note that the Beveridge-Nelson decomposition does not require that unexpected changes in the two components be uncorrelated – that is, they may be correlated. Note that the equations above imply:

$$\lim_{j\to\infty} E_t f_{t+j} = Ex_t + p_t \,.$$

9. For the stochastic process $f_t - f_{t-1} = \rho\left(f_{t-1} - f_{t-2}\right) + u_t$, where $0 < \rho < 1$ and u_t is a mean-zero, serially uncorrelated random variable, find the stationary and permanent components of f_t. (A possibly useful hint is that because there is no intercept term, the unconditional mean of the stationary component is zero, so $p_t = \lim_{j\to\infty} E_t f_{t+j}$.)

We can write out equation (1) for the exchange rate as:

$$(2) \qquad s_t = f_t + \beta E_t f_{t+1} + \beta^2 E_t f_{t+2} + \ldots$$

With those facts in mind, my basic intuition for the theorem was that when the discount factor is close to one, so markets put a lot of weight not just on the current fundamental but on future fundamentals as well, most of the change in the exchange rate is unexpected. It is unexpected because either there were unexpected changes in the current fundamental, $f_t - E_{t-1} f_t$, or, more importantly, changes in expectations of the future fundamentals

To see this, write $s_t - s_{t-1} = s_t - E_{t-1}s_t + \left[E_{t-1}s_t - s_{t-1}\right]$. The first term is the unexpected change in the exchange rate, and the second term is the expected change in the exchange rate. The Engel-West theorem in essence says that as the discount factor gets close to one, the first term becomes increasingly important relative to the second term: the unexpected change component in the exchange rate is much more important than the expected or forecastable component.

From (2), we get

$$s_t - E_{t-1}s_t = f_t - E_{t-1} f_t + \beta\left(E_t f_{t+1} - E_{t-1} f_{t+1}\right) + \beta^2\left(E_t f_{t+2} - E_{t-1} f_{t+2}\right) + \dots$$

Now, recalling $f_t = x_t + p_t$, and $p_t = p_{t-1} + \varepsilon_t$, we can write:

$$\begin{aligned}
&s_t - E_{t-1}s_t = A + B, \text{ where} \\
&A = x_t - E_{t-1}x_t + \beta\left(E_t x_{t+1} - E_{t-1}x_{t+1}\right) + \beta^2\left(E_t x_{t+2} - E_{t-1}x_{t+2}\right) + \dots \\
&B = p_t - E_{t-1}p_t + \beta\left(E_t p_{t+1} - E_{t-1}p_{t+1}\right) + \beta^2\left(E_t p_{t+2} - E_{t-1}p_{t+2}\right) + \dots
\end{aligned}$$

and

$$= \varepsilon_t + \beta\varepsilon_t + + \beta^2\varepsilon_t + \dots = \frac{1}{1-\beta}\varepsilon_t$$

Intuitively, the terms in parentheses in the infinite sum in *A* are converging toward zero, because x is stationary, so it is expected to converge to its unconditional mean. But we can see that *B* is quite large if β is large and goes to infinity as β goes to one. Intuitively, when markets get news about the fundamental, they may learn about the permanent component, but that can have a large effect on the current exchange rate when the discount factor is near one.

But both *A* and *B* are unpredictable changes. Taken together, they are much larger than the predictable component as $\beta \to 1$, because *B* goes to infinity, while the predictable component is finite:

$$\begin{aligned}
&E_{t-1}s_t - s_{t-1} = \left[E_{t-1} f_t + \beta E_{t-1} f_{t+1} + \beta^2 E_{t-1} f_{t+2} + \dots\right] - \left[f_{t-1} + \beta E_{t-1} f_t + \beta^2 E_{t-1} f_{t+1} + \dots\right] \\
&= E_{t-1}\left[\left(f_t - f_{t-1}\right) + \beta\left(f_{t+1} - f_t\right) + \beta^2\left(f_{t+2} - f_{t+1}\right) + \dots\right]
\end{aligned}$$

Since the first difference of f_t is stationary, the expected change in f_t is converging to zero, so the sum is finite.

In short, as the discount factor gets close to one, the present value formula puts more and more weight on the future. But changes in the exchange rate in that case are mostly driven by changes in the expectation of the permanent component – that is, “news” about the permanent component. So, the exchange rate change is “nearly” unpredictable.

The importance of this result is that many exchange-rate models fit, or nearly fit, the assumptions of the Engel-West theorem, but the implication is that if the model is correct then in fact the exchange rate should not be forecastable. That is, the failure of the models to forecast changes in the exchange rate is not a failure of the model, because the model itself says we should not be able to forecast exchange rate changes.

Ten Important Papers:

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Chahrour, R., Cormun, V., De Leo, P., Guerrón-Quintana, P. A., & Valchev, R. (2021). *Exchange Rate Disconnect Redux*. Boston College.

Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R. McGrattan. "Can sticky price models generate volatile and persistent real exchange rates?." *The review of economic studies* 69.3 (2002): 533-563.

Corsetti, Giancarlo, and Paolo Pesenti. "Welfare and macroeconomic interdependence." *The Quarterly Journal of Economics* 116.2 (2001): 421-445.

Dornbusch, Rudiger. "Expectations and exchange rate dynamics." *Journal of political Economy* 84.6 (1976): 1161-1176.

Gali, Jordi, and Tommaso Monacelli. "Monetary policy and exchange rate volatility in a small open economy." *The Review of Economic Studies* 72.3 (2005): 707-734.

Gopinath, Gita, et al. "Dominant currency paradigm." *American Economic Review* 110.3 (2020): 677-719.

Obstfeld, Maurice, and Kenneth Rogoff. "Exchange rate dynamics redux." *Journal of political economy* 103.3 (1995): 624-660.

Obstfeld, Maurice, and Kenneth Rogoff. "Risk and exchange rates." *Economic Policy in the International Economy: Essays in Honor of Assaf Razin* (2003): 74-120.

Schmitt-Grohé, Stephanie, and Martin Uribe. "Downward nominal wage rigidity, currency pegs, and involuntary unemployment." *Journal of Political Economy* 124.5 (2016): 1466-1514.

Answers to Questions on Simple NK Model

1. Set $\alpha = 0$ in the model at the top of page 460. Rewrite the matrix version of the model in a "block recursive" form: the first block is in terms of two variables, $\pi_t - \pi_t^*$ and q_t, the second block simply determines $i_t - i_t^*$ as a function of $\pi_t - \pi_t^*$.

<u>Answer:</u>

$$E_t \begin{bmatrix} \pi_{t+1} - \pi_{t+1}^* \\ q_{t+1} \\ i_t - i_t^* \end{bmatrix} = \begin{bmatrix} 1/\beta & -\delta/\beta & 0 \\ (\sigma\beta - 1)/\beta & (\beta+\delta)/\beta & 0 \\ \sigma & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \pi_t^* \\ q_t \\ i_{t-1} - i_{t-1}^* \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_{t+1} - \varepsilon_{t+1}^* \\ \varepsilon_{t+1} - \varepsilon_{t+1}^* \end{bmatrix}$$

2. Explain why this converts the system into two single equation difference equations, each a function of only one variable.

<u>Answer:</u>

Our original system was given by:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ y_t \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}.$$

As the assignment notes, we can write $A = Q\Lambda Q^{-1}$, where Q and Λ are defined in the article. It is important to note that Λ is diagonal. Then note by multiplying through by Q^{-1}, we can write the above system as:

$$Q^{-1} \begin{bmatrix} E_t x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \Lambda Q^{-1} \begin{bmatrix} x_t \\ y_t \end{bmatrix} + Q^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Let $Q^{-1} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$, then we can write this system as:

$$\begin{bmatrix} E_t z_{1t+1} \\ E_t z_{2t+1} \end{bmatrix} = \Lambda \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} + \begin{bmatrix} q_{11}u_{1t} + q_{12}u_{2t} \\ q_{21}u_{1t} + q_{12}u_{2t} \end{bmatrix}.$$

Λ is a diagonal matrix which we can write as $\Lambda = \begin{bmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{22} \end{bmatrix}$. Then writing out the system, we have:

$$E_t z_{1,t+1} = \lambda_{11} z_{1,t} + q_{11} u_{1t} + q_{12} u_{2t}$$
$$E_t z_{2,t+1} = \lambda_{22} z_{2,t} + q_{21} u_{1t} + q_{12} u_{2t} ,$$

which are both univariate difference equations.

3. With this reminder of how to solve dynamic models, show that if one of the diagonal elements of Λ above is <1 and one is >1, then the solution for x_t and y_t will in general have a "backward" and "forward" element. (Hint: After solving for the two variables in question 2, use the fact that $\begin{bmatrix} x_t \\ y_t \end{bmatrix} = Q \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix}$ to recover solutions for x_t and y_t.) What is the parameter that determines the speed of adjustment of the backward-looking component? What is the parameter that determines the "discount factor" of the forward-looking component?

Answer:

Once we have solved for $z_{1,t}$ and $z_{2,t}$, we can recover x_t and y_t using $Q \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$.

Let's write out *Q:* $Q = \begin{bmatrix} q^{11} & q^{12} \\ q^{21} & q^{22} \end{bmatrix}$.

Then:

$x_t = q^{11} z_{1,t} + q^{12} z_{2,t}$ and $y_t = q^{21} z_{1,t} + q^{22} z_{2,t}$

Since one of the two *z*'s solved forward and one is solved backward, x_t and y_t will in general have a "backward" and "forward" element. If $z_{1,t}$ is the variable solved backward, then λ_{11}, the eigenvalue that is less than one, determines the speed of adjustment, and the discount factor is given by $1/\lambda_{22}$, where λ_{22} is the eigenvalue greater than one.

4. Equation (20) has a solution for the real exchange rate. From that, we can see how a shock that implies a monetary contraction, a shock to the long-run real exchange rate, and a shock to the expected excess return on foreign bonds affect q_t. Say in words how each of these affect the real exchange rate, but say it clearly. In other words, don't write "a monetary shock leads to a real appreciation." You need to say something like "a monetary contraction in the home country leads to a home real appreciation." That is, you need to say clearly which direction of shock you are talking about and which direction of change of the real exchange rate. Then, compare your answers to the descriptions of the

effects of these shocks from the forward-looking, single-equation “model” that I gave on page 2 of these notes.

Answer:

A monetary contraction in the home country leads to a home real appreciation.

If the home liquidity return increases, there is a home real appreciation.

If there is an equilibrium home real appreciation, it translates into an immediate home real appreciation.

These are the same conclusion we reached in looking at the forward-looking single equation “model”, but now we incorporate the feedback effects to all the variables in equilibrium.

5. Note from equation (17) that the effect of a monetary shock on the nominal interest differential is ambiguous. Explain why that is the case.

Answer:

Suppose the shock is a monetary contraction. On the one hand, reading directly from the monetary policy rule, the nominal interest rate rises, holding inflation constant. But if the effect of the contraction is to lower expected inflation, then for a given real rate of interest, the nominal interest rate will fall. In other words, the monetary contraction raises the real interest rate and lowers expected inflation, so the effect on the nominal interest rate is ambiguous.

6. δ can be considered a measure of how much inflation responds to deviations of the real exchange rate from its long-run value. A small value of δ implies price adjustment is slower. How does the size of δ influence the response of the real exchange rate to a monetary shock?

Answer:

A smaller δ gives a larger response, thereby increasing the volatility of the real exchange rate. It does not affect the persistence.

7. What is delayed overshooting, and how could it account for the uncovered interest parity puzzle?

Answer:

The absolute value of the impulse response of the real exchange rate to a monetary policy shock does not reach its maximum at the time of the shock but rather several periods later. So, if the home interest rate increases, the home real exchange rate falls but then continues to fall in the next period. That means an increase in the home interest rate predicts an appreciation over the following period, which is in contrast to uncovered interest parity but consistent with the empirical findings of the UIP puzzle.

8. Why does news that home inflation is higher than expected lead to an appreciation of the home currency? In other words, it seems like if markets expect home inflation to be higher, the value of the currency should be lower and it should depreciate. So why isn't that true?

Answer:

The reason there is a real appreciation is that the stability condition holds, so that an increase in home inflation means that eventually the monetary policy maker's response is to raise the real interest rate, leading to a real appreciation.

9. For the stochastic process $f_t - f_{t-1} = \rho(f_{t-1} - f_{t-2}) + u_t$, where $0 < \rho < 1$ and u_t is a mean-zero, serially uncorrelated random variable, find the stationary and permanent components of f_t. (A possibly useful hint is that because there is no intercept term, the unconditional mean of the stationary component is zero, so $p_t = \lim_{j\to\infty} E_t f_{t+j}$.)

Answer:

From the statement of the problem, we know that $\lim_{j\to\infty} E_t f_{t+j} = p_t$. We can find p_t then by working from $E_t f_{t+1} = (1+\rho) f_t - \rho f_{t-1}$, then plugging in to $E_t f_{t+2} = (1+\rho) E_t f_{t+1} - \rho f_t$, use those two solutions to plug into $E_t f_{t+3} = (1+\rho) E_t f_{t+2} - \rho E_t f_{t+1}$, and so on to find $\lim_{j\to\infty} E_t f_{t+j} = p_t$.

Instead, I will do the following. I notice that $f_t - f_{t-1}$ is stationary. I have also noticed that we can write the original equation as: $f_t - \rho f_{t-1} = f_{t-1} - \rho f_{t-2} + u_t$. This means that $f_t - \rho f_{t-1}$ follows a random walk.

So, I conclude that the stationary component is proportional to $f_t - f_{t-1}$, and the permanent component is proportional to $f_t - \rho f_{t-1}$. A weighted average of those to variables equals f_t: $f_t = (1-b)(f_t - f_{t-1}) + b(f_t - \rho f_{t-1})$, where b is an undetermined coefficient to solve for. That equation implies that $0 = -(1-b) - b\rho$. So $b = \frac{1}{1-\rho}$. Hence, we have

$$p_t = \frac{1}{1-\rho}(f_t - \rho f_{t-1}) \text{ and } x_t = -\frac{\rho}{1-\rho}(f_t - f_{t-1}).$$

Assignment 4: Obstfeld and Rogoff, Foundations, Chapter 1, sections 1.1-1.3 and Chapter 2, sections 2.1-2.3 and 2.5

For the next five or five and a half weeks, we will be reading out of, first, the Obstfeld and Rogoff textbook, and then out of the Uribe and Schmitt-Grohe textbook. This is 5 to five and a half weeks out of a 13.5 week semester, so it is substantial.

None of what we will read here is cutting-edge, or frontier research. As the syllabus explains, international macro/finance is a wide-ranging field, and a substantial part of the objective of this course is to give you the background to be able to read the latest research. That means covering a lot of material.

These next 5-5.5 weeks are ones where you need to do a lot of reading. Being a PhD student is a full-time job, where you work a lot of overtime and don't get paid for any of it. During this period, you'll be sitting at your desk with the oil lamp burning and reading stuff. There's no programming, no proving theorems, no gathering data and doing empirical work. Just reading (though you will need to read with a pen and paper at hand, so you can work through any equations that you don't see right away.)

During this period, the assignments will have weekly due dates. My aim still is not to ask questions that are difficult – just ones that force you to do the readings with some comprehension.

The chapters from Obstfeld and Rogoff are long, though not that hard. For this book, I am going to list sections or subsections that you should read more carefully (labeled *) versus ones that you don't need to spend as much time on (labeled x). By the way, there is a lot of overlap between the two books, but the way they present ideas is quite different. We will go through OR first, and then, in essence, start over with USG, rather than syncing the readings by topic.

Here is the list of *ed and xed subsections, and then a few questions come at the end:

1.1.1 *	1.1.2 *	1.1.3 *	1.1.4 *	1.1.5 *	1.1.6 *
1,1,7 x	1.2.1 *	1.2.2 *	1.2.3 *	1.2.4 x	1.3.1 *
1.3.2 x	1.3.3 *	1.3.4 x			
2.1.1 *	2.1.2 *	2.1.3 *	2.1.4 x	2.2.1 *	2.2.2 *
2.3.1 *	2.3.2 *	2.3.3 *	2.3.4 *	2.3.5 x	2.5.1 *
2.5.2 *	2.5.3 x				

Questions from Chapter 1:

1. Explain the intuition of why the comparison of the autarky real interest rate with the world real interest rate determines whether a country runs a current account surplus or deficit.

2. There are four ways to define the current account balance: 1. National saving – investment; 2. National income – expenditure; 3. Minus the financial account balance; 4. Exports – imports of goods and services plus net foreign income. How does one express each of the first three of these using the notation of equation (12)? The fourth one is the way most people think of the current account balance, but why is it difficult to think of this definition in the context of this model?

3. Explain the curve shifts in Figure 1.8. Explain the changes in the equilibrium interest rate. What can we say about equilibrium current account balances?

Questions about Chapter 2:

4. Explain equation (20)
5. Explain the intuition of equation (37)
6. Why does it make sense to say an entrepreneur's objective should be to maximize the value of the firm? (I'm not looking for a political opinion here. I'm talking about a specific part of the Chapter 2 readings.)

Ten Important Papers

Aguiar, Mark, and Gita Gopinath. "Emerging market business cycles: The cycle is the trend." *Journal of political Economy* 115.1 (2007): 69-102.

Alfaro, Laura, Sebnem Kalemli-Ozcan, and Vadym Volosovych. "Why doesn't capital flow from rich to poor countries? An empirical investigation." *The review of economics and statistics* 90.2 (2008): 347-368.

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Chinn, Menzie D., and Eswar S. Prasad. "Medium-term determinants of current accounts in industrial and developing countries: an empirical exploration." *Journal of international economics* 59.1 (2003): 47-76.

Gourinchas, Pierre-Olivier, and Olivier Jeanne. "Capital flows to developing countries: The allocation puzzle." *Review of Economic Studies* 80.4 (2013): 1484-1515.

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Schmitt-Grohé, Stephanie, and Martın Uribe. "Closing small open economy models." *Journal of international Economics* 61.1 (2003): 163-185.

Zucman, Gabriel. "The missing wealth of nations: Are Europe and the US net debtors or net creditors?." *The Quarterly journal of economics* 128.3 (2013): 1321-1364.

Questions from Chapter 1 of Obstfeld-Rogoff

1. Explain the intuition of why the comparison of the autarky real interest rate with the world real interest rate determines whether a country runs a current account surplus or deficit.

Answer:

There are many appropriate answers to this question. Here is one. We can think of an analogy to classic trade theory. Countries export the good for which they have a comparative advantage, which means that the price of their export goods relative to their import goods is lower in autarky than under free trade.

In this case, the price of goods next period relative to goods this period is the inverse of the gross interest rate. So, the relative price of current period goods is higher when the interest rate is higher. If the autarky interest rate for a country is lower than the world interest rate, it has a comparative advantage in current period goods. That means it will export current period goods in exchange for future goods, which is a current account surplus. That is, it will be a lender to the rest of the world. When its autarky interest rate is higher than the world interest rate, it has a comparative advantage in future goods. It will export future goods in exchange for current goods, meaning it borrows and runs a current account deficit.

2. There are four ways to define the current account balance: 1. National saving – investment; 2. National income – expenditure; 3. Minus the financial account balance; 4. Exports – imports of goods and services plus net foreign income. How does one express each of the first three of these using the notation of equation (12)? The fourth one is the way most people think of the current account balance, but why is it difficult to think of this definition in the context of this model?

Answer:

Equation (12) says $CA_t = B_{t+1} - B_t = Y_t + r_t B_t - C_t - I_t - G_t$.

1. $CA_t = (Y_t + r_t B_t - C_t - G_t) - I_t$ Here, national saving equals private saving, $Y_t + r_t B_t - C_t - T_t$ plus government saving, $T_t - G_t$, where T_t is taxes.
2. $CA_t = (Y_t + r_t B_t) - (C_t + I_t + G_t)$
3. $CA_t = B_{t+1} - B_t$
4. There is only one good in the model, so at any time, the country is either importing and has a trade deficit, or is exporting and has a trade surplus.

3. Explain the curve shifts in Figure 1.8. Explain the changes in the equilibrium interest rate. What can we say about equilibrium current account balances?

Answer:

In the left-hand-side graph: A rise in future home productivity makes investment more profitable. For any given interest rate, the home country increases investment, shifting the II' curve to the right. Also, since future output will be higher, the home households save less for every level of the interest rate, shifting the SS' curve to the left.

The home interest rate must rise, but the increase in interest rates at home must also increase the interest rates in the foreign country, which induces higher saving there and lower investment. The difference between saving and investment in the foreign country, S*-I* must equal the shortfall in saving in the home country, I-S, because in world equilibrium, S+S*=I+I*. So the interest rate must settle at the point where S*-I* = I-S.

4. Explain equation (20)

Answer:

Assume the period utility function is given by $u(C_t) = \frac{1}{1-\sigma} C_t^{\frac{1}{1-\sigma}}$. Using the dynamic first-order condition, equation (15), and substituting that solution into the lifetime budget constraint, (14), we get the consumption function at time *t* given in equation (16):

$$C_t = \frac{r+\vartheta}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]., \text{ where } \vartheta = 1-(1+r)^{\sigma}\beta^{\sigma}$$

Then using the definition in equation (17), we can write this as

$$C_t = (r+\vartheta)\left[B_t + \frac{1}{r}\left(\tilde{Y}_t - \tilde{G}_t - \tilde{I}_t\right)\right] = (r+\vartheta)B_t + \left(1+\frac{\vartheta}{r}\right)\left(\tilde{Y}_t - \tilde{G}_t - \tilde{I}_t\right)$$

, which says that the optimal consumption is related to initial bond holdings plus the annuity value of income left over for consumers after taxes and investment (where the lifetime budget constraint holds for the government so the annuity value of lifetime government spending equals the annuity value of lifetime taxes.)

Initial wealth is given in equation (19) as

$$W_t = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)$$

Then,

$$CA_t = B_{t+1} - B_t = Y_t + r_t B_t - C_t - I_t - G_t$$

This gives us:

$$CA_t = B_{t+1} - B_t = Y_t - \left(\tilde{Y}_t - \tilde{G}_t - \tilde{I}_t\right) - I_t - G_t - \left[\vartheta B_t + \frac{\vartheta}{r}\left(\tilde{Y}_t - \tilde{G}_t - \tilde{I}_t\right)\right]$$
$$= Y_t - \tilde{Y}_t - \left(G_t - \tilde{G}_t\right) - \left(I_t - \tilde{I}_t\right) - \frac{\vartheta}{1+r} W_t$$

The current account is determined by current income, investment and government spending relative to their permanent levels, with a "twist" for how initial wealth affects consumption given the relationship between the utility discount factor and market discount factor. When current output is high relative to its permanent level, some output is saved, and the current account is increased. When government spending or investment are high relative to their permanent levels, there is more spending this period and the current account falls. If $\vartheta > 0$, it means $\beta < 1 + r$, so households put less weight on the future than the market does, which leads to more spending by households this period, which lowers the current account.

5. Explain the intuition of equation (37)

Answer:

Here we see that a temporary shock to output (a positive realization of ε_t) leads to an increase in the current account because households save some of the increased output, as their consumption depends on the present discounted value of the lifetime stream of income. If last period's output was below average, the country was prone to running a current account deficit, and borrowing from abroad. Subsequently, they must repay their loans, which contributes to a higher current account this period.

6. Why does it make sense to say an entrepreneur's objective should be to maximize the value of the firm? (I'm not looking for a political opinion here. I'm talking about a specific part of the Chapter 2 readings.)

Answer:

The objective of the firm is to maximize the utility of the firm owners. Firm owners gain from owning a firm because they get paid dividends. Current and future dividends matter to the firm owners. The utility of future dividends depends on the marginal rate of substitution between consumption in the future and consumption today. From the first-order conditions of the household, the marginal rate of substitution is equal to the interest rate (in the deterministic model), so households discount future dividends by the inverse of the gross interest rate. The utility of the firm is then equal to the present value of dividends the firm pays, and that determines the value of the firm.

Assignment 5: Obstfeld and Rogoff, Chapter 4, sections 4.1, 4.2.1-4.2.3 and 4.4; Chapter 5, sections 5.1.1-5.1.7, 5.2-5.4

We could spend a lot of time on these chapters. However, I want us to focus on the parts of the chapters that explain economic mechanisms that arise in a lot of models – they are, in a sense, more general – rather than the sections that are somewhat more model-specific.

In chapter 4, we look again at the Balassa-Samuelson model. This chapter presents a richer version than was in the paper that was on the earlier assignment by Berka, Devereux and me because it has production with capital. A key concept that comes up here is the discussion of how international trade in factors (in this case, capital) substitutes for trade in goods –the converse of the result in trade theory that trade in goods substitutes for trade in factors. The other section we read here is section 4.4. In that subsection, the important concept is that it is not only the intertemporal rate of substitution (which controls the overall saving rate), but also the substitutability between goods that matters for the current account determination.

We skip the part of chapter 4 that looks at the Ricardian model of terms of trade. We will come back to nontraded goods prices as well as terms of trade when we read chapters 7-8 in USG. The chapters in USG present models that are fairly typical of dynamic, quantitative models with nontraded goods, or with two (or more) traded goods that you see in the literature in recent years.

I think chapter 5 is a valuable chapter. It mostly ought to be review. The first part on complete markets should be a review of 1st-year micro theory. The latter parts on asset choice at an individual level are either review of things we covered in reading assignment 3. The chapter is worth reading and understanding well. It is well-written, and I think helps your and my understanding of how models of uncertainty in international economics work.

The first questions are from Chapter 4

1. Suppose that instead of having a traded and nontraded good, the model of section 4.2 was one of a small open economy that had positive production of two goods, Tea and Nuts (N and T), and it took the international relative price of those goods, *p*, as given. Suppose also that both labor and capital were mobile between sectors within the economy, and the total capital stock and labor supply were fixed, as the chapter assumes is true in the long-run. Show that the return to capital and labor are functions only of *p* and productivity in both sectors. Discuss how this relates to the factor-price equalization theorem in the Hecksher-Ohlin model of international trade (which in a 2-country model in which countries have identical technologies and otherwise have the same assumptions made above, that factor prices are equalized even if labor and capital are not mobile internationally.)

The whole reason that I want you to read section 4.4 is to see in this basic neoclassical model why the real exchange rate affects the current account. That is, the model presented here is not a Keynesian model where changes in exchange rates influence output through a Keynesian demand effect. But there still can be an effect of the real exchange rate on consumption if $\sigma \neq \theta$. I don't want you to get lost in the thicket of derivations in section 4.4. The important thing is to be able to answer this question:

2. If a country expects to have a real appreciation over time in the future, under what circumstances will that make consumption higher (than it would be if the real exchange rate were constant) and the current account balance lower? Explain.

I really like sections 5.1 and 5.2. There should not be anything new here if you have taken the 1st-year PhD sequence in micro, but the examples and closed-form solutions really help intuition. Also, it seems like in work in international macro, people often just take "complete markets" to mean an equilibrium condition that relates home and foreign consumption, but I think it is helpful to understand that there is an underlying model of trade in assets that leads to the equilibrium condition. When there is only a single traded good, complete markets implies the equilibrium condition $C_t = \kappa C_t^*$ in every state of the world at every time, where κ is a constant that depends on initial wealth. As we have discussed previously, and using notation for nominal exchange rates and prices, when the real exchange rate is not constant, in equilibrium complete markets implies the equilibrium condition $\frac{U'(C_t)}{P_t} = \kappa \frac{U'(C_t^*)}{S_t P_t^*}$, where the consumption variables refer to the exact consumption aggregate rather than consumption of a single good. This would be an equilibrium condition in the model of section 5.5 with nontraded goods (which is not an assigned reading), though that result is not derived in the book.

At the end of these notes, I've attached some old lecture slides about this section of Obstfeld-Rogoff, as well as some commentary on the *consumption-real-exchange-rate* anomaly.

Section 5.2.2 can be skipped for our purposes.

3. Suppose there are two agents trading in a market for contingent claims. Agent 1 has a constant income across all states and time, while agent 2 has income that varies across states and time. Agent 2 would benefit from being able to trade in a state-contingent claim market because he could insure his idiosyncratic risk. But what about Agent 1. Would he want to engage in this market and trade with Agent 2? Why or why not?

4. Here we will work through equilibrium allocations in a specific example. There is only one period, and two agents. There are two possible states of the world, and each has equal probability. Prior to the realization of the state, the agents can trade state contingent claims.

   Label the two stats A and B. The price of a claim on state A is $p(A)$ and the price of a claim on state B is $p(B)$. In this problem, we need to choose a numeraire, so let's take $p(B) = 1$.

Agent 1 receives an endowment of 2 in each of state A and B: $Y_A^1 = 2, Y_B^1 = 2$
Agent 2 receives an endowment of 1 in state A and 3 in state B: $Y_A^2 = 1, Y_B^2 = 3$
The superscript refers to the agent, and the subscript to the state.

The two agents have the same expected utility functions:
$U^i = \frac{1}{2}\ln\left(C_A^i\right) + \frac{1}{2}\ln\left(C_B^i\right),\ i = 1,2$

Each agent maximizes utility subject to its budget constraint:
$p(A)C_A^i + p(B)C_B^i = p(A)Y_A^i + p(B)Y_B^i,\ i = 1,2$

i. For each agent *i*, from the first-order conditions, derive the condition that relates the marginal rate of substitution of consumption across the two states to the relative price, $p(A)/p(B)$.

Next, we derive the equilibrium relative price. To do that, we need to use the equilibrium conditions that in each state total consumption equals total endowment:

$C_j^1 + C_j^2 = Y_j^1 + Y_j^2,\ \ j = A, B$

ii. From the first-order conditions for each of the two agents and the two equilibrium conditions, and using the actual numbers for the endowments in each state for each agent given above, derive $p(A)$ (recalling we've set $p(B) = 1$.)

iii. In each state of the world, we find in equilibrium $C_j^1 = \kappa C_j^2,\ \ j = A, B$. From the equations above, solve for κ.

iv. How does this problem relate to question 3? Is it correct to say that in a complete market for contingent claims, agents will smooth consumption?

Read sections 5.3 and 5.4. From these sections, answer this question:

5. Consider an endowment model. Call the "certainty equivalent" price of a claim to a country's stream of endowments the present discounted value of its expected endowment. The actual price differs from the certainty equivalent because of risk. In what way, and why? I am only asking if the equity price will be greater or less than the certainty equivalent, and a very short explanation of why (or what it depends on.)

Ten Important Papers

Backus, David K., and Gregor W. Smith. "Consumption and real exchange rates in dynamic economies with non-traded goods." *Journal of International Economics* 35.3-4 (1993): 297-316.

Benigno, Gianluca, and Christoph Thoenissen. "Equilibrium exchange rates and supply-side performance." *The Economic Journal* 113.486 (2003): C103-C124.

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Lustig, Hanno, and Adrien Verdelhan. "Does incomplete spanning in international financial markets help to explain exchange rates?." *American Economic Review* 109.6 (2019): 2208-2244.

Rogoff, Kenneth. "The purchasing power parity puzzle." *Journal of Economic literature* 34.2 (1996): 647-668.

Econ 872 – Spring 2008

Charles Engel

Model with complete markets

- Trade in state-contingent claims – allow for insurance.
- Is such a model realistic? We want to examine how opportunities for risk sharing affect our analysis.
- As an example, consider an endowment model with idiosyncratic transitory shocks. Suppose home country gets a positive shock:
  - Bonds only model, home runs a current account surplus. Acquires claims on foreign country
  - Risk-sharing, we “share” some of our bonus with foreign country. We run a trade balance surplus, but run a deficit on the net factor payments account, so that current account is zero.

Small-country, 2-period model, 2-state model

- Households take security prices as given, but the country as a whole will also.
- Households maximize

$$u(C_1) + \beta\left[\pi(1)u(C_2(1)) + \pi(2)u(C_2(2))\right] \qquad \pi(1) + \pi(2) = 1$$

- No uncertainty in the 1st-period, two states in 2nd
- An Arrow-Debreu security pays off one unit of consumption in state i
- For convenience, let p(i) be the price of a contingent claim at the beginning of period 2, so p(i)/(1+r) is the price at the beginning of period 1.
- But what is r?

More

- r is defined as the “certain” rate of return. If you pay 1 unit now, you get 1+r next period no matter which state is realized. You get that by buying 1+r claims on all states.
  - What is the cost of buying 1+r claims on all states?

    (1+r)*[(p(1)+p(2))/(1+r)] = p(1) + p(2)

    But this should equal 1: r is defined as the return from paying 1 now and getting 1+r no matter what state is realized.
  - So p(1) + p(2) = 1
    - The whole purpose of this normalization is to get this convenient result.

Budget constraint

- The household budget constraint in first period:

$$\left(\frac{p(1)}{1+r}\right)B_2(1)+\left(\frac{p(2)}{1+r}\right)B_2(2)=Y_1-C_1$$

- In second period:

$$C_2(s)=Y_2(s)+B_2(s) \qquad s=1,2$$

- Combining these two, we get that the present discounted value of consumption equals the pdv of income, where we discount using A-D security prices:

$$C_1+\frac{p(1)C_2(1)+p(2)C_2(2)}{1+r}=Y_1+\frac{p(1)Y_2(1)+p(2)Y_2(2)}{1+r}$$

1^{st}-order condtion

- M.R.S. equals relative price: $$\frac{\beta\pi(s)u'(C_2(s))}{u'(C_1)} = \frac{p(s)}{1+r}$$
- Under certainty we get back familiar condition

  plug in $\pi(s) = p(s) = 1$
- Sum the f.o.c. across all states and we get familiar condition from models with uncertainty:

$$\beta E\left[\frac{u'(C_2)}{u'(C_1)}\right] = \frac{1}{1+r}$$

where we have used $\sum \pi(s) = \sum p(s) = 1$

Equilibrium

- Economy takes prices as given $\dfrac{u'(C_2(1))}{u'(C_2(2))} = \dfrac{p(1)}{\pi(1)}\dfrac{1-\pi(1)}{1-p(1)}$

  If $p(1) > \pi(1)$ then $u'(C_2(1)) > u'(C_2(2))$

  which implies $C_2(1) < C_2(2)$

- Example $u(C) = \ln(C)$ then $C_2(s) = \dfrac{\beta\pi(s)(1+r)}{p(s)} C_1$

  substitute into the budget constraint:

$$CA_1 = Y_1 - C_1 = Y_1 - \frac{1+\frac{1}{1+r}}{1+\beta}\left[\frac{1}{1+\frac{1}{1+r}}\left(\frac{p(1)Y_2(1)}{1+r} + \frac{p(2)Y_2(2)}{1+r} + Y_1\right)\right]$$

2-country, 2-period, S-states model

- The objective function and the budget constraint for households generalizes in the obvious way to the case of S states.
- The same first-order conditions hold for each state.
- Assume utility function: $u(C_t) = \frac{1}{1-\rho} C_t^{1-\rho}$
- 1st-order condition (home country):

$$\frac{p(s)}{1+r} C_1^{-\rho} = \beta \pi(s) (C_2(s))^{-\rho} \qquad \forall s$$

Equilibrium

- Home country $C_2(s) = \left[\pi(s)\beta(1+r)/p(s)\right]^{1/\rho} C_1 \qquad \forall s$

- Foreign country $C_2^*(s) = \left[\pi(s)\beta(1+r)/p(s)\right]^{1/\rho} C_1^* \qquad \forall s$

- Resource constraint $C_1 + C_1^* = Y_1 + Y_1^* \equiv Y_1^W$

$$C_2(s) + C_2^*(s) = Y_2(s) + Y_2^*(s) \equiv Y_2^W(s)$$

- Combining, we get

$$Y_2^W(s) = \left[\pi(s)\beta(1+r)/p(s)\right]^{1/\rho} Y_1^W$$

Contingent claims prices

- Price in period 1

$$\frac{p(s)}{1+r} = \pi(s)\beta\left[\frac{Y_2^W(s)}{Y_1^W}\right]^{-\rho}$$

- Compare two states:

$$\frac{p(s)}{p(s')} = \left[\frac{Y_2^W(s)}{Y_2^W(s')}\right]^{-\rho}\frac{\pi(s)}{\pi(s')}$$

If $Y_2^W(s) < Y_2^W(s')$ then $\frac{p(s)}{p(s')} > \frac{\pi(s)}{\pi(s')}$

Interest rates

- Sum the expression for state-contingent prices across states to get: $$\sum_{s=1}^{S} \frac{p(s)}{1+r} = \frac{\beta}{\left(Y_1^W\right)^{-\rho}} \sum_{s=1}^{S} \pi(s) \left(Y_2^W(s)\right)^{-\rho}$$
- Solve:
$$1+r = \frac{\left(Y_1^W\right)^{-\rho}}{\beta \sum_{s=1}^{S} \pi(s) \left(Y_2^W(s)\right)^{-\rho}} = \frac{\left(Y_1^W\right)^{-\rho}}{\beta E\left(Y_2^W\right)^{-\rho}}$$
    - same as model without state-contingent bonds

Consumption correlation puzzle

- We can now derive

$$\pi(s)\beta\left[\frac{C_2(s)}{C_1}\right]^{-\rho} = \frac{p(s)}{1+r} = \pi(s)\beta\left[\frac{Y_2^W(s)}{Y_1^W}\right]^{-\rho}$$

- This implies

$$\frac{C_1}{Y_1^W} = \frac{C_2(s)}{Y_2^W(s)} = \mu \qquad \forall s$$

- Also

$$\frac{C_1^*}{Y_1^W} = \frac{C_2^*(s)}{Y_2^W(s)} = 1-\mu$$

- All idiosyncratic consumption risk is eliminated

Solve for μ

- Add up total consumption

$$C_1 + \sum_{s=1}^{S} \frac{p(s)C_2(s)}{1+r} = \mu \left[Y_1^W + \sum_{s=1}^{S} \frac{p(s)Y_2^W(s)}{1+r} \right]$$

- From budget constraint:

$$C_1 + \sum_{s=1}^{S} \frac{p(s)C_2(s)}{1+r} = Y_1 + \sum_{s=1}^{S} \frac{p(s)Y_2(s)}{1+r}$$

- So

$$\mu = \left[Y_1 + \sum_{s=1}^{S} \frac{p(s)Y_2(s)}{1+r} \right] \Big/ \left[Y_1^W + \sum_{s=1}^{S} \frac{p(s)Y_2^W(s)}{1+r} \right]$$

Consumption correlations

- The model implies that consumption levels will be perfectly correlated across countries
- What does the real world look like:
  - Backus, Kehoe, Kydland (1992) calculate correlation of HP-filtered consumption for 11 advanced countries relative to the U.S. The average consumption correlation is .19
  - Obstfeld-Rogoff's book reports average correlation of OECD countries with world consumption (35 "benchmark" countries) of .43.
  - For LDCs, it is -.10
- So what?

Output correlations

- In BKK data, the average output correlation is .31
- In Obstfeld-Rogoff, the average OECD country's output correlation with world benchmark is 0.52
- For LDCs it is .05
- In all these measures, consumption correlation is lower than output correlation.
- The puzzle is not that consumption levels are not perfectly correlated. The puzzle is that they are less correlated than output.
  - There is apparently <u>no</u> risk sharing going on.

Contrast complete markets model to bonds only

- Suppose all shocks are iid, and home gets a good shock
- In the bonds only model, in the home country current income rises more than permanent income. Home country saves and runs a CA surplus. The home country acquires claims on foreigners, and therefore is permanently wealthier.
- In the complete markets model, the home country would effectively have traded half of its output for half of foreign output in all states. Risk is completely pooled. So even though GDP is different in the countries, GNP is equal (and equal to half of world output) in all states. The current account is always balanced.

More

- Suppose all shocks are permanent
- It is still the case in the complete markets model that there is complete risk pooling. The current account is always zero.
- In the bonds only model, in the home country permanent output rises the same amount as current output. So consumption increases the same amount as current output, and the current account is zero. There is a difference between the bonds-only and the complete markets model, however. If the home output increase is greater, its wealth increases permanently relative to the foreign country.

When there is a complete market for state-contingent claims, the first-order condition for the home household for choosing (at the initial date) the consumption in state *j* at time *t* is given by:

$$prob(state_t = j,t)U'\left(C_{j,t}\right) = \lambda P_{j,t} Z_{j,t}$$

where $Z_{j,t}$ is the price of a claim on state *j* in period *t*. λ is the Lagrange multiplier on the home household's budget constraint, and can be interpreted as the marginal utility of an additional unit of wealth.

The analogous first-order condition for a foreign household is:

$$prob(state_t = j,t)U'\left(C^*_{j.t}\right) = \lambda^* S_{j,t} P^*_{j.t} Z_{j,t}$$

From these two first-order conditions, we arrive at an equilibrium condition:

$$\frac{U'\left(C_{j,t}\right)}{P_{j,t}} = \frac{\lambda}{\lambda^*} \frac{U^{*\prime}\left(C^*_{j,t}\right)}{S_{j,t} P^*_{j,t}}$$

This equality holds at every date and for every state. Henceforth, I will drop the state subscript. Here is how to interpret this. First, suppose

$\lambda = \lambda^*$, so the marginal utility of wealth is the same for home and foreign households.

Then, one dollar buys $\frac{1}{P_t}$ units of the home consumption basket. So, $\frac{U'(C_t)}{P_t}$ is the marginal utility of a dollar for a home household.

Similarly, one dollar buys $\frac{1}{S_t P_t^*}$ of the foreign consumption basket, so $\frac{U^{*\prime}(C_t^*)}{S_{j,t} P_t^*}$ is the marginal utility of a dollar for the foreign household.

Under complete markets in this case, the marginal utility of a dollar for a home household is equalized with the marginal utility of a dollar for the foreign household.

What is the sense of this? The idea here is that, first, contingent claims are settled in dollars (although it could be in euros and it would not make any difference.) That is, the contingent claim is not paid off in real units of consumption.

Why not? One possible reason is that the consumption basket contains non-traded goods. It would be impossible to settle claims by paying off in non-traded goods.

Another possibility is that there is pricing to market, so that producers have set different prices (when expressed in a common currency) for sale to home and to foreign consumers. If contingent claims could be settled with goods payments, it would circumvent this pricing. We don't want to allow that in the model.

The usual way we think of Arrow-Debreu securities is that they offer consumption insurance. Let's further specialize the example to assume that PPP holds: $P_t = S_t P_t^*$. Then, assuming $\lambda = \lambda^*$, the equilibrium condition implies $U'(C_t) = U^{*\prime}(C_t^*)$. Complete markets equalize the marginal utility of consumption between home and foreign consumers in every date and state.

A question that has arisen in the literature is whether financial markets provide consumption insurance at all. That is, if there were no financial markets at all, then each country would have to consume its own GDP. So the question arises of whether C_t and C_t^* are more highly correlated across country pairs than the corresponding Y_t and Y_t^* (all measured in per capita terms.)

The general finding in the literature is that the GDP correlations are actually higher than the consumption correlations. This is known as the ***consumption correlation puzzle***.

A good reference on work done on this puzzle is:

Obstfeld, Maurice and Kenneth Rogoff. 2000. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" *NBER Macroeconomics Annual* 15, 339-390.

Note that the test of the consumption correlation puzzle assumes PPP holds. In general, complete markets imply $\frac{U'\left(C_{j,t}\right)}{P_{j,t}} = \frac{\lambda}{\lambda^*}\frac{U^{*\prime}\left(C^*_{j,t}\right)}{S_{j,t}P^*_{j,t}}$. Why is there not perfect consumption correlation?

The intuition is that there are two different states where it makes sense to receive a state contingent payoff for a home resident. The first is when marginal utility of consumption is relatively high. The second is when prices are relative cheap at home ($P_{j,t}$ is low relative to $S_{j,t}P_{j,t}^{*}$.)

What are the testable implications? Assume home and foreign households have the same CRRA utility function: $U\left(C\right)\equiv\frac{1}{1-\theta}C^{1-\theta}$.

Then $U'\left(C\right)\equiv C^{-\theta}$.

In the risk sharing condition, we can write: $\frac{C_{j,t}^{-\theta}}{P_{j,t}}=\frac{\lambda}{\lambda^{*}}\frac{C_{j,t}^{*-\theta}}{S_{j,t}P_{j,t}^{*}}$. Taking logs and rearranging, we have:

$$c_{j,t} - c^*_{j,t} = k + \frac{1}{\theta}\left(s_{j,t} + p^*_{j,t} - p_{j,t}\right) .$$

This equation implies relative home to foreign consumption should be perfectly positively correlated with the real exchange rate if markets are complete. In fact, in the data for advanced countries, the correlation is very weak and frequently negative. This is know as the ***Backus-Smith puzzle*** or the ***consumption-real-exchange-rate anomaly***.

A good paper to read on this is:

Corsetti, Giancarlo: Luca Dedola; and, Sylvain Leduc. 2008. "International Risk Sharing and the Transmission of Productivity Shocks." *Review of Economic Studies* 75, 443-473.

Note that the test of this condition does not really offer us any insight into whether there is *any* risk sharing at all. It would be interesting to pursue this further, and try to develop a test of the degree of risk sharing when PPP does not hold.

The Obstfeld-Rogoff Handbook paper concentrates on a model in which only a bond is traded. That makes it seem similar to the non-stochastic model, but actually there is no risk to share under certainty. When only a bond is traded, idiosyncratic shocks lead to permanent redistribution of wealth.

Answers to Questions on Obstfeld-Rogoff, Chapters 4-5

1. Suppose that instead of having a traded and nontraded good, the model of section 4.2 was one of a small open economy that had positive production of two goods, Tea and Nuts (N and T), and it took the international relative price of those goods, *p*, as given. Suppose also that both labor and capital were mobile between sectors within the economy, and the total capital stock and labor supply were fixed, as the chapter assumes is true in the long-run. Show that the return to capital and labor are functions only of *p* and productivity in both sectors. Discuss how this relates to the factor-price equalization theorem in the Hecksher-Ohlin model of international trade (which in a 2-country model in which countries have identical technologies and otherwise have the same assumptions made above, that factor prices are equalized even if labor and capital are not mobile internationally.)

<u>Answer</u>:

Equations (2)-(5) on page 205 would still hold in this model:

(2) $A_T f'(k_T) = r$

(3) $A_T\left[f(k_T) - f'(k_T)k_T\right] = w$

(4) $pA_N g'(k_N) = r$

(5) $pA_N\left[f(k_N) - f'(k_N)k_N\right] = w$

In the Balassa-Samuelson model presented in the book, capital is mobile internationally, and the country takes the rate of return, *r*, as given. Good *N* is the nontraded good, and its price, *p*, along with the wage, *w*, and the capital labor ratio in each sector, k_T and k_N, are determined by these four equations as functions of *r*, A_T, and A_N.

In the question posed above, good *N* is a traded good, and the small open economy takes its price, *p*, relative to good *T* as given. Capital is not traded internationally, so its rate of return is determined within the country. The four equations determine *r*, *w*, k_T and k_N as functions of *p*, A_T, and A_N. In a two-country model, if the two countries have identical technologies (that is, the same *f* and *g* functions, and the same productivity levels, A_T, and A_N), and the goods are freely traded, so the two countries face the same *p*, then the four equations will determine identical *r*, *w*, k_T and k_N as functions of *p*, A_T, and A_N. This means that wages and returns to capital will be equal in the two countries.

2. If a country expects to have a real appreciation over time in the future, under what circumstances will that make consumption higher (than it would be if the real exchange rate were constant) and the current account balance lower? Explain.

<u>Answer</u>:

If the country expects future appreciation, the price of its nontraded goods is rising over time relative to the price of traded goods. If bonds are traded that pay off in real terms in units of the traded good, then the consumption real interest rate this country faces is lower than the international borrowing/lending rate. Maybe its easiest to think of this in a case where nominal prices of traded goods are completely stabilized by monetary policy. The real rate of return in terms of traded goods is then just equal to the nominal interest rate. Then the increase in the relative price of nontraded goods implies inflation in the consumer price index, which lowers the real rate of interest. The lower real interest rate encourages higher overall consumption today through intertemporal substitution.

But a rising price of nontraded goods implies that nontraded goods are cheaper today than they will be in the future. Consumers will buy relatively more nontraded goods today, and relatively fewer traded goods.

The consumption of traded goods is driven higher today through the intertemporal substitution effect, and lower today through the substitution of nontraded for traded consumption. If the intertemporal elasticity of substitution is higher than the intratemporal substitution, then the former effect dominates and consumption of tradables rises and the current account falls.

3. Suppose there are two agents trading in a market for contingent claims. Agent 1 has a constant income across all states and time, while agent 2 has income that varies across states and time. Agent 2 would benefit from being able to trade in a state-contingent claim market because he could insure his idiosyncratic risk. But what about Agent 1. Would he want to engage in this market and trade with Agent 2? Why or why not?

Answer:

Yes, Agent 1 would want to engage in trade. Trade in contingent claims will increase Agent 1's wealth. During bad times for Agent 2, the value of Agent 1's endowment in that state is higher, while in states where Agent 2 has high endowment, the value of Agent 1's endowment is lower. But the former dominates the latter because Agent 2 has concave utility. On the margin he values an additional unit of consumption more when his endowment is low. Agent 1 is happy to trade in the contingent claims market even though it makes his consumption more volatile because trading makes him wealthier and able to achieve higher levels of expected utility.

4. Here we will work through equilibrium allocations in a specific example. There is only one period, and two agents. There are two possible states of the world, and each has equal probability. Prior to the realization of the state, the agents can trade state contingent claims.

   Label the two stats A and B. The price of a claim on state A is $p(A)$ and the price of a claim on state B is $p(B)$. In this problem, we need to choose a numeraire, so let's take $p(B)=1$.

   Agent 1 receives an endowment of 2 in each of state A and B: $Y_A^1 = 2, Y_B^1 = 2$

Agent 2 receives an endowment of 1 in state A and 3 in state B: $Y_A^2 = 1, Y_B^2 = 3$
The superscript refers to the agent, and the subscript to the state.

The two agents have the same expected utility functions:
$U^i = \frac{1}{2}\ln\left(C_A^i\right) + \frac{1}{2}\ln\left(C_B^i\right),\ i = 1,2$

Each agent maximizes utility subject to its budget constraint:
$p(A)C_A^i + p(B)C_B^i = p(A)Y_A^i + p(B)Y_B^i,\ i = 1,2$

i. For each agent *i*, from the first-order conditions, derive the condition that relates the marginal rate of substitution of consumption across the two states to the relative price, $p(A)/p(B)$.

Next, we derive the equilibrium relative price. To do that, we need to use the equilibrium conditions that in each state total consumption equals total endowment:

$C_j^1 + C_j^2 = Y_j^1 + Y_j^2,\ j = A, B$

ii. From the first-order conditions for each of the two agents and the two equilibrium conditions, and using the actual numbers for the endowments in each state for each agent given above, derive $p(A)$ (recalling we've set $p(B) = 1$.)

iii. In each state of the world, we find in equilibrium $C_j^1 = \kappa C_j^2,\ j = A, B$. From the equations above, solve for κ.

iv. How does this problem relate to question 3? Is it correct to say that in a complete market for contingent claims, agents will smooth consumption?

<u>Answer</u>:

i. The first order conditions are $\frac{p(A)}{p(B)} = \frac{C_B^1}{C_A^1}$ and $\frac{p(A)}{p(B)} = \frac{C_B^2}{C_A^2}$.

ii. We have $C_A^1 + C_A^2 = Y_A^1 + Y_A^2 = 3$ and $C_B^1 + C_B^2 = Y_B^1 + Y_B^2 = 5$. Use the first-order conditions that tell us $C_B^1 = p(A)C_A^1$ and $C_B^2 = p(A)C_A^2$. Use these to substitute into the second equation to write it as $p(A)C_A^1 + p(A)C_A^2 = 5$. Divide this by the first equation, $C_A^1 + C_A^2 = 3$, and we find $p(A) = \frac{5}{3}$.

iii. The budget constraint for person 1 is:

$$\frac{5}{3}C_A^1 + C_B^1 = \frac{5}{3}2 + 2 = \frac{16}{3}$$

The budget constraint for person 2 is:

$$\frac{5}{3}C_A^2 + C_B^2 = \frac{5}{3} + 3 = \frac{14}{3}.$$

We can conclude that $C_j^1 = \frac{8}{7}C_j^2$ for $j = A, B$. We see that $\kappa = \frac{8}{7}$.

iv. Here, person 1 does want to participate in markets, even though his consumption becomes more volatile. Under autarky, his expected utility is $\ln(2) \cong 0.6931$. Under trade, in each state, person 1 gets $\frac{8}{15}$ of the world endowment, which translates into $\frac{24}{15}$ in state A and $\frac{40}{15}$ in state B. His expected utility is $\frac{1}{2}\ln\left(\frac{24}{15}\right) + \frac{1}{2}\ln\left(\frac{40}{15}\right) \cong 0.7254$, His expected utility is higher under free trade.

5. Consider an endowment model. Call the "certainty equivalent" price of a claim to a country's stream of endowments the present discounted value of its expected endowment. The actual price differs from the certainty equivalent because of risk. In what way, and why? I am only asking if the equity price will be greater or less than the certainty equivalent, and a very short explanation of why (or what it depends on.)

Answer:

Equation (59) gives the answer to this question. The price under uncertainty is equal to the certainty equivalent price plus a term involving the covariance of the marginal utility at time *s* with the endowment at time *s*. If this covariance is positive, the price is greater than the certainty equivalent price. The intuition is that the endowment acts like insurance in this case – endowments are high the marginal utility of consumption is high, so consumption is low (given the concavity of utility.) When the covariance is negative, the high endowment occurs when consumption is high, so the asset is risky, and the price is lower than the certainty equivalent price.

Obstfeld-Rogoff, chapters 9 and 10

In chapter 9, I only ask that you read through section 3.3. Actually, sections 3.4 and 3.5 touch on a number of really interesting topics: fixed versus floating exchange rates, optimal currency areas, determinants of monetary credibility, reputation, "conservative" central banker, political business cycles, central bank independence, pegging the exchange rate to gain credibility, international monetary policy coordination. But we just don't have time in the semester to get into those topics. If those are areas you want to do research in, it pays to read these sections. More recent papers have much better microfoundations, but these sections of Obstfeld-Rogoff will give you some good intuition without getting bogged down into some of the particulars of the way in which the microfounded models are specified.

As you read the material on the Dornbusch model, you will be able to see how similar the simple 3-equation New Keynesian model is. In fact, if we used the Dornbusch version of price adjustment instead of the Calvo version, the models would be almost identical. In the NK model, the monetary policy instrument is the interest rate, while in the Dornbusch model it is the money supply, but in the Dornbusch model, changes in the money supply affect the macroeconomy through their effects on real interest rates. One difference is that in the NK model, monetary policy is set by an instrument rule (that is, a rule that says how the monetary policy instrument, the interest rate, responds to inflation), while in the Dornbusch model the monetary policy is completely exogenous. However, it would be easy to modify the Dornbusch model to endogenize the monetary policy – for example, by positing that the growth in the money supply responds to inflation. (Actually, I deal with both differences in my 2019 Journal of Monetary Economics paper, "Real exchange rate convergence: The roles of price stickiness and monetary policy." The point of the paper is not to compare the NK model and the Dornbusch model, but I can see how that comparison was motivating my thoughts.)

When you get to the section on empirical fit of exchange rate models, I want to mention a few things. First, we know now from Engel-West that many exchange-rate models have the property that if the model is true, they cannot forecast the exchange rate out of sample. The first point is that you may wonder about the Meese-Rogoff exercise, which is perhaps better labeled "out of sample fit". It is described as forecasting but giving the forecaster the benefit of knowing the realization of the macro variables that are supposed to be driving the exchange rate. The Engel-West paper deals with pure forecasting, so this finding of Meese-Rogoff seems even more puzzling. However, note that if you are doing the "random walk forecast", which is the standard by which the models are measured, so the forecast of s_{t+1} is simply s_t, the forecaster does have the advantage of knowing s_t. The model-based forecasts never have this privilege. Any permanent shocks to unmeasured variables (that is, variables in the error term) are never accounted for, so the forecasts can go very wrong if we never let the forecaster know the current exchange rate.

I'm making an exception to the rule that I don't list any of my own papers in the Ten Important Papers section, because, though the paper is not really saying much original, it does compile some of the ways in which we can be less sad about the performance of exchange rate models. I am referring to the paper with Nelson Mark and Ken West called "Exchange Rate Models are Not as Bad as You Think." I make a related point about "forecasts" that give the forecaster the "advantage" of knowing the actual realization of the macro variables. In fact, I give a somewhat trivial and maybe frustrating example. In the Meese-Rogoff exercise, you are allowed to use the realization of interest rate differentials, $i_t - i_t^*$, to fit the exchange rate. Under covered interest rate parity (which held well in the data at the time this paper was written back in 2007), the interest differential equals the forward premium, $f_t - s_t$. With a little algebra, I show that if instead of letting the forecaster know the ex post realization of the forward premium, you let him know the ex post value of the forward rate itself, then the fit of the models out of sample is great.

The paper lists several ways in which we could test models, such as the response to announcements, looking at whether exchange rates forecast macro variables, etc. That paper was written a long time ago, but now I feel we know even more. News certainly plays a role in driving exchange rates, which this paper talks about. The Chahrour et al. paper, which was on the list of important papers for the section on the simple NK model, takes a promising approach to seeing how news might affect exchange rates.

Since we wrote the 2007 paper, there has been a lot of focus on financial markets. Risk premiums may play a big role in driving exchange rates, though I don't think a fully satisfactory general equilibrium model has been built yet. There is a lot of talk about "risk on, risk off" episodes. "Risk on" refers to periods in which investors are willing to take risky positions, while "risk off" is when they pull back from such positions. People talk about this as if it is a change in preferences, but I find that very unsatisfying. I think it is related to balance sheet constraints and how the business cycle affects the willingness of financial intermediaries to take on risk.

Another, related, strand of literature is on demand for liquidity, where again I think the most promising avenue is to think of liquidity demand by financial intermediaries.

While a full general equilibrium model with all three of these things going on (news, risk appetite and liquidity demand) is not out there, the field is moving in that direction. We will look at some of the recent work in the second half of the class. Empirically, the paper by Obstfeld and Zhou, which I've included on the ten important papers reading list, tries to incorporate all three of these channels, as well as the traditional interest rate effects. There are three important caveats: First, these channels are measured imperfectly. Second, their empirical work can be criticized as simply relating some endogenous financial variables to another endogenous financial variable (the exchange rate). Third, these financial variables have outsized explanatory power for advanced country exchange rates in the 21st century when financial shocks have played a major role. With those caveats in mind (which the authors talk about), it is striking to see that some of the simple regression models for explaining movements in exchange rates have R-squared values nearing 0.50. It seems like there is progress toward understanding exchange rate movements!

For your homework for Chapter 9, I only ask you to derive some of the equations in the chapter:

1. Derive the unnumbered equation following equation (17) of Chapter 9.
2. Derive equation (20) of Chapter 9.

3. Derive the unnumbered equation between equations (23) and (24) in Chapter 9.

In Chapter 10, only read through section 10.1 (that is, up through page 689.) The rest of the chapter is interesting, but we don't have time in the semester to go through the whole thing. Chapter 10 presents the "father" of New Keynesian open-economy models. The model presented is a version of Obstfeld and Rogoff's 1995 *JPE* paper. It differs in many ways from the New Keynesian models that have evolved since then: there are no explicitly random variables (there is perfect foresight); nominal prices are sticky, but are set each period, one period in advance (as compared to the Calvo price-setting mechanism); people hold money (as opposed to the "cashless" economies in most recent models); money supplies are the instrument of monetary policy (rather than interest rates); monetary policy is exogenous (rather than set endogenously according to some rule).

You shouldn't spend too much time trying to derive each equation. Even the authors warn you not to. If you have time, and are unsure where an equation came from, then certainly try to work it out. The questions I ask are aimed at teasing out aspects of this model that differ from more recent NK open-economy models; they are about economic mechanisms, not algebra.

4. Just to make sure we understand basic things, the real exchange rate is constant in this model even though nominal prices are sticky. What assumptions give us that?

5. The authors emphasize that markets are not complete in their model, and in fact are skeptical of models that assume complete markets. Yet there is no uncertainty, so at each date there is only one possible state. Since there is a bond traded each period, it seems like markets are complete – one bond, one state. What aspect of the analysis in this chapter makes this like an incomplete-markets model?

6. In the model, if prices are flexible, changes in the money supply are neutral (that is, they do not affect real variables.) If prices are sticky, of course money does affect real variables in the short run. But changes in the money supply also affect real variables in the long run. Why?

7. Related to the last question, suppose there were complete markets. Maybe think of the money supply as being a random variable that follows a random walk. Would a shock to the money supply affect real variables in the long run? Explain why or why not.

8. Why does an increase in the global money supply raise welfare in this model?

9. Does this mean that there is a (global) monetary policy rule that can be followed that will raise average output?

In terms of important papers to read, our section on the 3-equation model already listed some of the important NK open-economy papers. And those that pertain to optimal monetary policy will be on a later list. Here, besides the two papers I mentioned above, I am listing eight open-economy NK papers that are well-cited and widely read, but that are not optimal monetary policy papers.

Ten Important Papers

Adolfson, Malin, et al. "Bayesian estimation of an open economy DSGE model with incomplete pass-through." *Journal of International Economics* 72.2 (2007): 481-511.

Benigno, Gianluca. "Real exchange rate persistence and monetary policy rules." *Journal of Monetary Economics* 51.3 (2004): 473-502.

Carvalho, Carlos, and Fernanda Nechio. "Aggregation and the PPP puzzle in a sticky-price model." *American Economic Review* 101.6 (2011): 2391-2424.

Corsetti, Giancarlo, André Meier, and Gernot J. Müller. "Fiscal stimulus with spending reversals." *Review of Economics and Statistics* 94.4 (2012): 878-895.

Engel, Charles, et al. "Exchange rate models are not as bad as you think [with comments and discussion]." *NBER macroeconomics annual* 22 (2007): 381-473.

Kollmann, Robert. "The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation." *Journal of International Economics* 55.2 (2001): 243-262.

Monacelli, Tommaso. "Into the Mussa puzzle: monetary policy regimes and the real exchange rate in a small open economy." *Journal of International Economics* 62.1 (2004): 191-217.

Obstfeld, Maurice, and Haonan Zhou. *The global dollar cycle*. No. w31004. National Bureau of Economic Research, 2023.

Schmitt-Grohé, Stephanie, and Martín Uribe. "The effects of permanent monetary shocks on exchange rates and uncovered interest rate differentials." *Journal of International Economics* 135 (2022): 103560.

Smets, Frank, and Raf Wouters. "An estimated dynamic stochastic general equilibrium model of the euro area." *Journal of the European economic association* 1.5 (2003): 1123-1175.

Answers to questions from reading assignment #6:

1. Derive the unnumbered equation following equation (17) of Chapter 9.

Answer:

For any time after time 0 in this derivation, the relevant money supply is $\overline{m}'$. So, equation (16) should be written as:

$$e_t = \overline{m}' + \overline{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}\left(q_t - \overline{q}\right)$$

Then, for equation (13), translate time *s* into time *t*, and time *t* into time 0, so write (13) as

$$q_t - \overline{q} = \left(1-\psi\delta\right)^t \left(q_0 - \overline{q}\right)$$

Then, we have from (12) that $q_0 - \overline{q} = e_0 - \overline{m} - \overline{q}$, and equation (17) gives us:

$$e_0 - \overline{m} - \overline{q} = \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}\left(\overline{m}' - \overline{m}\right).$$

Making the series of substitutions from the equations above, we get:

$$\begin{aligned} e_t &= \overline{m}' + \overline{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}\left(1-\psi\delta\right)^t \left[\frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}\left(\overline{m}' - \overline{m}\right)\right] \\ &= \overline{m}' + \overline{q} + \frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}\left(1-\psi\delta\right)^t \left(\overline{m}' - \overline{m}\right) \end{aligned}$$

2. Derive equation (20) of Chapter 9.

Answer:

We have from (18):

$$e_0 - \left(e_0^{flex}\right)' = \frac{1-\phi\delta}{1+\psi\delta\eta}\left(q_0 - \overline{q}\right)$$

We'll use $q_0 = e_0 - p_0^{flex}$ and $\overline{q} = e_0^{flex} - p_0^{flex}$, so we can write $q_0 - \overline{q} = e_0 - e_0^{flex}$.
Then we have:

$$e_0 - \left(e_0^{flex}\right)' = \frac{1-\phi\delta}{1+\psi\delta\eta}\left(e_0 - e_0^{flex}\right)$$

Then rearrange:

$$\left(1+\psi\delta\eta\right)\left[e_0 - \left(e_0^{flex}\right)'\right] = \left(1-\phi\delta\right)\left(e_0 - e_0^{flex}\right)$$

$$\left(\phi\delta+\psi\delta\eta\right)e_0 - \left(1+\psi\delta\eta\right)\left(e_0^{flex}\right)' = -\left(1-\phi\delta\right)e_0^{flex}$$

$$\left(\phi\delta+\psi\delta\eta\right)e_0 - \left(1+\psi\delta\eta\right)\left(e_0^{flex}\right)' + \left(1-\phi\delta\right)\left(e_0^{flex}\right)' = -\left(1-\phi\delta\right)e_0^{flex} + \left(1-\phi\delta\right)\left(e_0^{flex}\right)'$$

$$\left(\phi\delta+\psi\delta\eta\right)\left(e_0 - \left(e_0^{flex}\right)'\right) = \left(1-\phi\delta\right)\left(\left(e_0^{flex}\right)' - e_0^{flex}\right)$$

$$e_0 - \left(e_0^{flex}\right)' = \frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}\left(\left(e_0^{flex}\right)' - e_0^{flex}\right)$$

3. Derive the unnumbered equation between equations (23) and (24) in Chapter 9.

<u>Answer</u>

In our answer to question (2), we derived:

$$e_0 - \left(e_0^{flex}\right)' = \frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}\left(\left(e_0^{flex}\right)' - e_0^{flex}\right)$$

Then, equation (22) gives us

$\left(e_0^{flex}\right)' - e_0^{flex} = \eta\left(\mu' - \mu\right)$, so we have:

$$e_0 - \left(e_0^{flex}\right)' = \eta\left(\frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}\right)\left(\mu' - \mu\right)$$

Set $t = 0$ in equation (23):

$$i_0 - \left(i^* + \mu\right) = \left(\mu' - \mu\right) - \psi\delta\left[e_0 - \left(e_0^{flex}\right)'\right]$$

Plug in the previous equation and we have

$$i_0 - \left(i^* + \mu\right) = \left(\mu' - \mu\right) - \psi\delta\eta\left(\frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}\right)\left(\mu' - \mu\right) = \phi\delta\left(\frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}\right)\left(\mu' - \mu\right)$$

In Chapter 10,

4. Just to make sure we understand basic things, the real exchange rate is constant in this model even though nominal prices are sticky. What assumptions give us that?

Answer:

The model assumes that the law of one price holds for both goods, and that the consumption baskets are identical in the two countries. Those imply purchasing power parity, or a constant real exchange rate.

5. The authors emphasize that markets are not complete in their model, and in fact are skeptical of models that assume complete markets. Yet there is no uncertainty, so at each date there is only one possible state. Since there is a bond traded each period, it seems like markets are complete – one bond, one state. What aspect of the analysis in this chapter makes this like an incomplete-markets model?

Answer:

It is because they look at the effects of a completely unanticipated shock to the money supply. There is no "insurance" against the wealth effects of this shock when there is only a non-state-contingent bond traded. You really could not introduce complete markets into this model easily, because this completely unanticipated event is like an event that had zero probability of occurring, but happened anyway.

6. In the model, if prices are flexible, changes in the money supply are neutral (that is, they do not affect real variables.) If prices are sticky, of course money does affect real variables in the short run. But changes in the money supply also affect real variables in the long run. Why?

Answer:

As the book explains, monetary shocks have real effects initially that might cause the country to run a current account surplus or deficit. But a current account imbalance changes the wealth distribution between home and foreign countries, and so has a permanent effect on real variables.

7. Related to the last question, suppose there were complete markets. Maybe think of the money supply as being a random variable that follows a random walk. Would a shock to the money supply affect real variables in the long run? Explain why or why not.

Answer:

In this case, the answer is there would be no long-run real effect. Under complete markets, there is no redistribution of wealth.

8. Why does an increase in the global money supply raise welfare in this model?

Answer:

In this model, output is produced at inefficiently low levels because it is produced by monopolists. A monetary shock that raises the output level has the potential to increase welfare as it raises output toward the efficient level.

9. Does this mean that there is a (global) monetary policy rule that can be followed that will raise average output?

Answer:

A rule would say how the monetary policy responds to variables in the economy. If the policymaker tried to follow a rule of setting higher money supplies in order to increase output, firms would simply set higher nominal prices in advance, leaving real variables unchanged. The policymaker can only increase output if it surprises markets with an increase, but they cannot surprise markets every period.

Chapters 2-4 of Uribe and Schmitt-Grohe

Chapter 2 (read the whole chapter)

This chapter is pretty straightforward and should not be a problem to read. I just want to make a couple of comments about the tests of the model presented at the very end of the chapter.

Under the assumptions of the model, we get a simple looking present-value formula for the current account, given in equation (2.27):

$$ca_t = -\sum_{j=1}^{\infty} \frac{E_t\left(\Delta y_{t+j}\right)}{\left(1+r\right)^j}$$

One implication of this formula is that the current account in the model is forward looking. It contains information that should help predict future changes in output. However, if the model is true, it is actually saying something much stronger, which is that if we want to predict the discounted sum of future growth rates of output, $-\sum_{j=1}^{\infty} \frac{\Delta y_{t+j}}{\left(1+r\right)^j}$, the only useful information is the current account at time *t*. In other words, if you knew the current account at time *t* and any other information, that other information would be superfluous if you were trying to predict $-\sum_{j=1}^{\infty} \frac{\Delta y_{t+j}}{\left(1+r\right)^j}$. That observation is the basis for the tests presented at the end of the chapter.

Think about this in relation to my paper with West, where we said that if the exchange rate can be represented as a present value of future fundamentals, we could use the exchange rate to help predict those fundamentals. But why did we now implement tests such as those in this chapter? If the model is true, the exchange rate should incorporate all of the information that is useful in predicting the present discounted value of the fundamentals that drive the exchange rate. Our view was that, unlike the simple model of the current account in this chapter, we needed to acknowledge that we did not know what all of the fundamentals are. In other words, in the model of this chapter, for the current account, the only thing that matters is output, and so ca_t is exactly minus the present discounted value of expected future growth rates of output. In the exchange rate model, the exchange rate is exactly the present discounted value of some linear combination of fundamentals, but since we admit we don't know all of the fundamentals that should be in that present value sum, we cannot implement the exact test like in this chapter. All we can say is that the exchange rate should be helpful in predicting some of the fundamentals.

The other thing that I want to point out is something people pointed out about these present value tests back in the late 1980s. Although they are intuitively interesting, they really are rather elaborate ways of testing the Euler equation. In this model, the Euler equation turns out to be just $E_t c_{t+1} = c_t$ (because the chapter assumes $\beta\left(1+r\right)=1$. You could test that by regressing c_{t+1} on c_t and testing the null that the intercept is equal to zero and the slope equals one. (By the way, here you would need to use the Dickey-Fuller statistic to test the slope coefficient.)

That leads to the first question:

1. The test of the model proposed in equation (2.31) seems to be a conclusion drawn from the present-value model. However, show that this test is equivalent to a test of $E_t c_{t+1} = c_t$, if we just make use of the national income accounting identities: $ca_t = -\left(d_t - d_{t-1}\right)$, and $ca_t = y_t - c_t - rd_{t-1}$.

Chapter 3 of Uribe and Schmitt-Grohe (read the whole chapter)

As with chapter 2, this chapter is pretty basic and is almost surely review. Let me just ask three simple questions to make sure you've read the chapter.

2. Consider the model with investment driven by productivity shocks that are not entirely transitory, and with no adjustment costs to changing the capital stock. If productivity rises today, there are two reasons why output is expected to be higher next period (relative to what was expected prior to realization of the shock.) What are they?

3. Explain Principle I on page 93.

4. Explain Principle II on page 99.

Chapter 4 of Uribe and Schmitt-Grohe. Read sections 4.1-4.9. You can skim section 4.10, but don't spend any time on the details. Skip section 4.11 entirely as we don't have time to cover it. Skim section 4.12. Read section 4.13.

Mostly there are not a lot of new models or theoretical insights in this chapter. It is mostly about how to operationalize a dynamic, stochastic model so that it can be examined quantitatively and compared to data. Presumably, you've already seen a lot of this material in your macro core class or experienced it yourself as you've written papers. However, there are a few new twists, and my questions concern those.

5. Explain why, as in section 4.1.1, the authors introduce a debt-elastic interest rate to "induce stationarity." Why do we need to introduce this feature in order to induce stationarity, and why do we want to induce stationarity?

6. In section 4.3, the paper uses the GHH form of preferences. One reason is "Because employment is procyclical, the prediction [that persistent positive productivity shocks result in a decline n employment] is counterfactual under the hypothesis that disturbances in total factor productivity are a major source of business cycles." Explain why these shocks result in a decline in employment. Also, another reason GHH preferences are widely used is for long-run considerations. As wealth in the U.S., for example, has increased greatly over the past century, what prediction about labor supply does this form of preferences capture (or, what prediction that is inconsistent with data does it avoid)?

7. In equation (4.42), on the left-hand-side of the equation, there is a term that represents the expected value of the portfolio of state-contingent bonds. Yet budget constraints must hold every period, and in every state in each period – that is, households always need to obey their budget constraint, not just their expected budget constraint. Explain the meaning of the expectation in this budget constraint.

8. Consider equation (4.49), which holds under complete markets. Suppose, however, that as we have discussed in class, there is more than one traded good, and purchasing power parity does not hold (that is, the real exchange rate is not constant.) How would this equation be modified? Next, suppose we made this modification, and preferences were modeled as GHH preferences. How might this case be offered as a possible resolution of the Backus-Smith puzzle?

9. The first equation on page 143 presents an equation for the current account balance when markets are complete. Clearly the current account balance need not equal zero. We often talk about the current account balance as referring to borrowing or lending, yet in the complete markets model, the relative wealth positions of the two countries is constant over time. How can you explain a non-zero current account in this case? To be concrete, suppose at time 0 there is no portfolio carried over from the past, so $s_0 = 0$. But suppose $s_1 < 0$ (where, as on the bottom of page 142, $s_t = E_t q_{t+1} b_{t+1}$), so the country runs a current account deficit in period 1. Why does this not imply that the country has become a debtor in period 1, so that its wealth falls relative to the lending country?

10. At the bottom of page 169, in the section on global solutions, the authors assume $\beta(1+r) < 1$. Explain in economic terms why this assumption has been made.

Finally, one thing that I want to note is that the two different cases this chapter looks like are either trade in a non-state-contingent bond, or trade in a complete set of contingent claims. There are obviously intermediate cases. Later in the semester, we will consider models of portfolio choice when markets are not complete. Also, the bonds that are traded in the incomplete markets case are all one-period bonds. In that case, there is no difference between the accumulation of (net) foreign assets and changes in the value of the country's net foreign asset position. I mean, there is no role for changes in the value of longer-lived assets to influence a country's net foreign asset position. In practice, these "valuation effects" may be important in understanding changes in net foreign asset positions.

Ten Important Papers:

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. "International real business cycles." *Journal of political Economy* 100.4 (1992): 745-775.

Baxter, Marianne, and Mario J. Crucini. "Business Cycles and the Asset Structure of Foreign Trade." *International Economic Review* 36.4 (1995): 821-854.

Fernández-Villaverde, Jesús, et al. "Risk matters: The real effects of volatility shocks." *American Economic Review* 101.6 (2011): 2530-2561.

Garcia-Cicco, Javier, Roberto Pancrazi, and Martin Uribe. "Real business cycles in emerging countries?." *American Economic Review* 100.5 (2010): 2510-2531.

Gourinchas, Pierre-Olivier, and Hélène Rey. "External adjustment, global imbalances, valuation effects." *Handbook of international economics*. Vol. 4. Elsevier, 2014. 585-645.

Lane, Philip R., and Gian Maria Milesi-Ferretti. "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004." *Journal of international Economics* 73.2 (2007): 223-250.

Mendoza, Enrique G. "Real business cycles in a small open economy." *The American Economic Review* (1991): 797-818.

Nason, James M., and John H. Rogers. "The present-value model of the current account has been rejected: Round up the usual suspects." *Journal of International Economics* 68.1 (2006): 159-187.

Neumeyer, Pablo A., and Fabrizio Perri. "Business cycles in emerging economies: the role of interest rates." *Journal of monetary Economics* 52.2 (2005): 345-380.

Zucman, Gabriel. "The missing wealth of nations: Are Europe and the US net debtors or net creditors?." *The Quarterly journal of economics* 128.3 (2013): 1321-1364.

Answers to questions on chapters 2-4 of Uribe and Schmitt-Grohe

1. The test of the model proposed in equation (2.31) seems to be a conclusion drawn from the present-value model. However, show that this test is equivalent to a test of $E_t c_{t+1} = c_t$, if we just make use of the national income accounting identities: $ca_t = -\left(d_t - d_{t-1}\right)$, and $ca_t = y_t - c_t - rd_{t-1}$.

Answer:

Equation (2.31) says:

$$E_t ca_{t+1} - (1+r) ca_t - E_t \Delta y_{t+1} = 0$$

This can be derived by first rearranging the identity $ca_t = y_t - c_t - rd_{t-1}$ as:

$$c_t = y_t - ca_t - rd_{t-1}$$

Then advancing the time period by one period and taking expectations, we have:

$$E_t c_{t+1} = E_t y_{t+1} - E_t ca_{t+1} - rd_t$$

Subtracting this equation from the one above it, we have:

$$c_t - E_t c_{t+1} = -E_t \Delta y_{t+1} + E_t ca_{t+1} - ca_t + r\left(d_t - d_{t-1}\right)$$

Replace the last term in this equation using the identity $ca_t = -\left(d_t - d_{t-1}\right)$:

$$c_t - E_t c_{t+1} = -E_t \Delta y_{t+1} + E_t ca_{t+1} - ca_t - r \cdot ca_t$$

The test of the Euler equation is the test that $c_t - E_t c_{t+1} = 0$,which from the equation above is equivalent to testing $-E_t \Delta y_{t+1} + E_t ca_{t+1} - (1+r) ca_t = 0$, which is equation (2.31).

Chapter 3 of Uribe and Schmitt-Grohe

2. Consider the model with investment driven by productivity shocks that are not entirely transitory, and with no adjustment costs to changing the capital stock. If productivity rises today, there are two reasons why output is expected to be higher next period (relative to what was expected prior to realization of the shock.) What are they?

Answer:

Because productivity shocks are not entirely transitory, if there is an increase in productivity today, it is expected to be higher next period. If productivity is expected to be higher next period, there is a direct effect of raising expected output for given factor inputs. In addition, when firms expect higher future productivity, they increase investment this period, which leads to a higher capital stock in the next period, which also increases output.

3. Explain Principle I on page 93.

Answer:

When productivity shocks are more persistent, the impacts on both consumption and investment this period are greater. With more persistence, the productivity increase has a greater effect on permanent income, which leads to a greater increase consumption. Also, if the productivity process is persistent, then the payoff to investment this period will persist for a longer duration, increasing the incentive to invest more this period. The larger the change in consumption and investment to a positive productivity shock, the more likely a current account deficit as it becomes more likely the increase in initial consumption plus investment exceeds the increase in initial income.

4. Explain Principle II on page 99.

Answer:

A higher cost of adjustment of the capital stock mutes the response of investment. Therefore, in response to a positive productivity shock, the increase in investment will be smaller and it is less likely the trade balance will deteriorate.

Chapter 4 of Uribe and Schmitt-Grohe.

5. Explain why, as in section 4.1.1, the authors introduce a debt-elastic interest rate to "induce stationarity." Why do we need to introduce this feature in order to induce stationarity, and why do we want to induce stationarity?

Answer:

In the linearized version of the model, a shock that leads the country to borrow or lend abroad would permanently alter the country's wealth if the interest rate it faced were constant. When we assume $\beta(1+r)=1$, we have seen that implies that consumption follows a random walk, even when productivity does not follow a random walk, because the effects on wealth are permanent. For example, a shock that causes the country to have a transitory increase in output will lead it to increase consumption permanently, but initially it lends to the rest of the world.

In order to have consumption return to its initial level eventually, the debt elastic interest rate has in this case that the rate of return from lending falls as lending increases. This induces higher consumption, and the claims on foreigners fall until the interest rate falls back into equality with the world rate of interest. This establishes a steady state level of claims or debt.

We want to induce stationarity simply because the solution method is to linearize the model around a non-stochastic steady state.

6. In section 4.3, the paper uses the GHH form of preferences. One reason is “Because employment is procyclical, the prediction [that persistent positive productivity shocks result in a decline n employment] is counterfactual under the hypothesis that disturbances in total factor productivity are a major source of business cycles.” Explain why these shocks result in a decline in employment. Also, another reason GHH preferences are widely used is for long-run considerations. As wealth in the U.S., for example, has increased greatly over the past century, what prediction about labor supply does this form of preferences capture (or, what prediction that is inconsistent with data does it avoid)?

Answer:

A more standard utility function has the property that demand for leisure increases as wealth increases. If positive productivity shocks are persistent, then wealth of households will rise with a positive productivity shock. That in turn implies households would like more leisure, so their supply of labor would fall during an expansion, which is counterfactual.

The long-run considerations are also counterfactual. Household wealth in the U.S. has grown greatly over the past century, but household labor supply has not declined.

7. In equation (4.42), on the left-hand-side of the equation, there is a term that represents the expected value of the portfolio of state-contingent bonds. Yet budget constraints must hold every period, and in every state in each period – that is, households always need to obey their budget constraint, not just their expected budget constraint. Explain the meaning of the expectation in this budget constraint.

Answer:

The expectation sign appears there because of a normalization.

That is, let $p\left(S^{t+1} \mid S_t\right)$ be the price of a claim in state S^{t+1} conditional on this period’s state being S^t. Then the value of claims bought is $\sum_{S^{t+1}|S^t} p\left(S^{t+1} \mid S^t\right) b\left(S^{t+1} \mid S^t\right)$. But then define $q_{t,t+1} \equiv \frac{p\left(S^{t+1} \mid S^t\right)}{\pi\left(S^{t+1} \mid S^t\right)}$, where $\pi\left(S^{t+1} \mid S^t\right)$ is the probability of state S^{t+1} conditional on this period’s state being S^t. We then have the value of the portfolio can be written as:

$$\sum_{S^{t+1}|S^t} p\left(S^{t+1} \mid S^t\right) b\left(S^{t+1} \mid S^t\right) = \sum_{S^{t+1}|S^t} \pi\left(S^{t+1} \mid S^t\right) q_{t,t+1} b\left(S^{t+1} \mid S^t\right) = E_t q_{t,t+1} b_{t+1}$$

8. Consider equation (4.49), which holds under complete markets. Suppose, however, that as we have discussed in class, there is more than one traded good, and purchasing power parity does not hold (that is, the real exchange rate is not constant.) How would this equation be modified? Next, suppose we made this modification, and preferences were modeled as GHH preferences. How might this case be offered as a possible resolution of the Backus-Smith puzzle?

Answer:

We would write it as $\frac{U_c\left(c_t, h_t\right)}{P_t} = \frac{\xi_0}{\xi_0^*} \frac{U_{c^*}\left(c_t^*, h_t^*\right)}{S_t P_t^*}$.

When we have GHH preferences, $U\left(c,h\right) = \frac{\left(c - \frac{h^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$, so, $U_c\left(c,h\right) = \left(c - \frac{h^\omega}{\omega}\right)^{-\sigma}$.

Hence, the complete market equilibrium condition can be written as:

$\frac{\left(c - \frac{h^\omega}{\omega}\right)^{-\sigma}}{P_t} = \frac{\xi_0}{\xi_0^*} \frac{\left(c^* - \frac{h^{*\omega}}{\omega}\right)^{-\sigma}}{S_t P_t^*}$. The Backus-Smith puzzle is that under complete markets, there should be a perfect positive correlation between relative home/foreign consumption and the real exchange rate. This may help resolve the problem, because the condition now involves a linear combination of consumption and labor in home relative to foreign.

(It may be helpful to remember this. Suppose we write utility as $U\left(c\left(c^A, c^B\right), h\right)$ where A and B are two goods, and $c\left(c_A, c_B\right)$ is a homothetic function of consumption of the two goods. Then by the way we define the ideal or exact price index, $P = \frac{P_A}{c_A\left(c^A, c^B\right)} = \frac{P_B}{c_B\left(c^A, c^B\right)}$. So, we have $\frac{\partial U / \partial c^A}{P_A} = \frac{U_c\left(c,h\right) c_A\left(c^A, c^B\right)}{P_A} = \frac{U_c\left(c,h\right)}{P}$ and so we could reinterpret the equation above as saying $\frac{\partial U / \partial c_t^A}{P_{A,t}} = \frac{\xi_0}{\xi_0^*} \frac{\partial U^* / \partial c_t^{*A}}{S_t P_{A,t}^*}$, for example.)

9. The first equation on page 143 presents an equation for the current account balance when markets are complete. Clearly the current account balance need not equal zero. We often talk about the current account balance as referring to borrowing or lending, yet in the complete markets model, the relative wealth positions of the two countries is constant over time. How can you explain a non-zero current account in this case? To be concrete,

suppose at time 0 there is no portfolio carried over from the past, so $s_0 = 0$. But suppose $s_1 < 0$ (where, as on the bottom of page 142, $s_t = E_t q_{t+1} b_{t+1}$), so the country runs a current account deficit in period 1. Why does this not imply that the country has become a debtor in period 1, so that its wealth falls relative to the lending country?

Answer:

If $s_1 < 0$, the country expects to make state contingent payments next period, but those will be exactly the payments that keeps its wealth constant relative to the other country.

10. At the bottom of page 169, in the section on global solutions, the authors assume $\beta(1+r) < 1$. Explain in economic terms why this assumption has been made.

Answer:

The country has a precautionary motive for saving. If $\beta(1+r) = 1$, for example, then it would save every period. In that case, the weight it puts on future utility is equal to the market discount factor, so the household would have no desire to alter its time path of consumption for reasons of intertemporal substitution, but would save for precautionary reasons. In this case, the country would be piling up claims on the rest of the world forever without bound. By assuming $\beta(1+r) < 1$, the precautionary motive for saving is balanced by the consumption "tilt" motive to bring consumption forward.

Put another way, if $\beta(1+r) = 1$, then the Euler equation implies $E_t U'(C_{t+1}) = U'(C_t)$. But by Jensen's inequality, if $U'' < 0$, then $U'(E_t C_{t+1}) > E_t U'(C_{t+1})$, so the Euler equation would imply $U'(E_t C_{t+1}) > U'(C_t)$, which means $E_t C_{t+1} > C_t$. Consumption rises forever as the country gets increasingly wealthy from its bond accumulation.

Readings from Uribe and Schmitt-Grohe, chapters 7 and 8.

From chapter 7, read the whole chapter, but there are parts you should not spend too much time on, specifically sections 7.3.1, 7.4, 7.6.

At the bottom of page 330 and top of page 332, the chapter mentions seven ways in which terms of trade changes could affect the trade balance. Let the imported good be the numeraire. Let's write imports as $m(\tau)\cdot C - Y_M$, where $m(\tau)$ is the fraction of consumption spent on imports, which may depend on the terms of trade, and Y_M is the amount of the imported good produced within the country. Write exports as $\tau\left(Y_X - \left(1-m(\tau)\right)C\right)$, where Y_X is the amount of the exported good produced. In units of the imported good, the trade balance is given by $\tau\left(Y_X - \left(1-m(\tau)\right)C\right) - \left[m(\tau)\cdot C - Y_M\right] = Y_M + \tau Y_X - m(\tau)\cdot C - \tau\left(1-m(\tau)\right)C$.

1. With those equations in mind, how would you write each of these ways that the terms of trade might influence the trade balance? (That is, which things are being held constant, and which are varying and in what way, in each of these statements? In some of these, output or total consumption may change with the terms of trade – I did not make those explicit functions of the terms of trade in the equations above to save on notation.)

a. Holding constant the quantities of goods imported and exported, an increase in the relative price of exports should improve the trade balance measured in terms of imports or exports.
b. If, in addition, the higher relative price of exports induces domestic firms to produce more exportables, holding demands and production of importables constant, the improvement in the trade balance would be reinforced.
c. If consumers substitute importable goods for exportable goods as the latter become more expensive, all other things equal, the trade balance would tend to deteriorate.
d. If the increase in the terms of trade makes households feel richer, the demand for consumption goods will go up. If consumption is mostly concentrated on importable goods, the income effect will tend to deteriorate the trade balance.
e. If an improvement in the terms of trade is perceived as the beginning of further future improvements, aggregate demand may experience a strong expansion, causing the trade balance to deteriorate.
f. if the terms of trade improvement is expected to die out quickly, households may choose to save much of the increased income it generates, causing the trade balance to improve.
g. The improvement in the terms of trade could trigger a surge in investment in physical capital, which would also tend to deteriorate the trade balance in the short run. The size of the investment surge will in general depend on the perceived persistence of the terms-of-trade improvement. (For this question, reinterpret *C* in the above equations to be total expenditure, that is, consumption plus investment.)

The next part of this chapter (section 7.2) is a somewhat confusing explanation of estimating a structural VAR. Let's suppose we are interested in estimating some structural model that has two equations and takes this form:

(1) $A_0\begin{bmatrix} x_t \\ y_t \end{bmatrix} = A_1\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$.

Let's write this out as:

$$\begin{bmatrix} 1 & a_{12}^0 \\ a_{21}^0 & 1 \end{bmatrix}\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix}\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$$

But we cannot estimate this model without some identifying assumptions. The actual VAR we estimate looks like this:

(2) $\begin{bmatrix} x_t \\ y_t \end{bmatrix} = B_1\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$.

How can we recover the structural model? One way is to assume that we can write:

(3) $\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = C\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$, where *C* is lower triangular. Normalize $c_{11} = 1$.

That means that x_t is not influenced by the ε_{2t} shock. That is, we have simply

(4) $x_t = b_{11}x_{t-1} + b_{12}y_{t-1} + v_{1t}$, where b_{ij} is the ij element of *B*. From equation (1), we can write this as $x_t = a_{11}^1 x_{t-1} + a_{12}^1 y_{t-1} + v_{1t}$. a_{ij}^k is the ij element of matrix A_k. We now have $a_{12}^0 = 0$ by making this assumption on *C*, and we see that $v_{1t} = u_{1t}$.

The second equation in system (1) is given by:

$$\begin{aligned} y_t &= -a_{21}^0 x_t + a_{21}^1 x_{t-1} + a_{22}^1 y_{t-1} + v_{2t} \\ &= -a_{21}^0\left(a_{11}^1 x_{t-1} + a_{12}^1 y_{t-1} + u_{1t}\right) + a_{21}^1 x_{t-1} + a_{22}^1 y_{t-1} + v_{2t} \\ &= \left(-a_{21}^0 a_{11}^1 + a_{21}^1\right)x_{t-1} + \left(a_{22}^1 - a_{21}^0 a_{12}^1\right)y_{t-1} - a_{21}^0 u_{1t} + v_{2t} \end{aligned}$$

Let $\mathrm{var}\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} \sigma_{u,1}^2 & \sigma_{u,12} \\ \sigma_{u,12} & \sigma_{u,2}^2 \end{bmatrix} = \Omega$, while $\mathrm{var}\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} = \begin{bmatrix} \sigma_{v,1}^2 & 0 \\ 0 & \sigma_{v,2}^2 \end{bmatrix}$.

Now we can recover all the parameters of the structural model from the reduced form model. Basically, the structural model introduces one parameter to solve for in the contemporaneous relationship, which is a_{21}^0, but has one fewer parameter in the covariance matrix.

2. Recover a^0_{21}, a^1_{21}, a^1_{22}, and $\sigma_{v,2}$ as functions of b_{21}, b_{22}, b_{11}, b_{12}, $\sigma^2_{u,1}$, $\sigma^2_{u,2}$ and $\sigma_{u,12}$.

Sections 7.3.2 and 7.3.4 may seem a bit odd. In the models of that section, the small open economy does not produce the good that it imports, and it does not consume the good that it exports. The terms of trade, as a result, don't induce any substitution on either the production side or the consumption side. A shock to the terms of trade acts only as a wealth or income shock by increasing or decreasing the value of exports. This is an important channel through which terms of trade changes affect emerging markets and developing economies, particularly ones that rely heavily on the export of commodities (for which they are price-takers in the global economy, hence the assumption that the terms of trade are exogenous.)

3. Briefly explain Figure 7.3, and make sure you cover both the effects of the terms of trade shock on consumption and investment.

In the MX model of sections 7.5 and 7.7, both goods are produced and consumed. As question 1 (above) hints at, there can be a lot of effects of a terms of trade shock.

4. In Figure 7.6, we see that the positive terms of trade shock leads both to an increase in imports and exports. Why do both rise? (Hint: It is more than just the explanation that starts at the bottom of page 372 and continues on page 375. Referring to question 1 above, that discussion focuses on effects b and c. There is also a, and d-g.) Why do you think the effect on imports is greater so that the trade balance falls (here, the book may not give us enough information to answer with certainty, so what do you think is happening)?

As with chapter 7, I would like you to read all of chapter 8. However there are some sections that you should not invest a lot of time on: 8.2.4-8.2.5, 8.4.9.

The TNT model of section 8.2 is like the model in the first part of chapter 7, in that there is no opportunity for substitution on the production side or consumption side between importables and exportables when the terms of trade change. There might be opportunities for substitution in production between exportables and nontradables production, except that those are assumed to be endowments. There is, though, an opportunity to substitute in consumption between nontraded goods and imports.

The MXN model allows production of all these goods, consumption of all the goods, investment in all the goods, with cost of investment in all these goods. The chapter investigates the effect of a terms of trade shock in this model in a calibrated numerical exercise, but it offers very little intuition or insight into what's driving the results.

The questions here are meant to make you exercise your intuition.

5. In the TNT model, how does an increase in the terms of trade (an increase in the price of exports relative to imports) affect the real exchange rate? What is the mechanism that leads to this result? What is the initial effect on consumption of nontraded goods.

Now, consider the following change in the TNT model. We still have that imported goods are not produced, only exported goods and nontraded goods. We assume the same preferences as in the TNT model. But instead of assuming that output is an endowment, suppose it is produced with labor: $Y_N = L_N$, $Y_X = L_X$. Both goods markets are competitive. There is a competitive labor market that pays a wage *w*, where the numeraire is the price of the imported good. Let p_X be the terms of trade (price of the exported good in terms of the numeraire) and p_N be the price of the nontraded good (in terms of the numeraire.) There is a fixed total supply of labor, and labor is freely mobile between sectors: $L = L_X + L_N$.

6. What are the first-order conditions for maximizing profit in each sector? Now, suppose p_X , which is exogenous, rises. What happens to *w* and p_N ? What happens to L_X and L_N ?
   What happens to consumption of the nontraded good? Explain the economic reasons for the differences with your answers to question 5.

Next, let's complicate the model by having capital in the production process in each sector. Assume no costs of adjustment of the capital stock. However, capital in each sector at time *t* is predetermined. That is, at time *t*, it does not respond to shocks, but then it can adjust fully by time $t+1$. Assume that agents can freely borrow or lend at the world interest rate *r* (expressed in units of the imported good.) At time $t+1$, in both the exported sector and the nontraded sector, the marginal products of labor and capital equal their respective prices:

$$p_{X,t+1} f'\left(k_{X,t+1}\right) = r$$
$$p_{X,t+1}\left[f\left(k_{X,t+1}\right) - f'\left(k_{X,t+1}\right) k_{X,t+1} \right] = w_{t+1}$$

$$p_{N,t+1} g'(k_{N,t+1}) = r$$
$$p_{N,t+1}\left[f(k_{N,t+1}) - f'(k_{N,t+1}) k_{N,t+1} \right] = w$$

The lower case *k*s are capital/labor ratios. These equations should be familiar to you as we saw them in Obstfeld-Rogoff. In comparison to the model in OR, there are three goods, though only two are produced. I've set productivity levels constant, but the relative price of exported to imported goods is exogenous. As in that model, total labor supply is fixed but labor is freely mobile between sectors. (Because the households can borrow or lend with the rest of the world, and adjust the capital stock after one period, you can think of this as the physical capital being freely mobile between sectors and internationally.)

7. Suppose $p_{X,t}$ rises, and the increase will persist at least until period $t+1$. What happens to w_{t+1}, $p_{N,t+1}$, $k_{X,t+1}$, $k_{N,t+1}$, $L_{X,t+1}$, $L_{N,t+1}$ and consumption in the nontraded sector at time $t+1$ (all relative to the situation with no change in $p_{X,t}$ or $p_{X,t+1}$.)

Ten Important Papers

Backus, David K., and Mario J. Crucini. "Oil prices and the terms of trade." *Journal of international Economics* 50.1 (2000): 185-213.

Backus, D. K., P. J. Kehoe, and F. E. Kydland. "Dynamics of the trade balance and the terms of trade: the J-curve?." *American Economic Review* 84.1 (1994): 84-103.

Burstein, Ariel, Christopher Kurz, and Linda Tesar. "Trade, production sharing, and the international transmission of business cycles." *Journal of Monetary Economics* 55.4 (2008): 775-795.

Fernández, Andrés, Andrés González, and Diego Rodriguez. "Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies." *Journal of International Economics* 111 (2018): 99-121.

Fernández, Andrés, Stephanie Schmitt-Grohé, and Martín Uribe. "World shocks, world prices, and business cycles: An empirical investigation." *Journal of International Economics* 108 (2017): S2-S14.

Mendoza, Enrique G. "The terms of trade, the real exchange rate, and economic fluctuations." *International Economic Review* (1995): 101-137.

Obstfeld, Maurice. "Aggregate spending and the terms of trade: Is there a Laursen-Metzler effect?." *The Quarterly Journal of Economics* 97.2 (1982): 251-270.

Schmitt-Grohé, Stephanie, and Martín Uribe. "How Important are Terms-Of-Trade Shocks?." *International Economic Review* 59.1 (2018): 85-111.

Stockman, Alan, and Linda Tesar. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements." *American Economic Review* 85.1 (1995): 168-85.

Svensson, Lars EO, and Assaf Razin. "The terms of trade and the current account: The Harberger-Laursen-Metzler effect." *Journal of political Economy* 91.1 (1983): 97-125.

Answers to questions from chapters 7-8 of Uribe and Schmitt-Grohe

At the bottom of page 330 and top of page 332, the chapter mentions seven ways in which terms of trade changes could affect the trade balance. Let the imported good be the numeraire. Let's write imports as $m(\tau)\cdot C - Y_M$, where $m(\tau)$ is the fraction of consumption spent on imports, which may depend on the terms of trade, and Y_M is the amount of the imported good produced within the country. Write exports as $\tau\left(Y_X - \left(1-m(\tau)\right)C\right)$, where Y_X is the amount of the exported good produced. In units of the imported good, the trade balance is given by $\tau\left(Y_X - \left(1-m(\tau)\right)C\right) - \left[m(\tau)\cdot C - Y_M\right] = Y_M + \tau Y_X - m(\tau)\cdot C - \tau\left(1-m(\tau)\right)C$.

1. With those equations in mind, how would you write each of these ways that the terms of trade might influence the trade balance? (That is, which things are being held constant, and which are varying and in what way, in each of these statements? In some of these, output or total consumption may change with the terms of trade – I did not make those explicit functions of the terms of trade in the equations above to save on notation.)

a. Holding constant the quantities of goods imported and exported, an increase in the relative price of exports should improve the trade balance measured in terms of imports or exports.

<u>Answer</u>

This statement holds $m(\tau)\cdot C$ and $\left(1-m(\tau)\right)C$ constant, as well as Y_M and Y_X, so in the expression $\tau\left(Y_X - \left(1-m(\tau)\right)C\right) - \left[m(\tau)\cdot C - Y_M\right]$, only the term $\tau\left(Y_X - \left(1-m(\tau)\right)C\right)$ is changing and only because τ changes and not the part in parentheses. The change in the trade balance is $Y_X - \left(1-m(\tau)\right)C$.

b. If, in addition, the higher relative price of exports induces domestic firms to produce more exportables, holding demands and production of importables constant, the improvement in the trade balance would be reinforced.

<u>Answer</u>

Now, in addition to the effect above, we may have an increase in Y_X and a decrease in Y_M, which also work to increase the trade balance. The change in the trade balance is

$$Y_X - \left(1-m(\tau)\right)C + \tau\frac{dY_X}{d\tau}$$

c. If consumers substitute importable goods for exportable goods as the latter become more expensive, all other things equal, the trade balance would tend to deteriorate.

<u>Answer</u>

An additional effect on consumption arises as τ rises. We are holding total consumption, *C*, constant. First, on the export side, without the consumption effect, we had the change exports equaled $Y_X - m(\tau)C + \tau\frac{dY_X}{d\tau}$. Now the total change in exports is larger:

$Y_X - m(\tau)C + \tau\frac{dY_X}{d\tau} + \tau m'(\tau)C$, where $m'(\tau) > 0$. Also, imports are larger. Without the consumption effect, we had no effect on imports (because part b held production and consumption of importables constant.) Now imports, $m(\tau)\cdot C$ rise because $m'(\tau) > 0$.

d. If the increase in the terms of trade makes households feel richer, the demand for consumption goods will go up. If consumption is mostly concentrated on importable goods, the income effect will tend to deteriorate the trade balance.

Answer

Here, I believe what the book means is to add the effect of $\frac{\partial C}{\partial \tau} > 0$ but to hold constant the output effect, though I guess you could interpret it in other ways. Now we get that the total effect is $Y_X - (1-m(\tau))C + \tau\frac{dY_X}{d\tau} + (\tau-1)m'(\tau)C - \left(m(\tau) + \tau(1-m(\tau))\right)\frac{dC}{d\tau}$.

e. If an improvement in the terms of trade is perceived as the beginning of further future improvements, aggregate demand may experience a strong expansion, causing the trade balance to deteriorate.

Answer

Now hold the expenditure switching effect, so *m* is constant and hold output of the goods constant. Then the effect on the trade balance is given by the effect on $Y_M + \tau Y_X - m\cdot C - \tau(1-m)C$. The derivative with respect to τ is given by $-\left(m + \tau(1-m)\right)\frac{dC}{d\tau} + Y_X - (1-m)C$. With $\frac{dC}{d\tau}$ sufficiently large, this derivative is negative compared to the effect holding *C* constant.

f. if the terms of trade improvement is expected to die out quickly, households may choose to save much of the increased income it generates, causing the trade balance to improve.

Answer

Here, the change is the same as in the last part, but now $\frac{dC}{d\tau}$, while positive, is small.

The improvement in the terms of trade could trigger a surge in investment in physical capital, which would also tend to deteriorate the trade balance in the short run. The size of the investment surge will in general depend on the perceived persistence of the terms-of-trade improvement. (For this question, reinterpret *C* in the above equations to be total expenditure, that is, consumption plus investment.)

Answer

This would be like the answers to e and f. Something that I want to clarify is this. In chapter 4, we saw

$$c_t = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{A_{t+j}F\left(k_{t+j}\right) - i_{t+j} - \frac{1}{2}\left(i_{t+j}^2 / k_{t+j}\right)}{(1+r)^j}.$$

You may think from this equation that when A_{t+j} increases, that the effect on consumption is reduced by investment. That is without investment, we would have

$$\frac{dc_t}{dA_t} = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{\frac{dA_{t+j}}{dA_t}F\left(k_{t+j}\right)}{\left(1+r\right)^j},$$

and so it looks like the consumption effect is reduced by investment when you take into account the other terms. But that ignores the fact that investment increases the capital stock.

We are concerned about the effect of the increase in productivity on the present discounted value of

$$A_{t+j}F\left(k_{t+j}\right)-i_{t+j}-\tfrac{1}{2}\left(i_{t+j}^2 / k_{t+j}\right)$$

But the optimality conditions for choosing investment implies this will be zero, so in fact the increase in investment does not detract from the direct effect on consumption. The same logic holds for terms of trade changes.

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The next part of this chapter (section 7.2) is a somewhat confusing explanation of estimating a structural VAR. Let's suppose we are interested in estimating some structural model that has two equations and takes this form:

(1) $A_0\begin{bmatrix} x_t \\ y_t \end{bmatrix} = A_1\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}.$

Let's write this out as:

$$\begin{bmatrix} 1 & a_{12}^0 \\ a_{21}^0 & 1 \end{bmatrix}\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix}\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$$

But we cannot estimate this model without some identifying assumptions. The actual VAR we estimate looks like this:

(2) $\begin{bmatrix} x_t \\ y_t \end{bmatrix} = B_1\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}.$

How can we recover the structural model? One way is to assume that we can write:

(3) $\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = C\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$, where C is lower triangular. Normalize $c_{11} = 1$.

That means that x_t is not influenced by the ε_{2t} shock. That is, we have simply

(4) $x_t = b_{11}x_{t-1} + b_{12}y_{t-1} + v_{1t}$, where b_{ij} is the ij element of *B*. From equation (1), we can write this as $x_t = a_{11}^1 x_{t-1} + a_{12}^1 y_{t-1} + v_{1t}$. a_{ij}^k is the ij element of matrix A_k. We now have $a_{12}^0 = 0$ by making this assumption on *C*, and we see that $v_{1t} = u_{1t}$.

The second equation in system (1) is given by:

$$
\begin{aligned}
y_t &= -a_{21}^0 x_t + a_{21}^1 x_{t-1} + a_{22}^1 y_{t-1} + v_{2t} \\
&= -a_{21}^0 \left(a_{11}^1 x_{t-1} + a_{12}^1 y_{t-1} + u_{1t} \right) + a_{21}^1 x_{t-1} + a_{22}^1 y_{t-1} + v_{2t} \\
&= \left(-a_{21}^0 a_{11}^1 + a_{21}^1 \right) x_{t-1} + \left(a_{22}^1 - a_{21}^0 a_{12}^1 \right) y_{t-1} - a_{21}^0 u_{1t} + v_{2t}
\end{aligned}
$$

Let $\operatorname{var}\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} \sigma_{u,1}^2 & \sigma_{u,12} \\ \sigma_{u,12} & \sigma_{u,2}^2 \end{bmatrix} = \Omega$, while $\operatorname{var}\begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} = \begin{bmatrix} \sigma_{v,1}^2 & 0 \\ 0 & \sigma_{v,2}^2 \end{bmatrix}$.

Now we can recover all the parameters of the structural model from the reduced form model. Basically, the structural model introduces one parameter to solve for in the contemporaneous relationship, which is a_{21}^0, but has one fewer parameter in the covariance matrix.

2. Recover a_{21}^0, a_{21}^1, a_{22}^1, and $\sigma_{v,2}$ as functions of b_{21}, b_{22}, b_{11}, b_{12}, $\sigma_{u,1}^2$, $\sigma_{u,2}^2$ and $\sigma_{u,12}$.

<u>Answer</u>

We have:

$$b_{11} = a_{11}^1,\ b_{12} = a_{12}^1,\ \sigma_{v,1}^2 = \sigma_{u,1}^2,$$

$$b_{21} = -a_{21}^0 b_{11} + a_{21}^1,\ b_{22} = a_{22}^1 - a_{21}^0 b_{12},\ \sigma_{u,2}^2 = \left(a_{21}^0 \sigma_{u,1}\right)^2 + \sigma_{v,2}^2,\ \sigma_{u,12} = -a_{21}^0 \sigma_{u,1}^2$$

The last four equations allow us to recover a_{21}^0, a_{21}^1, a_{22}^1, and $\sigma_{v,2}$ as functions of b_{21}, b_{22}, b_{11}, b_{12}, $\sigma_{u,1}^2$, $\sigma_{u,2}^2$ and $\sigma_{u,12}$. Specifically, $a_{21}^0 = -\sigma_{u,12} / \sigma_{u,1}^2$. Then with this solution, we get $\sigma_{v,2}^2 = \sigma_{u,2}^2 - \left(a_{21}^0 \sigma_{u,1}\right)^2$, $a_{21}^1 = b_{21} + a_{21}^0 b_{11}$, and $a_{22}^1 = b_{22} + a_{21}^0 b_{12}$.

Sections 7.3.2 and 7.3.4 may seem a bit odd. In the models of that section, the small open economy does not produce the good that it imports, and it does not consume the good that it exports. The terms of trade, as a result, don't induce any substitution on either the production side or the consumption side. A shock to the terms of trade acts only as a wealth or income shock by increasing or decreasing the value of exports. This is an important channel through which terms of trade changes affect emerging markets and developing economies, particularly ones that rely heavily on the export of commodities (for which they are price-takers in the global economy, hence the assumption that the terms of trade are exogenous.)

3. Briefly explain Figure 7.3, and make sure you cover both the effects of the terms of trade shock on consumption and investment.

Answer

As the terms of trade effect becomes more persistent, first it increases the present discounted value of the country's income, which increases consumption more. Also, it increases the returns to investment more, as new capital is more productive for a longer period of time. Both effects work to reduce the trade balance more.

In the MX model of sections 7.5 and 7.7, both goods are produced and consumed. As question 1 (above) hints at, there can be a lot of effects of a terms of trade shock.

4. In Figure 7.6, we see that the positive terms of trade shock leads both to an increase in imports and exports. Why do both rise? (Hint: It is more than just the explanation that starts at the bottom of page 372 and continues on page 375. Referring to question 1 above, that discussion focuses on effects b and c. There is also a, and d-g.) Why do you think the effect on imports is greater so that the trade balance falls (here, the book may not give us enough information to answer with certainty, so what do you think is happening)?

Answer

As in the answer to question 1, first there is the direct effect of the terms of trade stock, which raises the value of exports. The persistent improvement in the terms of trade raises income and so increases consumption of both goods, which raises both imports and exports. Overall, it is clear by looking at the graphs that the main effects are coming through an increase in investment in the exportable sector which more than offsets the drop in investment in the importable sector, so the overall level of investment rises a lot. I'm guessing that under these calibrations, the exportable sector is large relative to the importable sector, and that is the main reason for this effect.

5. In the TNT model, how does an increase in the terms of trade (an increase in the price of exports relative to imports) affect the real exchange rate? What is the mechanism that leads to this result? What is the initial effect on consumption of nontraded goods?

Answer

The increase in the terms of trade raises income, which raises consumption of both goods. This must mean that, given the fixed endowment of the nontraded goods, the price of the nontraded good must rise, so there is a real appreciation.

Now, consider the following change in the TNT model. We still have that imported goods are not produced, only exported goods and nontraded goods are. We assume the same preferences as in the TNT model. But instead of assuming that output is an endowment, suppose it is produced with labor: $Y_N = L_N$, $Y_X = L_X$. Both goods markets are competitive. There is a competitive labor market that pays a wage *w*, where the numeraire is the price of the imported good. Let p_X be the terms of trade (price of the exported good in terms of the numeraire) and p_N be the price of the nontraded good (in terms of the numeraire.) There is a fixed total supply of labor, and labor is freely mobile between sectors: $L = L_X + L_N$.

6. What are the first-order conditions for maximizing profit in each sector? Now, suppose p_X, which is exogenous, rises. What happens to *w* and p_N? What happens to L_X and L_N? What happens to consumption of the nontraded good? Explain the economic reasons for the differences with your answers to question 5.

Answer

This is like the model in Berka, Devereux and Engel that we looked at earlier. Letting the importable be the numeraire, the profit maximizing condition in the exportable sector is $w = p_X$, and in the nontraded sector, it is $w = p_N$. When p_X rises, *w* must rise, which means that p_N must rise. Note that the real appreciation is independent of the consumption decision. That is because of the mobility of labor between sectors. The increase in the price of the nontraded good will discourage consumption of the good, but the increase in the income from exports will have a positive wealth effect on consumption of nontraded goods. The overall effect on employment and output in the nontraded sector is ambiguous without further assumptions, so that makes the changes in L_X and L_N ambiguous. But relative to the previous model, consumption of nontradables must certainly increase less because of the expenditure switching effect.

Next, let's complicate the model by having capital in the production process in each sector. Assume no costs of adjustment of the capital stock. However, capital in each sector at time *t* is predetermined. That is, at time *t*, it does not respond to shocks, but then it can adjust fully by time $t+1$. Assume that agents can freely borrow or lend at the world interest rate *r* (expressed in units of the imported good.) At time $t+1$, in both the exported sector and the nontraded sector, the marginal products of labor and capital equal their respective prices:

$$p_{X,t+1} f'(k_{X,t+1}) = r$$
$$p_{X,t+1}\left[f(k_{X,t+1}) - f'(k_{X,t+1}) k_{X,t+1} \right] = w_{t+1}$$
$$p_{N,t+1} g'(k_{N,t+1}) = r$$
$$p_{N,t+1}\left[f(k_{N,t+1}) - f'(k_{N,t+1}) k_{N,t+1} \right] = w$$

The lower case *k*s are capital/labor ratios. These equations should be familiar to you as we saw them in Obstfeld-Rogoff. In comparison to the model in OR, there are three goods, though only two are produced. I've set productivity levels constant, but the relative price of exported to imported goods is exogenous. As in that model, total labor supply is fixed but labor is freely mobile between sectors. (Because the households can borrow or lend with the rest of the world, and adjust the capital stock after one period, you can think of this as the physical capital being freely mobile between sectors and internationally.)

7. Suppose $p_{X,t}$ rises, and the increase will persist at least until period $t+1$. What happens to w_{t+1}, $p_{N,t+1}$, $k_{X,t+1}$, $k_{N,t+1}$, $L_{X,t+1}$, $L_{N,t+1}$ and consumption in the nontraded sector at time $t+1$ (all relative to the situation with no change in $p_{X,t}$ or $p_{X,t+1}$.)

<u>Answer</u>

From the first equation, $p_{X,t+1} f'\left(k_{X,t+1}\right) = r$, an increase in $p_{X,t+1}$ increases $k_{X,t+1}$.

From the second equation, an increase in $p_{X,t+1}$ and $k_{X,t+1}$ increases w_{t+1}.

As in the Balassa-Samuelson model, the solution to the last two equations requires that $p_{N,t+1}$ and $k_{N,t+1}$ rise.

Again, note that the real exchange rate is determined independently of consumption because of the mobility of factors. As in the previous question, the actual change in consumption of nontrade goods depends on income and substitution effects, but it is definitely less than in the TNT model. This makes the employment in the two sectors ambiguous.

Optimal Monetary Policy

We will read two papers in this section:

Engel, Charles. "Currency misalignments and optimal monetary policy: a reexamination." *American Economic Review* 101.6 (2011): 2796-2822.

Bengui, Julien, and Louphou Coulibaly. *Stagflation and Topsy-Turvy Capital Flows*. No. w30652. National Bureau of Economic Research, 2022.

In this section, we are reading a couple of papers that have a lot of analytical results. There is a large literature on optimal monetary policy in open economies, and much of the recent literature is in the context of very rich models where the optimal policy is described using numerical solutions. (However, almost all that literature examines optimal “instrument rules” rather than “targeting rules”. I explain the difference below.) We are reading my paper because, first, of course, I know it well. But actually, a benefit of reading it is that it lays out all the derivations (including in the appendix) and makes clear some important points relative to the literature. The second paper is a neat recent contribution.

Beginning with my paper, it is helpful to understand the origins of this paper. There are a couple of papers, listed in the “Ten Important Papers” section, that investigated optimal monetary policy in two-country New Keynesian models under the assumption of producer currency pricing (PCP), so that the law of one price holds for traded goods. (Clarida, Gali, Gertler (2002) and Benigno and Benigno (2002).) These papers are open-economy extensions of the closed-economy monetary policy analysis, as explained in detail in Woodford’s book. These papers show how there can be a tradeoff between objectives of targeting inflation and targeting the output gap in models with Calvo price setting. In the Clarida et al. paper, in addition to the price stickiness distortion, there is a monopoly distortion in the supply of labor that is time-varying and stochastic. In that paper, if there were only the price-stickiness distortion, then policies that purely target inflation would be optimal (so it is the introduction of the labor supply distortion that leads to the tradeoff between inflation and output gap targets.) In that setting, if the nominal price level were kept constant by monetary policy, the flexible-price equilibrium could be replicated (and this would be first-best if, in addition, there was the correct constant subsidy rate to monopolists to correct the distortion from underproduction by firms with monopoly power.) The Benigno and Benigno paper shows that even in the absence of the labor supply distortion, pure inflation targeting might not be optimal. Those cases may arise when the elasticity of substitution between home and foreign goods in consumption is more general than the Clarida et al. assumption of unitary elasticity.

Concurrently with those papers, I had some papers co-authored with Devereux that examined optimal monetary policy when there is local-currency pricing (LCP). That paper was in a different setting than the papers above, because, like the original Obstfeld and Rogoff model, we assumed prices were set one period in advance and were reset every period (rather than assuming a Calvo price-setting mechanism.) We did find under this assumption that, like the Clarida et al. finding when there is no labor market distortion, and when the elasticity of substitution between home and foreign aggregates is one, that under PCP, monetary policy would replicate the flexible-price equilibrium. Under PCP, the terms of trade can change in response to shocks, because the exchange rate is flexible, even though goods are priced in the currency of the producer. But, we found, under LCP, the relative prices that households faced could not respond to shocks, because nominal prices were set a period in advance. In fact, we found that the optimal monetary policy led to fixed exchange rate. The reason is that fixed exchange rates eliminated any deviations from the law of one price that could occur under LCP when exchange rates fluctuate. So, the optimal policy could not replicate the flexible price equilibrium (because relative prices consumers faced could not adjust contemporaneously to shocks), but it could eliminate the pricing to market that occurs under LCP.

However, our set-up did not allow for the distortions present in the Clarida et al. framework. In the Calvo model, because price-setting is staggered, inflation leads to misalignment of prices within each country. And the labor market distortion introduces a "cost-push" shock into inflation determination. Neither of those are present in my paper with Devereux, so it raises the question of whether the objective of closing the distortion caused by deviations from the law of one price is a separate objective than the inflation and output gap objectives. I wrote my paper adopting the Clarida et al. framework closely. I made a few minor changes: Clarida et al. allow the countries to have different populations, but there was not much to be learned from that, so I set my countries to have equal populations. Clarida et al. had the two countries with identical preferences, but I introduced home bias in preferences which made the results a little richer. As we will see, for the LCP model, I set the Frisch elasticity of labor supply to be infinite (that is, utility is quasilinear in disutility of work.)

One objective is to derive an optimal "targeting rule" for monetary policy. A targeting rule describes the tradeoffs that monetary policymakers choose, but does not prescribe a particular function for the policy instrument as an "instrument rule" would do. First, note that both of these are policy "rules", which are policies under commitment. A targeting rule would be expressed in terms of the targets of monetary policy. A simple rule would be an inflation target of zero: $\pi_t = 0$. Another type of rule would state how the objectives of minimizing the output gap and minimizing inflation should trade off when policy cannot set both to zero: $\tilde{y}_t + \xi\pi_t = 0$. A targeting rule would say how the monetary policy instrument, such as the interest rate, should respond to variables such as inflation and the output gap: $i_t = \tilde{r}_t + \alpha\pi_t + \gamma\tilde{y}_t$. Why do I want to find the targeting rule instead of the instrument rule? First, in the real world, central bank policymakers think in terms of targeting rules. They want to know what the tradeoffs are – for example, what factors determine the size of ξ in the targeting rule above – and they adjust the instrument until they achieve the desired tradeoff. Second, the targeting rule generally does not change when the data generating processes of the exogenous variables changes – that is, when there is a change in regime – while the parameters of the targeting rule do depend on that. In that sense, the targeting rule is more robust.

In addition, I was trying to find a way to express the optimal targeting rule in a way that was simple enough to tell to a policymaker, and in terms of variables that the policymaker would think were reasonable. The targeting rule from above, for example, expresses the optimal policy in terms of two variables that policy makers are familiar with, inflation and the output gap. It is a linear rule, and there are no complicated lags or other strange terms. I was hoping to derive a simple rule of that nature that included the objective of minimizing the deviation from the law of one price.

The targeting rule is the first-order condition that arises from the policymaker's optimization problem. In the NK literature, we get linear rules when we use an LQ (linear-quadratic) framework. Let me demonstrate two ways to get linear first-order conditions from an optimization problem. The first simply involves taking the first-order condition from the general problem, then linearizing it. The second is more complicated, but has some advantages, though the linearized first-order condition is the same as from the first method.

Take a simple two-variable problem. The objective is to maximize $f(x)+g(y)$ subject to $y=h(x)$

<u>Method 1:</u> Derive the f.o.c. then do linear approximation:

Maximize $f(x)+g(h(x))$.
f.o.c.: $f'(x)+g'(h(x))\cdot h'(x)=0$

Do first-order approximation around some point $\tilde{x}$:

$$f'(\tilde{x})+g'(h(\tilde{x}))\cdot h'(\tilde{x})+f''(\tilde{x})(x-\tilde{x})+g'(h(\tilde{x}))h''(\tilde{x})(x-\tilde{x})+g''(h(\tilde{x}))\left[h'(\tilde{x})\right]^2(x-\tilde{x})=0$$

The notation here is that the variable in the equation is *x*, and the point around which the function is being approximated is $\tilde{x}$. Keep in mind that *x* is a variable, but $\tilde{x}$ is simply a number. When I write something like $f(\tilde{x})$, that should be interpreted as the value of the function $f(x)$ when $x=\tilde{x}$. So, the equation directly above is linear in *x*, because the things multiplying *x* are numbers, not functions.

Note that if $\tilde{x}$ is the optimal value of *x*, then using the first-order condition evaluated at $x=\tilde{x}$, the linearized first-order condition reduces to:

$$f''(\tilde{x})(x-\tilde{x})+g'(h(\tilde{x}))h''(\tilde{x})(x-\tilde{x})+g''(h(\tilde{x}))\left[h'(\tilde{x})\right]^2(x-\tilde{x})=0$$

<u>Method 2:</u> Do 2nd-order approximation of constraint and substitute into 2nd-order approximation of objective, then maximize subject to first-order approximation of constraint:

2nd order approximation of constraint:

$$y=h(\tilde{x})+h'(\tilde{x})(x-\tilde{x})+\tfrac{1}{2}h''(\tilde{x})(x-\tilde{x})^2$$

2nd-order approximation of objective:

$$f(\tilde{x})+g(\tilde{y})+f'(\tilde{x})(x-\tilde{x})+g'(\tilde{y})(y-\tilde{y})+\tfrac{1}{2}\left(f''(\tilde{x})(x-\tilde{x})^2+g''(\tilde{y})(y-\tilde{y})^2\right)$$

Now substitute 2nd-order approximation of constraint in for *y*. In doing this, we will ignore terms that are higher than 2nd-order. That is, we'll omit terms that are 3rd or 4th order:

$$f(\tilde{x})+g(\tilde{y})+f'(\tilde{x})(x-\tilde{x})+g'(\tilde{y})\left(h'(\tilde{x})(x-\tilde{x})+\tfrac{1}{2}h''(\tilde{x})(x-\tilde{x})^2\right)$$
$$+\tfrac{1}{2}\left(f''(\tilde{x})(x-\tilde{x})^2+g''(\tilde{y})\left(h'(\tilde{x})(x-\tilde{x})\right)^2\right)$$

LQ problem: Maximize this quadratic objective subject to first-order approximation of constraint:
$y=h(\tilde{x})+h'(\tilde{x})(x-\tilde{x})$.

The f.o.c. is given by:

$$f'(\tilde{x})+g'(\tilde{y})h'(\tilde{x})+g'(\tilde{y})h''(\tilde{x})(x-\tilde{x})+f''(\tilde{x})(x-\tilde{x})+g''(\tilde{y})\left[h'(\tilde{x})\right]^2(x-\tilde{x})$$
$$=f'(\tilde{x})+g'(\tilde{y})h'(\tilde{x})+g'(h(\tilde{x}))h''(\tilde{x})(x-\tilde{x})+f''(\tilde{x})(x-\tilde{x})+g''(h(\tilde{x}))\left[h'(\tilde{x})\right]^2(x-\tilde{x})=0$$

In deriving this first-order condition, you need to remember that *x* is a variable, but $\tilde{x}$ is simply a number. We are choosing *x* to maximize the function.

If $\tilde{x}$ is the optimum value of *x*, then from the f.o.c., $f'(\tilde{x})+g'(h(\tilde{x}))h'(\tilde{x})=0$, and this reduces to

$$g'(h(\tilde{x}))h''(\tilde{x})(x-\tilde{x})+f''(\tilde{x})(x-\tilde{x})+g''(h(\tilde{x}))\left[h'(\tilde{x})\right]^2(x-\tilde{x})=0.$$

In fact, if $\tilde{x}$ is the optimum value of *x*, the objective function simplifies to:

$$f(\tilde{x})+g(\tilde{y})+\tfrac{1}{2}g'(\tilde{y})\left(h''(\tilde{x})(x-\tilde{x})^2\right)+\tfrac{1}{2}\left(f''(\tilde{x})(x-\tilde{x})^2+g''(\tilde{y})(y-\tilde{y})^2\right)$$

Note that Method 1 and Method 2 arrive at the same approximated first-order conditions. The second method seems more complicated, but there is an advantage in that the objective function has been rewritten as a quadratic function. That advantage will become clear shortly.

Now look at this second method. Suppose we had started just by taking a quadratic (i.e., second order) approximation of the objective and a first-order approximation of the constraint rather than the more cumbersome approach we took. Would we have gotten the same result? If you go back through the steps, you'll see that an important term would be missing from the approximated first-order condition: $g'(\tilde{y})h''(\tilde{x})(x-\tilde{x})$. That term needs to be in there for the linearized first-order condition to give us the correct second-order solution to the optimization problem.

Suppose that we were to approximate the objective function around the optimum point. Then as noted above, the objective function becomes

$$f(\tilde{x})+g(\tilde{y})+\tfrac{1}{2}g'(\tilde{y})\left(h''(\tilde{x})(x-\tilde{x})^2\right)+\tfrac{1}{2}\left(f''(\tilde{x})(x-\tilde{x})^2+g''(\tilde{y})(y-\tilde{y})^2\right)$$

That is, when evaluated at the optimum, the linear terms drop out of the approximated objective function and only the second-order terms remain. This is an application of the envelope theorem. Then if we maximize this function subject to linearized constraints, we will get the correct first-order condition for this case:

$$g'\left(h(\tilde{x})\right)h''(\tilde{x})(x-\tilde{x})+f''(\tilde{x})(x-\tilde{x})+g''\left(h(\tilde{x})\right)\left[h'(\tilde{x})\right]^2(x-\tilde{x})=0\,.$$

That is what I do in my paper. The policymaker is assumed to maximize the expected utility of home and foreign households. This is a way to represent the optimal cooperative monetary policy – the central banks of the two countries get together and choose policies to maximize the sum of the utilities of households in the countries. (Given that the countries are of equal size and their initial wealth is the same, a sensible objective function is the sum of welfare of households in the two countries.) I then approximate the utility function around the optimal point, so that I get a second-order approximation of the utility function, and then I find the optimal choices using first-order approximations of the constraints. (Following the tradition in the literature, I express the objective as a "loss" function – the difference between the utility under a fully efficient allocation of resources and the utility from the actual economy. The objective is then to minimize the loss.)

Note that while utility is a function of consumption and work effort in the two countries, I write the loss function in terms of output gaps, inflation and the deviation from the law of one price. Why do I do that? First, if I want to talk to policymakers, I don't think I can give them a policy rule expressed in terms of consumption and hours of work. That would not be realistic, while a policy rule in terms of output gaps, inflation, and the exchange rate (which is what determines the deviation from the law of one price in the LCP model) would make sense. Also, however, to apply the envelope theorem, the objective needs to be expressed in terms of variables that are at their unconstrained minimum in the steady state around which we approximate (that is, the utility is at the unconstrained steady-state maximum.) The optimal policy in steady state does not tell us to maximize the utility of consumption, or to maximize the utility of leisure. Instead, we rewrite in terms of the squares of inflation, output gap and deviation from the law of one price. Each of these things causes distortions in the economy unless they are at their unconstrained minimum (which is zero in all three cases). That is, optimally, we want inflation, the output gap and the deviation from the law of one price to each be at its minimum to get the optimal utility. The idea is to rewrite expected utility as a loss function and express the loss function in terms of the squares of variables that each represent a distortion in the economy.

Hence the approach in this paper and many like it is to approximate the model around a non-stochastic undistorted steady state. We take a second-order approximation of the loss function of the policymaker and a first-order approximation of the equations of the model.

With that background, here are some questions about some aspects of my paper:

1. S_t is defined at the bottom of page 2802. Is this the terms of trade for the home country? If not, write out the formula for the terms of trade.

2. Suppose we were looking at a closed economy, for simplicity. Then from equation (2), we have $Y_t = C_{Ht}$, but from equation (5) we have $A_t N_t = Y_{Ht} V_{Ht}$, where $V_{Ht} = \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df$. Why don't we have $Y_t = A_t N_t$ since for each firm *f* we have $Y_t(f) = A_t N_t(f)$? In particular, show that the measure of GDP is equal to nominal GDP, $\int_0^1 \left(P_{Ht}(f) C_{Ht}(f) \right) df$, divided by the closed-economy consumer price index. Explain why we would use such a measure.

That previous question makes me wonder, though, if a better definition of Y_t in equation (2) would have been $Y_t = \frac{P_{Ht} C_{Ht}}{P_t} + \frac{E_t P_{Ft}^* C_{Ft}^*}{P_t} = \frac{\nu}{2} C_t + \left(\frac{E P_t^*}{P_t} \right) \left(1 - \frac{\nu}{2} \right) C_t^*$. That is, maybe it should be nominal GDP in the home country's currency divided by the home country consumer price level.

3. Suppose we did use this alternate definition, and utility was logarithmic in consumption. Show this would imply $Y_t = C_t$.

4. Why is there a z_t^2 term in equation (15), but it is no longer there in equation (29)?

5. Consider the price setting decisions of home firms under either PCP or LCP (first equation on pages 2808 and 2809, respectively.) These optimization problems use the home stochastic discount factor to discount expected future dividends of the firm. Suppose instead the home firms were owned by foreigners and the foreign s.d.f. were used to discount future profits. How, if at all, would the pricing decisions be different?

Consider the period-by-period loss, given in equation (15). First, let me address the question of why this representation of the loss function has the cross-section variance of prices, but the loss function in equation (29) has squared inflation rates. As the paper explains briefly (in the PCP case), Ψ_t is not the period-by-period loss function, but it is true that $E_t \sum_{j=0}^{\infty} \beta^j X_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \Psi_{t+j}$, where X_t is the actual period loss function (and E_t here means expectation, not exchange rate.) The remarkable insight comes from Woodford. When price are set under the Calvo mechanism, the staggered price setting leads to dispersion of prices over time. It turns out that when you sum over time the discounted cross-sectional variances of prices you get the sum over time of the discounted squared inflation rates.

But then, on page 2806, I ask this question: consider the period loss function. Suppose the output gaps were somehow zero and suppose the price dispersion were zero. The zero output gap would imply the overall production level of the economy was efficient. If there were zero dispersion of prices, that would furthermore imply that every firm was producing at the efficient level. In that case, there would be no distortion in output levels of any firm. So why are the terms relating to the deviation from the law of one price, m_t^2 and z_t^2, in loss function. Let's not focus on the z_t^2 term, since question (4) above tells you it drops out. But what about the m_t^2 term? As I explain, if that is not zero, consumption allocations are inefficient.

The model assumes markets are complete in that a complete set of nominal state-contingent claims are traded. But when there are deviations from the law of one price, the complete markets are giving us an inefficient allocation. *Ceteris paribus*, under complete markets, the country whose prices are lower in some state (due to LCP and nominal exchange rate fluctuations) will receive transfers from the other country. This is not an efficient allocation of resources. An omniscient planner (not a Ramsey planner, but a planner not subject to the constraints imposed by market outcomes) would not allocate goods to households in some country under these circumstances. The pricing to market may lead to inefficient consumption allocations under complete markets.

The Corsetti, Dedola and Leduc (2010) paper generalizes my paper in two ways (it was written after my paper, though published first, which is why my less general paper was published in AER.) One way it is generalized is, in one sense, not very important, but in another sense is. The paper assumes a general CES utility function for the home and foreign aggregate consumption goods, rather than the Cobb-Douglas function I assume. Maybe for the question I was addressing, this generalization is not so important. If you look at the loss function that paper derives (their equation (50)), things are not much changed when you introduce this elasticity. From my perspective, I was trying to hew closely to Clarida et al. I didn't want anyone to think that my result on targeting exchange rate fluctuations somehow depended on making that elasticity different than one.

In other contexts, assuming an elasticity different than one matters. Terms of trade fluctuations have wealth effects, except under unitary elasticities, as in the optimal tariff literature. That matters in questions of non-cooperative policy or incomplete markets.

That brings me to the second contribution, which really was to reinterpret my Δ_t variable. The easiest way to see this is start with the complete markets condition (and I'll set relative wealth levels to one for simplicity):

$$\frac{U'\left(C_t^*\right)}{U'\left(C_t\right)} = Q_t,$$

Where Q_t is the real exchange rate. You could rewrite this in my paper as:

$$\frac{U'\left(C_t^*\right)}{U'\left(C_t\right)} = \tilde{Q}_t \Delta_t$$

Here, $\tilde{Q}_t$ is the real exchange rate if there were no law of one price distortion, which is not equal to one because there is home bias in consumption. In my paper, the actual real exchange rate is different than the efficient one because of the deviation from the law of one price, Δ_t. Corsetti et al.'s point was that in the loss function, Δ_t could stand in for any deviation from the efficient complete markets allocation. It could be the pricing to market distortion I looked at, which still maintains a complete market in nominal contingent claims. But it could be any source of market incompleteness.

Solving the optimal targeting rules under different assumptions about which assets are traded can be very difficult. These authors' forthcoming *JIE* paper solves the problem when only a non-state contingent bond is traded. That was a very difficult problem to solve – it took hundreds of pages of algebra!

By the way, how might we interpret the consumption distortion in my paper – the Δ_t that arises because of pricing to market? Here is one way to think about it. My paper with Matsumoto (*American Economic Journal*, 2009) looks at similar models under LCP, but with prices set one period in advance rather than Calvo pricing. To a first-order approximation, the complete-markets allocation is replicated with only two non-state-contingent bonds traded, one denominated in each currency. When markets are complete, the home country tends to be allocated more consumption when its currency depreciates. Those are the states of the world in which its price level is relatively low compared to the foreign country. If only nominal bonds were traded, but each country borrowed in its own currency, the same thing would happen. (This is why trade in only these two bonds replicates complete markets.) If the U.S. borrowed in debt denominated in dollars, and held foreign currency debt, then a depreciation of the dollar would be a relative wealth gain for the U.S. Now, the point of my paper (the monetary policy paper that we are reading) is that such a shift in relative wealth is not generally efficient. Why should the U.S. become relatively wealthier, from an efficiency perspective, if U.S. monetary policy is expansionary and the dollar depreciates? Yet that is what happens under LCP and fluctuating nominal exchange rates.

6. Compare the targeting rules for PCP under commitment (equation (27)) and discretion (equation (28)). How is the Fed's decision in 2019 to pursue "average inflation" targeting rather than inflation targeting related to the difference between (27) and (28)?
7. In deriving the optimal policies under LCP, as the paper explains, I make the additional assumption that the parameter on labor supply in the utility function equals zero ($\phi = 0$.) One reason I did this is that it simplifies the analysis under discretion. We can solve for optimal policy under discretion by solving the Bellman equation. Write out the Bellman equation for the cooperative policymaker for the general case (that is, where we don't necessarily have $\phi = 0$), and then explain why this assumption simplifies things.

Moving on to the Bengui and Coulibaly paper, there are a few differences in the set-up of the model from my paper, but not many. First, the model is set in continuous time and with no uncertainty, though those things don't end up mattering. The fact that the assumption of no uncertainty doesn't matter is somewhat subtle and I'll mention it below. The model assumes

PCP. The utility function is both less and more general. It is less general in that it assumes log utility of consumption. It is more general in that it does not assume a unitary elasticity of substitution between home and foreign aggregates.

The key assumption in the model is that markets are not necessarily complete. The variable Θ_t plays a similar role to Δ_t in my decomposition above, $\frac{U'\left(C_t^*\right)}{U'\left(C_t\right)} = \tilde{Q}_t \Delta_t$. In their model, $\frac{U'\left(C_t^*\right)}{U'\left(C_t\right)} = Q_t \Theta_t$. They have the actual real exchange rate, because, where in my formulation, $\tilde{Q}_t$ is the real exchange rate if there is no law of one price distortion. They have a PCP model of pricing. But as in Corsetti et al. (2010), we can interpret Θ_t as any deviation arising from market incompleteness.

That general formulation might give us a model that is very difficult to solve (such as Corsetti et al.'s forthcoming *JIE* paper), but then Bengui and Coulibaly do something very clever. They say that in addition to the two monetary policy instruments that monetary policymakers have when solving the optimal monetary policy problem, they also have one macroprudential tool that allows them to control Θ_t. That assumption makes Θ_t exogenous when choosing the monetary policy instruments. Given that assumption, choosing optimal monetary policy in this setting is like choosing monetary policy under complete markets. That makes the monetary policy problem like the PCP case in my paper (that is, like the problem solved by Clarida et al. (2002).) Their solution, (21) and (22), is the same as in my paper under PCP, in (27)

8. This is a very simple question: show my (27) is the same as Bengui and Coulibaly's (21) and (22). This is essentially a question of whether you correctly interpret my (27).

By the way, it is the fact that Θ_t is chosen by policymakers that makes it irrelevant that the model does not have explicit uncertainty. When there is uncertainty, allocations are affected by how asset markets are set up to allocate risk. But in this case, no matter what the actual asset market structure, the risk-sharing outcome is determined by the policymaker. The policymaker can, in a sense, undo anything the asset markets do to reallocate wealth, and instead choose the wealth allocation he prefers.

It is actually a very relevant problem to ask how macroprudential policy should be set in conjunction with monetary policy. The emphasis of the paper is that, in essence, under optimal macroprudential policy, and when monetary policy is optimal, wealth transfers should be made to the country that is experiencing a boom. That is somewhat surprising. In this model, the source of fluctuations is the cost-push shocks (arising from the mark-up fluctuations in the labor market.) Suppose that the home country, elasticity of demand for workers falls, causing a positive cost-push shock. This type of shock worsens the inflation-output tradeoff. Suppose a country experiences a cost-push shock, which is inflationary. Under optimal monetary policy, there is a contraction. That is, holding the capital flow policy constant, the monetary policymaker trades off the inflation shock with the output gap goal, pushing down inflation some but also having output fall below potential. The optimal capital flow policy (the optimal capital flow management or rebalancing of portfolios) is to effect transfers from the country experiencing inflation. This means reducing consumption in that country, which eases pressure on inflation. The country suffering from stagflation should use capital controls to reduce inflows.

9. What are “topsy-turvy” capital flows? Exactly what do the authors mean by this? Explain how this happens in the Cole-Obstfeld case.

Ten Important Papers

Benigno, Gianluca, and Pierpaolo Benigno. "Price stability in open economies." *The Review of Economic Studies* 70.4 (2003): 743-764.

Benigno, Gianluca, et al. "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective." *Journal of monetary economics* 84 (2016): 147-165.

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Optimal Monetary Policy

With that background, here are some questions about some aspects of my paper:

1. S_t is defined at the bottom of page 2802. Is this the terms of trade for the home country? If not, write out the formula for the terms of trade.

Answer:

S_t is defined as $S_t = \frac{P_{Ft}}{P_{Ht}}$, the price home consumers pay for imported goods divided by the price they pay for home-produced goods. The terms of trade involves the relative price of imports to exports (or, more commonly, the relative price of exports to imports.) The price of exports, expressed in home currency, is $E_t P_{Ht}^*$, so we can write the terms of trade as $\frac{P_{Ft}}{E_t P_{Ht}^*}$ (or, its inverse, $\frac{E_t P_{Ht}^*}{P_{Ft}}$,)

2. Suppose we were looking at a closed economy, for simplicity. Then from equation (2), we have $Y_t = C_{Ht}$, but from equation (5) we have $A_t N_t = Y_{Ht} V_{Ht}$, where $V_{Ht} = \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df$. Why don't we have $Y_t = A_t N_t$ since for each firm *f* we have $Y_t(f) = A_t N_t(f)$? In particular, show that the measure of GDP is equal to nominal GDP, $\int_0^1 \left(P_{Ht}(f) C_{Ht}(f) \right) df$, divided by the closed-economy consumer price index. Explain why we would use such a measure.

Answer:

We are using C_{Ht} as our measure of GDP in the closed economy case. Since we have $C_{Ht} V_{Ht} = \int_0^1 C_{Ht}(f)$ from the paper, then $C_{Ht} = \frac{\int_0^1 C_{Ht}(f)}{V_{Ht}}$. This equation in the paper follows simply from the fact that, as equation (A10) in the appendix shows, $C_{Ht}(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} C_{Ht}$. The question is asking us to show that this measure of GDP is equivalent to taking total nominal expenditure and dividing by the nominal price index, P_{Ht}.

In the first place, note that while nominal GDP is the sum of the expenditure on all goods for all consumers, $\int_0^1 \left(P_{Ht}(f) C_{Ht}(h, f) \right) dhdf$, because we have a representative agent model, this just equals $\int_0^1 \left(P_{Ht}(f) C_{Ht}(f) \right) df$.

But we have derived the demand curve, equation (A10) in the appendix, $P_{Ht}(f)C_{Ht}(f)=\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{1-\xi}P_{Ht}C_{Ht}$. Substituting this into the integral above, we get:

$$\int_0^1\left(P_{Ht}(f)C_{Ht}(f)\right)df=\int_0^1\left(\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{1-\xi}P_{Ht}C_{Ht}\right)df=P_{Ht}^{\xi}C_{Ht}\int_0^1\left(\left(P_{Ht}(f)\right)^{1-\xi}\right)df\ .$$

But equation (A7) tells us $\int_0^1\left(\left(P_{Ht}(f)\right)^{1-\xi}\right)df=P_{Ht}^{1-\xi}$. Plugging this into the previous equation, we conclude $\int_0^1\left(P_{Ht}(f)C_{Ht}(f)\right)df=P_{Ht}C_{Ht}$.

Therefore, we have $C_{Ht}=\frac{\int_0^1\left(P_{Ht}(f)C_{Ht}(f)\right)df}{P_{Ht}}=\frac{\int_0^1 C_{Ht}(f)}{V_{Ht}}$

3. Suppose we did use this alternate definition, and utility was logarithmic in consumption. Show this would imply $Y_t=C_t$.

Answer:

Here, using equation (2) from the paper, the proposed alternative measure is:

$$Y_t=\frac{P_{Ht}C_{Ht}+E_tP_{Ht}^{*}C_{Ht}^{*}}{P_t}=\frac{\nu}{2}C_t+\left(1-\frac{\nu}{2}\right)\frac{E_tP_t^{*}C_t^{*}}{P_t}\ .$$

From the risk sharing condition when utility is logarithmic, $E_tP_t^{*}C_t^{*}=P_tC_t$, so we get:

$$Y_t=\frac{P_{Ht}C_{Ht}+E_tP_{Ht}^{*}C_{Ht}^{*}}{P_t}=\frac{\nu}{2}C_t+\left(1-\frac{\nu}{2}\right)\frac{P_tC_t}{P_t}=C_t$$

4. Why is there a z_t^2 term in equation (15), but it is no longer there in equation (29)?

Answer:

As the paper explains, if we start at a point in which $z_{-1}=0$, then to a first-order approximation, $z_t=0$ for all periods $t\geq 0$. Since only the square of z_t matters for the loss, only the first-order values of z_t matter for z_t^2, and therefore $z_t^2\approx 0$.

5. Consider the price setting decisions of home firms under either PCP or LCP (first equation on pages 2808 and 2809, respectively.) These optimization problems use the home stochastic discount factor to discount expected future dividends of the firm. Suppose instead the home firms were owned by foreigners and the foreign s.d.f. were used to discount future profits. How, if at all, would the pricing decisions be different?

Answer:

The pricing decisions would be the same because by the risk-sharing condition under complete markets, the s.d.f. of home and foreign households is the same.

6. Compare the targeting rules for PCP under commitment (equation (27)) and discretion (equation (28)). How is the Fed's decision in 2019 to pursue "average inflation" targeting rather than inflation targeting related to the difference between (27) and (28)?

Answer:

Equation (27) is a price-level targeting rule. In this case, the objective is to target the price level at its initial level in period -1. If the price rises above that level, then inflation on average should be negative until it returns to that level, and vice-versa if the price level falls below its initial level. Equation (28) says to target an inflation rate of 0 from time *t* onward, no matter what the actual outcome of prices is at time *t*. In essence, equation (27) is a rule that says to target an average inflation rate of zero, much like the Fed's 2019 rule.

7. In deriving the optimal policies under LCP, as the paper explains, I make the additional assumption that the parameter on labor supply in the utility function equals zero ($\phi = 0$.) One reason I did this is that it simplifies the analysis under discretion. We can solve for optimal policy under discretion by solving the Bellman equation. Write out the Bellman equation for the cooperative policymaker for the general case (that is, where we don't necessarily have $\phi = 0$), and then explain why this assumption simplifies things.

Answer:

The point here is that when $\phi = 0$, the s_t term is no longer a state variable. That is, in general, we have

$$V(s)$$
$$= \max_{\tilde{y}^R, \tilde{y}^W, m, \pi^R, \pi^W} \left(\frac{\sigma}{D} \left(\tilde{y}^R \right)^2 + \sigma \left(\tilde{y}^W \right)^2 + \frac{\nu(2-\nu)}{4D} m^2 \right) + \frac{\xi}{\delta} \left(\left(\pi^R \right)^2 + \left(\pi^W \right)^2 + \frac{\nu(2-\nu)}{4} \left(\left[\frac{2\sigma}{D} y^R - \frac{(\nu-1)}{D} m \right] - s \right) \right)$$
$$+\beta EV(s')$$

subject to the constraints imposed by the Phillips curves.

But when $\phi = 0$, the policymaker no longer has control over *s*, and it is just exogenous to the policymaker's problem. In this case, there are no state variables, and the policymaker's problem under discretion is just a static maximization problem.

8. This is a very simple question: show my (27) is the same as Bengui and Coulibaly's (21) and (22). This is essentially a question of whether you correctly interpret my (27).

Answer:

All I am asking you to recognize here is that when I take the first-difference of equation (27), I don't update the time subscript on the last term. That is, we have:

$\tilde{y}_t + \xi \left(p_{Ht} - p_{H-1} \right) = 0$ and $\tilde{y}_{t+1} + \xi \left(p_{Ht+1} - p_{H-1} \right) = 0$. So, taking differences, we have

$$\tilde{y}_{t+1} - \tilde{y}_t + \xi \pi_{Ht+1} = 0$$

9. What are "topsy-turvy" capital flows? Exactly what do the authors mean by this? Explain how this happens in the Cole-Obstfeld case.

Answer:

This means is that under the optimal policy, the trade is the opposite of the direction under free capital mobility. Under optimal controls, the trade balance moves the opposite direction of the relative output gaps. The country that is relatively booming runs a trade deficit, and the relatively more depressed country runs a trade surplus, assuming $\eta > 1$. We see the opposite relationship under free capital mobility. In the Cole-Obstfeld case, where $\eta = 1$, we get the case in which trade is always balanced under free capital mobility, but this is not constrained efficient. Even in this case, the optimal capital control policy is to reduce inflows for the country that is relatively booming. In terms of equation (26), the real wage effect still dominates the influence of the cost push shock on marginal costs.

Portfolio Choice

In this section, we will try to get some insight into international portfolio holdings and capital flows. A successful model would integrate the determination of portfolios with the asset pricing puzzles we discussed in the second section of readings (as well as the puzzles we did not discuss that are covered in the Ten Important Papers for that section.)

Returns but not portfolios

Finance research – as done by financial economists in finance departments and published in finance journals – has shown little interest in modeling portfolio choices in the past couple of decades. There are probably a few reasons for that. First, the puzzles on asset prices themselves are difficult and so it may be too much to build models that explain both returns and portfolios. Second, going back to the CAPM, models usually don't embed much heterogeneity in the preferences of investors. The CAPM allowed for people with different levels of risk aversion, and the famous result is that all investors allocate their risky assets in a mutual fund of the market portfolio and differ only in how they divide their wealth between the riskless asset and the risky mutual fund. In a closed economy, of course, macro models have for years assumed a representative agent, so this lack of heterogeneity is in accord with macro modeling. But in the open economy, as we have noted, investors in different countries necessarily are heterogeneous because they value asset returns differently, unless the real exchange rate is constant. I suspect a third reason for neglect of modeling of portfolios in the open-economy context in the finance field arises from the lack of success of standard models, a problem to which I will turn to in a bit.

In the international context, we certainly care about portfolios and not just returns because it may matter who owns which assets. For normative questions about monetary policy, macroprudential policy, capital flow management policies, the home/foreign ownership of assets matters. Moreover, from a positive standpoint of trying to understand the effects of shocks on the economy and how they influence GDP, inflation, exchange rates and the gross and net flows of assets, it will be important to get insight into the demand for different assets.

Capital flows

There has been a lot of empirical literature in the past 20 years on capital flows and asset flows among countries. I fear that a lot of the reduced-form regressions that are estimated here – while useful as guides to modeling open economies – have been subject either explicitly or implicitly to misinterpretation. Thinking about why the studies have been subject to misinterpretation helps us to understand what we need to get out of models.

Here are a couple of well-known findings about capital flows: When the Fed is raising interest rates, capital flows out of emerging markets and into the U.S., and vice-versa when the Fed is lowering rates. During times of global financial stress, there is "retrenchment" – investors in all countries bring their assets home.

These are statements about how investors alter their portfolios in response to shocks to the economy that might have some exogenous cause. But we don't really have a well-settled model of portfolio rebalancing. Start with the simplest case, which we should consider to be a benchmark of sorts – a representative world investor. Suppose that rate of return go up in the

U.S. What will happen to portfolios? If all investors are the same, they will not change their portfolios – who would they trade with, since everyone is the same? Instead, in response to the interest rate shock, some other return or asset price will have to change. Maybe it will be the exchange rate, so that the expected return on foreign assets changes in response to the change in U.S. interest rates (even if foreign interest rates themselves do not change.) Or, if global risk has changed, perhaps the price of all risky assets will fall.

That example illustrates two things: First, investors' portfolios rebalance in equilibrium even if the investor does nothing because the values of assets change. Second, for there to be capital flows (that is, actual trading of assets) across countries, exogenous changes must have some differential effect on the home and foreign demand for assets. Keep in mind, however, that even though there could be a differential change in demand for assets, that still does not necessarily imply asset trade will occur because home and foreign investors value returns in different currencies. That is, the exchange rate may adjust in response, so there may not be a necessity for capital flows.

Let me give another example to help illustrate the issues. Retrenchment is often interpreted as evidence that during times of global financial uncertainty, foreign investors sell their foreign assets because investors in each country now believe assets held in other countries have become riskier for them. But just looking at the flows is not enough to tell you that is the reason for retrenchment. For every seller of an asset, there must be a buyer, and so we need to know something more about not just why investors are selling foreign assets but why they are buying their own country's assets. Here is an example that cannot be ruled out just by looking at flows: perhaps risk does not increase for anybody during these times, but maybe home investors have private information about asset returns. Maybe during these times, home investors get a stronger signal about expected returns on their assets than foreign investors do, so they want to buy more of their own assets – because the relative expected return has risen from their perspective, not because foreign assets have become relatively riskier.

International portfolio choice

That leads us to models of portfolio choice, but unfortunately these models are hard. In the first place, it is difficult to build models that have convincing answers for the long-run average portfolios that are held globally. Secondly, it is even harder to explain how those portfolios may shift over time as the environment changes. And third, it is hardest to be able to also account for the international asset pricing puzzles.

I would say that this field is in its infancy. That may mean it's a great area to work in, though keep in mind that international finance economists have been struggling with these questions since the 1950s, so this infant is getting pretty old.

Two major puzzles in the long run

There are two major puzzles in the data that are hard to resolve. One is the home bias in equity puzzle, and the other is the "exorbitant privilege" puzzle. The home bias in equity puzzle asks why countries hold a disproportionate share of their equity in their own country's equities. An "unbiased" portfolio would say that if a country holds *x* percent of the worlds' equities, it will hold x percent of every country's equities. Instead, we find countries hold much more than x percent of their own country's equities and a smaller percentage of other countries'.

Several of the papers on the Ten Important Papers try to build models to address this issue. As the Baxter and Jermann paper points out, maybe countries should hold an even less than proportional share of their own equities, since foreign equities may be a hedge against non-capitalized income (such as labor income) that is positively correlated with the return to home equities. Some of the other papers argue, in essence, that this non-capitalized income may be negatively correlated with equity income, giving an incentive for home bias.

Sometimes papers refer to home bias in asset holdings in general, including bonds. There is certainly ambiguity of what that means. In many of our models, there is only one type of bond traded, perhaps denominated in units of traded goods. The borrowing country is short in the bonds and the lending country is long. In this context, what does home bias mean? That a country's net foreign assets are positive? For home bias to have any meaning, there must be a difference between bonds issued by the foreign country and by the home country. Even there, it still is not clear to me what home bias means. I guess it means that an individual lender in a country lends more to residents of his own country than to foreign countries, but in that context we must then be assuming heterogeneity of agents within a country (so we have borrowers and lenders within each country.) One possibility is that these models refer to government bonds – home agents hold a disproportionate share of home government bonds. However, we get back into the confusion of who is a lender and who is a borrower when we recognize that the supply of bonds by the government is not purely an exogenous asset issued by the government. These bonds are backed by a future tax liability for home residents. If a small home country holds 90% of its country's government bonds but is subject to 100% of the future tax liability, can we really say that there is home bias in this country's holdings of government bonds? We need to be on the lookout in partial equilibrium models for how tax liabilities are accounted for.

The second major puzzle is that over long periods of time, the U.S. has earned more on its foreign investments than foreigners have earned on investments in the U.S. This is called an "exorbitant privilege" of the U.S., and Gourinchas and Rey have written on it extensively. As a rough approximation, this difference in returns stems mainly from the fact that U.S. investments abroad are concentrated in riskier assets (equities and, notably, foreign direct investment), while foreign investments in the U.S. are more heavily weighted toward U.S. government bonds. As we will see over the rest of the semester, there are two good explanations for why U.S. government bonds pay a lower return on average than riskier investments. First, the dollar tends to appreciate during times of global downturn, making it a good hedge. However, the leading issue here has been to built plausible models of why the dollar appreciates during these times. The other explanation is that the U.S. government bonds can pay a lower return on average because they are useful for their liquidity. That is, they pay a non-monetary liquidity return.

Solving portfolio models in general equilibrium

Even a static portfolio allocation problem is difficult to solve in a general equilibrium macroeconomic model. Models of portfolio selection, such as CAPM, have the investor choosing a portfolio taking the moments of returns as given. We usually look at models where only the first two moments matter. In a simple case, imagine an investor choosing between home and foreign bonds, and assume the nominal interest rates are given, and the current exchange rate is known, so only the future exchange rate is uncertain. The portfolio choice problem involves allocating wealth between home and foreign bonds taking the mean and variance of the future exchange rate as given. (In the international context, the parameters of the problem may be different for home and foreign investors.)

In general equilibrium, there are exogenous forces driving these moments of the exchange rate, for example, monetary policy shocks, productivity shocks, etc. We can take the moments of the exogenous shocks as exogenously specified. But the moments of the endogenous variables, such as the exchange rate, must be solved for in the equilibrium of the model.

The problem is that the moments of the endogenous variables depend on the portfolio choices of the investors in equilibrium. On the one hand, investors' choices depend on the moments of the exchange rate, for example, but on the other hand, the moments of the exchange rate depend on the investors' choices.

Finding the solution is a fixed-point problem. In models that are simple enough to solve algebraically, we might "guess" that the moments of the exchange rate are functions of the moments of the underlying exogenous processes. We might use an undetermined coefficients method to solve the model.

Time-varying portfolios and moments

If we want to understand capital flows and portfolio adjustments, we need to allow the moments of returns to vary over time. For example, the demand for dollar assets arising from their risk properties might depend on the covariance of the exchange rate with consumption. But for the demand of agents to change over time in this case, we need a model in which the covariance changes over time.

A related problem is building models with time-varying risk premiums. For example, the UIP puzzle can be interpreted as saying that the foreign exchange risk premium on foreign short term bonds rises as the foreign nominal interest rate increases. In order for the risk premium to change, the covariance between exchange rates and consumption (for example) must vary over time.

Of course, another obvious hurdle with dynamic models is that the dynamic optimization problem and finding the equilibrium over time is inherently more difficult than a static problem.

The papers by Devereux and Sutherland introduce methods of approximation that allow us to solve these models. One way to think of the fundamental problem is, around which point are equations being approximated? In the monetary policy literature, for example, we approximate the stochastic model around a non-stochastic steady state. But a non-stochastic steady state is not helpful if we are trying to characterize portfolio choice. So what do we do? Just in order to get the long-run point around which we approximate, in essence the Devereux-Sutherland method proposes something like solving the long-run portfolio problem using unconditional moments, and then taking the limit as *k* goes to zero as all the exogenous shocks are scaled down by *k*.

The paper by Sauzet is a new entry into the field, which solves an international macro model with portfolio choice using global solution methods. This is quite a heroic effort, and I guess how far we can go using global methods to solve more complex models will depend on advances in computing power and programming innovations.

Finally, building a model that can account for both portfolio flows and asset returns puzzles is quite challenging, at least if we want to rely on models that are fully general equilibrium, dynamic, quantitatively plausible, and in which agents have rational expectations and maximize utility. In the end, at least so far, such models have not yielded realistic predictions about portfolios. It just doesn't seem like we can describe portfolios well when expected returns are driven entirely by covariances of returns with s.d.f.s, when the latter is based on utility functions and the former (the covariances) are derived in realistic macro models.

Our readings for this section

The two papers that I have assigned for this section do not fit the description in the previous paragraph. However, they are important attempts to build models – even if not fully general equilibrium, dynamic, optimizing, etc. – that can help rationalize what we see in the data. We can't just keep building more complex models and hope they resolve the puzzles, so these papers offer insights into what kinds of forces might be at work:

Gourinchas, Pierre-Olivier, Walker D. Ray, and Dimitri Vayanos. *A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers*. No. w29875. National Bureau of Economic Research, 2022.

Gourinchas, Pierre-Olivier, and Helene Rey. "Exorbitant privilege and exorbitant duty." (2022).

The first reading is the main focus of this section.

I realize that the paper might be hard to read because it makes use of continuous-time stochastic processes, which you may not be familiar with. Also, to be honest, anytime someone starts using matrix notation, it is harder to follow the equations. Maybe a good start, and sufficient for following this paper, is the Wikipedia entry on Ito's Lemma. The Wikipedia entry on Ito Calculus will take you farther. What you would need to know to read this paper is not very much different than, and not much harder than doing approximations in discrete time as we've already done this semester.

So, the continuous time shouldn't throw you off, though it makes the paper hard to work through line by line. Also, dynamics of course make things harder. But the real key to getting the neat, insightful conclusions from this paper comes from the introduction of investors with preferred habitats. In addition, note that the only optimizing agent – the arbitrageur – is not maximizing a present discounted value of expected future utility, but instead maximizing an instantaneous mean-variance function of returns. The mean-variance framework can fall out of assuming that the arbitrageur has constant absolute risk aversion (though this has weird implications, essentially meaning that people become more averse to variance as they get wealthier), but they still are only concerned with instantaneous utility.

Here, I'm going to give you a simple set-up from which you will see the intuition of some of the main results. I will only look at the special case in which there is an arbitrageur that is in international markets, or an arbitrageur that is in each local market (arbitraging between short- and long-term bonds) though it would not be too hard to think of the more general case in which the arbitrageur is in all markets. This example is only useful for intuition, and not for doing quantitative exercises as the paper does in section 4.

The mean-variance problem for the arbitrageur means that when there is a deviation from uncovered interest parity, the demand for bonds is finite – that is, they do not want to take an infinitely long position in one country's short-term bond and an infinitely short position in the other country's because of their aversion to risk. We can call their holdings of home minus foreign short-term bonds B_A^r, where the subscript *A* stands for arbitrageurs, and the superscript *r* means "relative" home to foreign bond holdings. If the arbitrageur has zero wealth, his long position in one asset is matched by a short position in the other asset.

Consider this a one-period model. The arbitrageur chooses a portfolio at the beginning of the period, when the home interest rate, *i*, the foreign interest rate, i^*, and the initial exchange rate, *s*, are known. The end of the period exchange rate (beginning of "next" period) is not known, and uncertain. The home and foreign interest rates are exogenous, chosen by the policymaker in each country. The current exchange rate can change in response to shocks. We have:

$$B_A^r = B_A^r\left(i - \left(i^* + s^e - s\right)\right), \qquad B_A^{r\prime} > 0,\ B_A^{r\prime\prime} < 0$$

s^e is the expected future exchange rate. The arbitrageur holds more home bonds and fewer foreign bonds the higher the expected return on home bonds, but because of risk aversion, $B_A^{r\prime\prime} < 0$.

If arbitrageurs were risk neutral, $B_A^{r\prime} = \infty$, $B_A^{r\prime\prime} = 0$, and u.i.p. would hold:

$$i - \left(i^* + s^e - s\right) = 0 .$$

There are also investors that have a "preferred habitat" position in foreign exchange markets. Their relative demand for home bonds depends on the level of the exchange rate – the more expensive is the foreign currency, the more home relative to foreign short-term bonds they will hold:

$$B_{PH}^r = B_{PH}^r\left(s\right), \qquad B_{PH}^{r}{}' > 0,\ B_{PH}^{r}{}'' < 0.$$

Let $\bar{B}^r$ be an exogenous supply of home bonds relative to foreign bonds. Perhaps this represents the supply of bonds coming from the government. Here, we would have to be assuming that nobody considers the future tax liabilities imposed by these bonds to be something any investor considers – or at least not something that our arbitrageurs and preferred habitat investors are concerned with.

Equilibrium in this market requires:
$B_A^r\left(i-\left(i^*+s^e-s\right)\right)+B_{PH}^r\left(s\right)=\bar{B}^r$.

Now for some questions about monetary policy and sterilized intervention:

1. Suppose the home monetary authority raises the home interest rate, holding the foreign interest rate, the total supply of bonds, and the expected future exchange rate constant.
    a. Derive and sign $\frac{ds}{di}$
    b. Derive and sign $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$
    c. Compare the magnitude of $\frac{ds}{di}$ to the size of $\frac{ds}{di}$ when there are no preferred habitat traders, so that $B_{PH}^r=0$.
    d. Compare the magnitude of $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$ to the size of $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$ when there are no preferred habitat traders, so that $B_{PH}^r=0$.
    e. Also compare $\frac{ds}{di}$ and $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$ in this model to the case in which u.i.p. holds.

You can see that your answers to these questions correspond to the qualitative findings of the model in the paper.

2. Next, let's consider sterilized intervention, in which the home country government sells short-term home bonds to the market and buys short-term foreign bonds, thereby increasing $\bar{B}^r$. In this exercise we hold the foreign and home interest rates constant, and the expected future exchange rate constant.
    a. Derive and sign $\frac{ds}{d\bar{B}^r}$

b. Derive and sign $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$

c. Compare the magnitude of $\dfrac{ds}{d\bar{B}^{r}}$ to the size of $\dfrac{ds}{d\bar{B}^{r}}$ when there are no preferred habitat traders, so that $B_{PH}^{r}=0$.

d. Compare the magnitude of $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$ to the size of $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$ when there are no preferred habitat traders, so that $B_{PH}^{r}=0$.

e. Also compare $\dfrac{ds}{d\bar{B}^{r}}$ and $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$ in this model to the case in which u.i.p. holds.

Now let's turn to the home market and assume that there is an arbitrageur that chooses between short-term and long-term domestic bonds, as well as a preferred habitat trader in that market. Long-term bonds are zero-coupon bonds, and their expected return is approximated by $p^{e}-p$, where p is the price of the long-term bond. The expected return on the short-term bond compared to the long-term bond is therefore $i-\left(p^{e}-p\right)$. The relative demand for short-term to long-term bonds by the arbitrageur is given by:

$$B_{AL}^{r}=B_{AL}^{r}\left(i-\left(p^{e}-p\right)\right), \qquad B_{AL}^{r}{}'>0,\ B_{AL}^{r}{}''<0$$

If arbitrageurs were risk neutral, $B_{AL}^{r}{}'=\infty$, $B_{AL}^{r}{}''=0$, and expectations hypothesis would hold:

$$i-\left(p^{e}-p\right)=0.$$

There are also investors that have a "preferred habitat" position in short- vs. long-term bonds. Their relative demand for short-term bonds depends on the level of the long-term bond price – the more expensive is the long-term bond, the more short- relative to long-term bonds they will hold:

$$B_{PHL}^{r}=B_{PHL}^{r}\left(p\right), \qquad B_{PHL}^{r}{}'>0,\ B_{PHL}^{r}{}''<0.$$

Let $\bar{B}_{L}^{r}$ be an exogenous supply of short-term bonds relative to long-term bonds. Perhaps this represents the supply of bonds coming from the government. Here, we would have t

Equilibrium in this market requires:

$$B_{AL}^{r}\left(i-\left(p^{e}-p\right)\right)+B_{PHL}^{r}\left(p\right)=\bar{B}_{L}^{r}.$$

Now for some questions about conventional and unconventional monetary policy:

3. Suppose the home monetary authority raises the home interest rate, holding the the total supply of bonds, and the expected future price of long-term bonds constant.
    a. Derive and sign $\frac{dp}{di}$
    b. Derive and sign $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$
    c. Compare the magnitude of $\frac{dp}{di}$ to the size of $\frac{dp}{di}$ when there are no preferred habitat traders, so that $B_{PHL}^r = 0$.
    d. Compare the magnitude of $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$ to the size of $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$ when there are no preferred habitat traders, so that $B_{PHL}^r = 0$.
    e. Also compare $\frac{dp}{di}$ and $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$ in this model to the case in which the expectations hypothesis holds.

4. Next, let's consider unconventional monetary policy, in which the home country government sells short-term home bonds to the market and buys long-term bonds, thereby increasing $\bar{B}_L^r$. In this exercise we hold the home interest rate constant, and the expected future price of long-term bonds constant.
    a. Derive and sign $\frac{dp}{d\bar{B}_L^r}$
    b. Derive and sign $\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$
    c. Compare the magnitude of $\frac{dp}{d\bar{B}_L^r}$ to the size of $\frac{dp}{d\bar{B}_L^r}$ when there are no preferred habitat traders, so that $B_{PHL}^r = 0$.
    d. Compare the magnitude of $\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$ to the size of $\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$ when there are no preferred habitat traders, so that $B_{PHL}^r = 0$.
    e. Also compare $\frac{dp}{d\bar{B}_L^r}$ and $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$ in this model to the case in which the expectations hypothesis holds.

You should be able to see the parallel between the results from this simple model and the results described in the paper, and, I hope, get some intuition of what is driving the results. Now we can also put the previous questions together to ask about the effects of policies on long-run uncovered interest rate parity. By long run interest rate parity, I mean:

$$p^e - p - \left(p^{*e} - p^* + s^e - s\right).$$

To keep things simple, let's assume the foreign market is populated only by foreign bond aribtrageurs and foreign preferred habitat traders, and that monetary policy, sterilized intervention, and unconventional monetary policy have no effects on expected long-term foreign bond returns. For simplification purposes, then, set $p^{*e} - p^* = 0$. Otherwise, assume that the domestic bond market has its arbitrageurs and preferred habitat traders (as in questions 3 and 4), and the foreign exchange market has a separate set of arbitrageurs and preferred habitat traders (as in questions 1 and 2.) Setting $p^{*e} - p^* = 0$, we are interested in the effects of policies on:

$$p^e - p - \left(s^e - s\right).$$

Note that this equals $i - \left(s^e - s\right) - \left[i - \left(p^e - p\right)\right]$, (and we'll set $i^* = 0$ since we won't change foreign monetary policy.)

If we are changing some exogenous variable, *x*, we have

$$\frac{d\left\{i - \left(s^e - s\right) - \left[i - \left(p^e - p\right)\right]\right\}}{dx} = \frac{d\left[i - \left(s^e - s\right)\right]}{dx} - \frac{d\left[i - \left(p^e - p\right)\right]}{dx}$$

5. Compare the effect of an increase in the domestic interest rate, *i*, on deviations from short-term uncovered interest-rate parity to deviations from long-term uncovered interest rate parity. That is, compare $\frac{d\left[i - \left(s^e - s\right)\right]}{di}$ to $\frac{d\left[p^e - p - \left(s^e - s\right)\right]}{di}$, holding expected future exchange rates and long-term bond prices constant.
6. Suppose that the monetary authority sells short term bonds to the public in exchange for equal amounts of long-term bonds and foreign short-term bonds, such that $d\bar{B} \equiv d\bar{B}^r = d\bar{B}_L^r$. Compare $\frac{d\left[p^e - p - \left(s^e - s\right)\right]}{d\bar{B}}$ to $\frac{d\left[i - \left(s^e - s\right)\right]}{d\bar{B}^r}$.

The paper goes on, in the analytic section, to draw interesting conclusions about various shocks when the arbitrageur is active in all the markets. That would be a little more complicated to do in this static setting but it could be done. I think the intuitive effects of many of the comparative exercises done in the paper can be well understood with this simple description, but not the quantitative exercises of section 5.

I would ask that you read sections 1, 2, and 4 carefully. You can skip the derivations in section 3 entirely, but then also read the quantitative results of section 5.

Given the time constraints we now face, I don't ask that you spend a lot of time on the Gourinchas-Rey paper. The first half of the paper neatly summarizes, with some new empirical facts, some arguments that these two authors have made over the past two decades.

Let's focus a bit on the model, and what it is trying to achieve. As the data suggests, dollar assets are a good hedge during times of global financial stress. It is an asset that acts like insurance. Its expected return is less than that of other assets because it is a hedge.

But the question is, why does the dollar appreciate during these times of global fragility? You might be tempted to say that people flee to the safe asset during these times, and that in fact is sort of what this paper says, but you need to be careful. Is it worthwhile to buy fire insurance once your house has caught fire? Probably not, because the cost of the insurance at that point might be the whole value of the house. That is, once the global crisis starts, it may be too late to buy the insurance.

The idea of the model of this paper, which I think is realistic, is that people might still want to buy the insurance when things turn bad, because things could get even worse. So there are three possible states – normal, fragile and disaster. In the fragile state, the probability of transition to disaster is higher than from the normal state, but it still might make sense to buy the dollar assets at that point. So, what we are observing in the data – the dollar appreciates in bad times – is actually the dollar appreciating in the fragile state (not the disaster state.) To draw the analogy to fire insurance again, once your house catches on fire, it may not burn to the ground with certainty. There is a higher probability it will than if the house wasn't on fire, but it still might make sense to buy insurance at that time. The insurance will be more expensive than in normal times because your house is on fire, and that is exactly the analogy to the dollar becoming more expensive in fragile times in this model.

I am not entirely sure how they go about solving the model. Here is what I think is done: The model is solved first assuming complete markets. From that, they can derive consumption, and therefore stochastic discount factors. They can solve for the returns on home equity, and foreign equity or global equity as the returns to trees in each country (which they can derive from the payoffs of the trees and the s.d.f.s) The home and foreign bonds are assumed to pay the local price index each period, so the rates of return from those can also be calculated using the output from the complete markets model. They can calculate the return on each country's wealth simply by seeing how much their wealth rose over previous unconsumed wealth.

Then with some algebra, they show how they can recover time-varying portfolio weights in each country by running a regression. But this is just an approximation to the complete markets outcome.

Here is what I found a disappointing part of the paper: I thought the story was going to be that in fragile times, the home currency appreciated because people were buying insurance against the disaster state. That much is true, but I thought the story was going to be that in the disaster state, the dollar appreciated, so it was a good hedge against global disaster. But in fact, the dollar depreciates in the disaster state. The insurance aspect is that the foreign bond partially defaults in the disaster state.

The reason the dollar depreciates in the disaster state is this: First, the U.S. holds more global equities, and is a dollar debtor. The global disaster lowers equity prices, which hurts the U.S. relatively more. Because its wealth falls, its consumption drops. That leads to a drop in its nontraded prices, which means a depreciation.

Note also that if the foreigners' risk aversion did not increase in fragile times, the excess return on foreign bonds would be much lower, and the real exchange rate does not change between normal times and fragile times. The reason the real exchange rate does not change is that even though in the fragile state we are more likely to fall into disaster, that increases the precautionary motive of both foreign and home, holding risk aversion constant, so has no effect on the real exchange rate.

Also, note that the disaster state here is very bad, and occurs relatively frequently.

But we should consider this as a step toward understanding why investors indeed do seem to buy dollar assets in a downturn. Another reason that recent literature has mentioned is that dollar bonds are valued for their liquidity, and the value of that liquidity rises during times of uncertainty or greater financial constraints.

Let me ask one question about the paper:

7. Can you think of a model set-up where, even when the U.S. income or wealth declines relative to the rest of the world, there is a real appreciation? In other words, the drop in wealth for the U.S. does not necessitate a drop in the price of the consumer basket in the U.S. relative to the price of the consumer basket in the rest of the world.

Ten (+1) Important Papers

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Coeurdacier, Nicolas, and Pierre-Olivier Gourinchas. "When bonds matter: Home bias in goods and assets." *Journal of Monetary Economics* 82 (2016): 119-137.

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Devereux, Michael B., and Alan Sutherland. "Country portfolios in open economy macro-models." *Journal of the european economic Association* 9.2 (2011): 337-369.

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Gourinchas, Pierre-Olivier, Hélene Rey, and Maxime Sauzet. "The international monetary and financial system." *Annual Review of Economics* 11 (2019): 859-893.

Heathcote, Jonathan, and Fabrizio Perri. "The international diversification puzzle is not as bad as you think." *Journal of Political Economy* 121.6 (2013): 1108-1159.

Koijen, Ralph SJ, and Motohiro Yogo. *Exchange rates and asset prices in a global demand system*. No. w27342. National Bureau of Economic Research, 2020.

Sauzet, Maxime. "Asset prices, global portfolios, and the international financial system." Working paper, Boston University (2022).

<u>Portfolio Choice</u>

Consider this a one-period model. The arbitrageur chooses a portfolio at the beginning of the period, when the home interest rate, *i*, the foreign interest rate, i^*, and the initial exchange rate, *s*, are known. The end of the period exchange rate (beginning of "next" period) is not known, and uncertain. The home and foreign interest rates are exogenous, chosen by the policymaker in each country. The current exchange rate can change in response to shocks. We have:

$$B_A^r = B_A^r\left(i - \left(i^* + s^e - s\right)\right), \qquad B_A^{r\prime} > 0,\ B_A^{r\prime\prime} < 0$$

s^e is the expected future exchange rate. The arbitrageur holds more home bonds and fewer foreign bonds the higher the expected return on home bonds, but because of risk aversion, $B_A^{r\prime\prime} < 0$.

If arbitrageurs were risk neutral, $B_A^{r\prime} = \infty$, $B_A^{r\prime\prime} = 0$, and u.i.p. would hold:

$$i - \left(i^* + s^e - s\right) = 0 .$$

There are also investors that have a "preferred habitat" position in foreign exchange markets. Their relative demand for home bonds depends on the level of the exchange rate – the more expensive is the foreign currency, the more home relative to foreign short-term bonds they will hold:

$$B_{PH}^r = B_{PH}^r\left(s\right), \qquad B_{PH}^r{}' > 0,\ B_{PH}^r{}'' < 0.$$

Let $\bar{B}^r$ be an exogenous supply of home bonds relative to foreign bonds. Perhaps this represents the supply of bonds coming from the government. Here, we would have to be assuming that nobody considers the future tax liabilities imposed by these bonds to be something any investor considers – or at least not something that our arbitrageurs and preferred habitat investors are concerned with.

Equilibrium in this market requires:

$$B_A^r\left(i - \left(i^* + s^e - s\right)\right) + B_{PH}^r\left(s\right) = \bar{B}^r .$$

Now for some questions about monetary policy and sterilized intervention:

1. Suppose the home monetary authority raises the home interest rate, holding the foreign interest rate, the total supply of bonds, and the expected future exchange rate constant.

a. Derive and sign $\frac{ds}{di}$

Answer:

$$B_A^r\left(i-\left(i^*+s^e-s\right)\right)+B_{PH}^r\left(s\right)=\bar{B}^r$$

$$B_A^{r\prime}\left(1+\frac{ds}{di}\right)+B_{PH}^r{}'\left(\frac{ds}{di}\right)=0$$

$$\frac{ds}{di}=\frac{-B_A^{r\prime}}{B_A^{r\prime}+B_{PH}^r{}'}<0$$

b. Derive and sign $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$

Answer:

$$\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}=1+\frac{ds}{di}=\frac{B_{PH}^r{}'}{B_A^{r\prime}+B_{PH}^r{}'}>0$$

c. Compare the magnitude of $\frac{ds}{di}$ to the size of $\frac{ds}{di}$ when there are no preferred habitat traders, so that $B_{PH}^r=0$.

Answer:

	Model	$B_{PH}^r=0$
$\frac{ds}{di}$	$\frac{-B_A^{r\prime}}{B_A^{r\prime}+B_{PH}^r{}'}$	-1

The absolute value of the appreciation is greater when there are no preferred habitat traders.

d. Compare the magnitude of $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$ to the size of $\frac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$ when there are no preferred habitat traders, so that $B_{PH}^r=0$.

Answer:

	Model	$B_{PH}^r = 0$
$\dfrac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$	$\dfrac{B_{PH}^{r}{}'}{B_A^{r}{}'+B_{PH}^{r}{}'}$	0

Obviously, the absolute value of the effect on expected excess returns is larger in the model with preferred habitat traders.

e. Also compare $\dfrac{ds}{di}$ and $\dfrac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$ in this model to the case in which u.i.p. holds.

Answer:

	Model	U.I.P.
$\dfrac{ds}{di}$	$\dfrac{-B_A^{r}{}'}{B_A^{r}{}'+B_{PH}^{r}{}'}$	-1
$\dfrac{d\left[i-\left(i^*+s^e-s\right)\right]}{di}$	$\dfrac{B_{PH}^{r}{}'}{B_A^{r}{}'+B_{PH}^{r}{}'}$	0

The absolute value of the appreciation is greater under uncovered interest parity, but obviously, the absolute value of the effect on expected excess returns is larger in the model with preferred habitat traders.

2. Next, let's consider sterilized intervention, in which the home country government sells short-term home bonds to the market and buys short-term foreign bonds, thereby increasing $\bar{B}^r$. In this exercise we hold the foreign and home interest rates constant, and the expected future exchange rate constant.

    a. Derive and sign $\dfrac{ds}{d\bar{B}^r}$

Answer:

$$B_A^r\left(i-\left(i^*+s^e-s\right)\right)+B_{PH}^r\left(s\right)=\bar{B}^r$$

$$B_A^{r}{}'\frac{ds}{d\bar{B}^r}+B_{PH}^{r}{}'\frac{ds}{d\bar{B}^r}=1$$

$$\frac{ds}{d\bar{B}^r}=\frac{1}{B_A^{r}{}'+B_{PH}^{r}{}'}>0$$

b. Derive and sign $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$

Answer:

$$\frac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}=\frac{ds}{d\bar{B}^{r}}=\frac{1}{B_{A}^{r}{}'+B_{PH}^{r}{}'}>0$$

c. Compare the magnitude of $\dfrac{ds}{d\bar{B}^{r}}$ to the size of $\dfrac{ds}{d\bar{B}^{r}}$ when there are no preferred habitat traders, so that $B_{PH}^{r}=0$.

Answer:

	Model	$B_{PH}^{r}=0$
$\dfrac{ds}{d\bar{B}^{r}}$	$\dfrac{1}{B_{A}^{r}{}'+B_{PH}^{r}{}'}$	$\dfrac{1}{B_{A}^{r}{}'}$

The effect is larger in the model without preferred habitat traders.

d. Compare the magnitude of $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$ to the size of $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$ when there are no preferred habitat traders, so that $B_{PH}^{r}=0$.

Answer:

	Model	$B_{PH}^{r}=0$
$\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$	$\dfrac{1}{B_{A}^{r}{}'+B_{PH}^{r}{}'}$	$\dfrac{1}{B_{A}^{r}{}'}$

The effect is larger in the model without preferred habitat traders.

e. Also compare $\dfrac{ds}{d\bar{B}^{r}}$ and $\dfrac{d\left[i-\left(i^{*}+s^{e}-s\right)\right]}{d\bar{B}^{r}}$ in this model to the case in which u.i.p. holds.

Answer:

	Model	U.I.P.
$\dfrac{ds}{d\bar{B}^r}$	$\dfrac{1}{B_A^{r\,\prime} + B_{PH}^{r}{}'}$	0
$\dfrac{d\left[i-\left(i^* + s^e - s\right)\right]}{d\bar{B}^r}$	$\dfrac{1}{B_A^{r\,\prime} + B_{PH}^{r}{}'}$	0

There is no effect on returns or the exchange rate of sterilized intervention under uncovered interest parity.

Now let's turn to the home market and assume that there is an arbitrageur that chooses between short-term and long-term domestic bonds, as well as a preferred habitat trader in that market. Long-term bonds are zero-coupon bonds, and their expected return is approximated by $p^e - p$, where p is the price of the long-term bond. The expected return on the short-term bond compared to the long-term bond is therefore $i-\left(p^e - p\right)$. The relative demand for short-term to long-term bonds by the arbitrageur is given by:

$$B_{AL}^r = B_{AL}^r\left(i-\left(p^e - p\right)\right), \qquad B_{AL}^{r}{}' > 0,\ B_{AL}^{r}{}'' < 0$$

If arbitrageurs were risk neutral, $B_{AL}^{r}{}' = \infty$, $B_{AL}^{r}{}'' = 0$, and expectations hypothesis would hold:

$$i-\left(p^e - p\right) = 0.$$

There are also investors that have a "preferred habitat" position in short- vs. long-term bonds. Their relative demand for short-term bonds depends on the level of the long-term bond price – the more expensive is the long-term bond, the more short- relative to long-term bonds they will hold:

$$B_{PHL}^r = B_{PHL}^r\left(p\right), \qquad B_{PHL}^{r}{}' > 0,\ B_{PHL}^{r}{}'' < 0.$$

Let $\bar{B}_L^r$ be an exogenous supply of short-term bonds relative to long-term bonds. Perhaps this represents the supply of bonds coming from the government. Here, we would have t

Equilibrium in this market requires:

$$B_{AL}^r\left(i-\left(p^e - p\right)\right) + B_{PHL}^r\left(p\right) = \bar{B}_L^r.$$

Now for some questions about conventional and unconventional monetary policy:

3. Suppose the home monetary authority raises the home interest rate, holding the the total supply of bonds, and the expected future price of long-term bonds constant.

a. Derive and sign $\frac{dp}{di}$

Answer:

$$B_{AL}^{r}\left(i-\left(p^{e}-p\right)\right)+B_{PHL}^{r}\left(p\right)=\bar{B}_{L}^{r}$$

$$B_{AL}^{r}{}'\left(1+\frac{dp}{di}\right)+B_{PHL}^{r}{}'\left(\frac{dp}{di}\right)=0$$

$$\frac{dp}{di}=-\frac{B_{AL}^{r}{}'}{B_{AL}^{r}{}'+B_{PHL}^{r}{}'}<0$$

b. Derive and sign $\frac{d\left[i-\left(p^{e}-p\right)\right]}{di}$

Answer:

$$\frac{d\left[i-\left(p^{e}-p\right)\right]}{di}=1+\frac{dp}{di}=\frac{B_{PHL}^{r}{}'}{B_{AL}^{r}{}'+B_{PHL}^{r}{}'}>0$$

c. Compare the magnitude of $\frac{dp}{di}$ to the size of $\frac{dp}{di}$ when there are no preferred habitat traders, so that $B_{PHL}^{r}=0$.

Answer:

	Model	$B_{PHL}^{r}=0$
$\frac{dp}{di}$	$\frac{-B_{AL}^{r}{}'}{B_{AL}^{r}{}'+B_{PHL}^{r}{}'}$	-1

The absolute value of the price change is greater when there are no preferred habitat traders.

d. Compare the magnitude of $\frac{d\left[i-\left(p^{e}-p\right)\right]}{di}$ to the size of $\frac{d\left[i-\left(p^{e}-p\right)\right]}{di}$ when there are no preferred habitat traders, so that $B_{PHL}^{r}=0$.

Answer:

	Model	$B_{PHL}^{r}=0$
$\frac{d\left[i-\left(p^{e}-p\right)\right]}{di}$	$\frac{B_{PH}^{r}{}'}{B_{A}^{r}{}'+B_{PH}^{r}{}'}$	0

Obviously, the effect is larger when there are preferred habitat traders.

e. Also compare $\frac{dp}{di}$ and $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$ in this model to the case in which the expectations hypothesis holds.

Answer:

	Model	Expectations Hypothesis
$\frac{dp}{di}$	$\frac{-B_A^{r\prime}}{B_A^{r\prime}+B_{PH}^{r}{}'}$	-1
$\frac{d\left[i-\left(p^e-p\right)\right]}{di}$	$\frac{B_{PH}^{r}{}'}{B_A^{r\prime}+B_{PH}^{r}{}'}$	0

The absolute value of the price change is greater under the expectations hypothesis, but obviously, the absolute value of the effect on expected excess returns is larger in the model with preferred habitat traders.

4. Next, let's consider unconventional monetary policy, in which the home country government sells short-term home bonds to the market and buys long-term bonds, thereby increasing $\bar{B}_L^r$. In this exercise we hold the home interest rate constant, and the expected future price of long-term bonds constant.

   a. Derive and sign $\frac{dp}{d\bar{B}_L^r}$

Answer:

$$B_{AL}^r\left(i-\left(p^e-p\right)\right)+B_{PHL}^r\left(p\right)=\bar{B}_L^r$$

$$\frac{dp}{d\bar{B}_L^r}B_{AL}^r{}'+\frac{dp}{d\bar{B}_L^r}B_{PHL}^r{}'=1$$

$$\frac{dp}{d\bar{B}_L^r}=\frac{1}{B_{AL}^r{}'+B_{PHL}^r{}'}$$

   b. Derive and sign $\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$

Answer:

$$\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}=\frac{dp}{d\bar{B}_L^r}=\frac{1}{B_{AL}^r{}'+B_{PHL}^r{}'}>0$$

c. Compare the magnitude of $\frac{dp}{d\bar{B}_L^r}$ to the size of $\frac{dp}{d\bar{B}_L^r}$ when there are no preferred habitat traders, so that $B_{PHL}^r = 0$.

Answer:

	Model	$B_{PH}^r = 0$
$\frac{dp}{d\bar{B}_L^r}$	$\frac{1}{B_{AL}^r{}' + B_{PHL}^r{}'}$	$\frac{1}{B_{AL}^r{}'}$

The effect is larger in the model without preferred habitat traders.

d. Compare the magnitude of $\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$ to the size of $\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$ when there are no preferred habitat traders, so that $B_{PHL}^r = 0$.

Answer:

Answer:

	Model	$B_{PH}^r = 0$
$\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$	$\frac{1}{B_{AL}^r{}' + B_{PHL}^r{}'}$	$\frac{1}{B_{AL}^r{}'}$

The effect is larger in the model without preferred habitat traders.

e. Also compare $\frac{dp}{d\bar{B}_L^r}$ and $\frac{d\left[i-\left(p^e-p\right)\right]}{di}$ in this model to the case in which the expectations hypothesis holds.

Answer:

	Model	Expectations Hypothesis
$\frac{dp}{d\bar{B}_L^r}$	$\frac{1}{B_{AL}^r{}' + B_{PHL}^r{}'}$	0
$\frac{d\left[i-\left(p^e-p\right)\right]}{d\bar{B}_L^r}$	$\frac{1}{B_{AL}^r{}' + B_{PHL}^r{}'}$	0

There is no effect on returns or the long-term bond price of quantitative easing under the expectations hypothesis.

You should be able to see the parallel between the results from this simple model and the results described in the paper, and, I hope, get some intuition of what is driving the results. Now we can also put the previous questions together to ask about the effects of policies on long-run uncovered interest rate parity. By long run interest rate parity, I mean:

$$p^e - p - \left(p^{*e} - p^* + s^e - s \right).$$

To keep things simple, let's assume the foreign market is populated only by foreign bond aribtrageurs and foreign preferred habitat traders, and that monetary policy, sterilized intervention, and unconventional monetary policy have no effects on expected long-term foreign bond returns. For simplification purposes, then, set $p^{*e} - p^* = 0$. Otherwise, assume that the domestic bond market has its arbitrageurs and preferred habitat traders (as in questions 3 and 4), and the foreign exchange market has a separate set of arbitrageurs and preferred habitat traders (as in questions 1 and 2.) Setting $p^{*e} - p^* = 0$, we are interested in the effects of policies on:

$$p^e - p - \left(s^e - s \right).$$

Note that this equals $i - \left(s^e - s \right) - \left[i - \left(p^e - p \right) \right]$, (and we'll set $i^* = 0$ since we won't change foreign monetary policy.)

If we are changing some exogenous variable, *x*, we have

$$\frac{d\left\{ i - \left(s^e - s \right) - \left[i - \left(p^e - p \right) \right] \right\}}{dx} = \frac{d\left[i - \left(s^e - s \right) \right]}{dx} - \frac{d\left[i - \left(p^e - p \right) \right]}{dx}$$

5. Compare the effect of an increase in the domestic interest rate, *i*, on deviations from short-term uncovered interest-rate parity to deviations from long-term uncovered interest rate parity. That is, compare $\frac{d\left[i - \left(s^e - s \right) \right]}{di}$ to $\frac{d\left[p^e - p - \left(s^e - s \right) \right]}{di}$, holding expected future exchange rates and long-term bond prices constant.

<u>Answer:</u>

We had

$$\frac{d\left[i - \left(i^* + s^e - s \right) \right]}{di} = \frac{B_{PH}^{r}{}'}{B_A^{r}{}' + B_{PH}^{r}{}'}$$ and then we have:

$$\frac{d\left[p^e - p - \left(s^e - s\right)\right]}{di} = \frac{d\left[i - \left(s^e - s\right)\right]}{di} - \frac{d\left[i - \left(p^e - p\right)\right]}{di}$$

$$= \frac{B_{PH}^{r}{}'}{B_A^{r}{}' + B_{PH}^{r}{}'} - \frac{B_{PHL}^{r}{}'}{B_{AL}^{r}{}' + B_{PHL}^{r}{}'} < \frac{d\left[i - \left(s^e - s\right)\right]}{di}$$

6. Suppose that the monetary authority sells short term bonds to the public in exchange for equal amounts of long-term bonds and foreign short-term bonds, such that $d\bar{B} \equiv d\bar{B}^r = d\bar{B}_L^r$. Compare $\frac{d\left[p^e - p - \left(s^e - s\right)\right]}{d\bar{B}}$ to $\frac{d\left[i - \left(s^e - s\right)\right]}{d\bar{B}^r}$.

Answer:

We had $\frac{d\left[i - \left(i^* + s^e - s\right)\right]}{d\bar{B}^r} = \frac{1}{B_A^{r}{}' + B_{PH}^{r}{}'}$, and then we have:

$$\frac{d\left[p^e - p - \left(s^e - s\right)\right]}{d\bar{B}} = \frac{d\left[i - \left(s^e - s\right)\right]}{d\bar{B}^r} - \frac{d\left[i - \left(p^e - p\right)\right]}{d\bar{B}_L^e}$$

$$= \frac{1}{B_A^{r}{}' + B_{PH}^{r}{}'} - \frac{1}{B_{AL}^{r}{}' + B_{PHL}^{r}{}'} < \frac{d\left[i - \left(s^e - s\right)\right]}{d\bar{B}^r}$$

7. Can you think of a model set-up where, even when the U.S. income or wealth declines relative to the rest of the world, there is a real appreciation? In other words, the drop in wealth for the U.S. does not necessitate a drop in the price of the consumer basket in the U.S. relative to the price of the consumer basket in the rest of the world.

Answer:

My opinion is that we do not need to tie the real exchange rate very closely to changes in the relative prices of goods within a country (either the price of nontraded goods to traded goods, or the price of home-produced traded goods to foreign produced traded goods.) I think most fluctuations of the real exchange rate are deviations from the law of one price, as under LCP. Therefore, while there may be a wealth transfer from the U.S. during global downturns, that can easily be reconciled with a real U.S. appreciation under LCP.

Assignment for last two weeks of class.

Given the time constraint in the class, I will simply lecture on these papers and my lecture slides are included:

Bianchi, Javier. "Overborrowing and systemic externalities in the business cycle." *American Economic Review* 101.7 (2011): 3400-3426.
Gabaix, Xavier, and Matteo Maggiori. "International liquidity and exchange rate dynamics." *The Quarterly Journal of Economics* 130.3 (2015): 1369-1420.
Itskhoki, Oleg, and Dmitry Mukhin. "Exchange rate disconnect in general equilibrium." *Journal of Political Economy* 129.8 (2021): 2183-2232.
Devereux, Michael B., Charles Engel, and Steve Pak Yeung Wu. "Collateral advantage: Exchange rates, capital flows, and global cycles." (2023).

For your final assignment, I want you to write a referee report on the last paper (the one by Devereux, Wu and me.) Call this paper DEW below. This should be around 1000-1500 words (for example, 3-4 pages in Word, 12-point font, 1.5 space.)

There are two things I'd like you to do in the referee report:

- Compare and contrast the paper with the other three papers. What features do those papers have that are desirable that DEW does not? What features does DEW have that are desirable that the others do not.
- What suggestions for further work or improvements in DEW do you have?

You can bring up both matters of substance and any sort of technical matter you think is relevant, as referee reports do.

The papers by Gabaix-Maggiori and Itskhoki-Mukhin obviously have a connection to DEW, since all three papers model financial intermediaries that choose portfolios. The connection to Bianchi is less obvious but I'd like you to make comparisons. The Bianchi paper is for a small-open economy, while the other three are two-country models. In the Bianchi model, household borrowing for consumption faces a constraint, while in the other papers, it is a financial intermediary that is constrained. (However, keep in mind that, in DEW, households own the bank and therefore own the balance sheet of the bank. Also, effectively, governments are borrowing on behalf of the households since they transfer the proceeds of borrowing to households, and tax households to make interest payments.) You might comment on the pros and cons of each approach for different economic issues.

Ten Important Papers related to Bianchi (2011)

Benigno, Gianluca, et al. "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective." *Journal of monetary economics* 84 (2016): 147-165.

Bianchi, Javier, and Enrique G. Mendoza. "Optimal time-consistent macroprudential policy." *Journal of Political Economy* 126.2 (2018): 588-634.

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Caballero, Ricardo J., and Arvind Krishnamurthy. "International and domestic collateral constraints in a model of emerging market crises." *Journal of monetary Economics* 48.3 (2001): 513-548.

Dávila, Eduardo, and Anton Korinek. "Pecuniary externalities in economies with financial frictions." *The Review of Economic Studies* 85.1 (2018): 352-395.

Dornbusch, Rudiger, et al. "Currency crises and collapses." *Brookings papers on economic activity* 1995.2 (1995): 219-293.

Farhi, Emmanuel, and Iván Werning. "A theory of macroprudential policies in the presence of nominal rigidities." *Econometrica* 84.5 (2016): 1645-1704.

Korinek, Anton. "Regulating capital flows to emerging markets: An externality view." *Journal of International Economics* 111 (2018): 61-80.

Lorenzoni, Guido. "Inefficient credit booms." *The Review of Economic Studies* 75.3 (2008): 809-833.

Mendoza, Enrique G. "Sudden stops, financial crises, and leverage." *American Economic Review* 100.5 (2010): 1941-1966.

Ten Important Papers related to Gabaix-Maggiori

Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe. "Money, interest rates, and exchange rates with endogenously segmented markets." *Journal of political Economy* 110.1 (2002): 73-112.

Banerjee, Ryan, Michael B. Devereux, and Giovanni Lombardo. "Self-oriented monetary policy, global financial markets and excess volatility of international capital flows." *Journal of International Money and Finance* 68 (2016): 275-297.

Branson, William H., and Dale W. Henderson. "The specification and influence of asset markets." *Handbook of international economics* 2 (1985): 749-805.

Bruno, Valentina, and Hyun Song Shin. "Capital flows and the risk-taking channel of monetary policy." *Journal of monetary economics* 71 (2015): 119-132.

Bruno, Valentina, and Hyun Song Shin. "Cross-border banking and global liquidity." *The Review of Economic Studies* 82.2 (2015): 535-564.

Dedola, Luca, Peter Karadi, and Giovanni Lombardo. "Global implications of national unconventional policies." *Journal of Monetary Economics* 60.1 (2013): 66-85.

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Maggiori, Matteo. "Financial intermediation, international risk sharing, and reserve currencies." *American Economic Review* 107.10 (2017): 3038-3071.

Miranda-Agrippino, Silvia, and Hélène Rey. *The global financial cycle*. No. w29327. National Bureau of Economic Research, 2021.

Rey, Hélène. "International channels of transmission of monetary policy and the Mundellian trilemma." *IMF Economic Review* 64.1 (2016): 6-35.

Ten Important Papers Related to Itskhoki-Mukhin

Chahrour, R., Cormun, V., De Leo, P., Guerrón-Quintana, P. A., & Valchev, R. (2021). *Exchange Rate Disconnect Redux*. Boston College.

Devereux, Michael B., and Charles Engel. "Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect." *Journal of Monetary economics* 49.5 (2002): 913-940.

Itskhoki, Oleg, and Dmitry Mukhin. *Mussa puzzle redux*. No. w28950. National Bureau of Economic Research, 2021.

Jeanne, Olivier, and Andrew K. Rose. "Noise trading and exchange rate regimes." *The Quarterly Journal of Economics* 117.2 (2002): 537-569.

Kalemli-Özcan, Ṣebnem. *US monetary policy and international risk spillovers*. No. w26297. National Bureau of Economic Research, 2019.

Kollmann, Robert. "Monetary policy rules in the open economy: effects on welfare and business cycles." *Journal of Monetary Economics* 49.5 (2002): 989-1015.

Kollmann, Robert. "The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation." *Journal of International Economics* 55.2 (2001): 243-262.

Lilley, Andrew, et al. "Exchange rate reconnect." *Review of Economics and Statistics* 104.4 (2022): 845-855.

Miyamoto, Wataru, Thuy Lan Nguyen, and Hyunseung Oh. "In Search of Dominant Drivers of the Real Exchange Rate." *Available at SSRN 4251909* (2022).

Obstfeld, Maurice, and Haonan Zhou. *The global dollar cycle*. No. w31004. National Bureau of Economic Research, 2023.

Ten Papers Related to Devereux, Engel, Wu

Bianchi, Javier, Saki Bigio, and Charles Engel. *Scrambling for dollars: International liquidity, banks and exchange rates*. No. w29457. National Bureau of Economic Research, 2021.

Broner, Fernando, et al. "Gross capital flows: Dynamics and crises." *Journal of monetary economics* 60.1 (2013): 113-133.

Engel, Charles, and Steve Pak Yeung Wu. *Liquidity and exchange rates: An empirical investigation*. No. w25397. National Bureau of Economic Research, 2018.

Forbes, Kristin J., and Francis E. Warnock. "Capital flow waves: Surges, stops, flight, and retrenchment." *Journal of international economics* 88.2 (2012): 235-251.

Gopinath, Gita, and Jeremy C. Stein. "Banking, trade, and the making of a dominant currency." *The Quarterly Journal of Economics* 136.2 (2021): 783-830.

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Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. *Dollar safety and the global financial cycle*. No. w27682. National Bureau of Economic Research, 2020.

Krishnamurthy, Arvind, and Hanno N. Lustig. "Mind the gap in sovereign debt markets: The us treasury basis and the dollar risk factor." *2019 Jackson Hole Economic Symposium, Stanford University Graduate School of Business Research Paper*. No. 3443231. 2019.

Kekre, Rohan, and Moritz Lenel. *The flight to safety and international risk sharing*. No. w29238. National Bureau of Economic Research, 2021.

Valchev, Rosen. "Bond convenience yields and exchange rate dynamics." *American Economic Journal: Macroeconomics* 12.2 (2020): 124-66.

Collateral Constraints and Sudden Stops

Econ 872

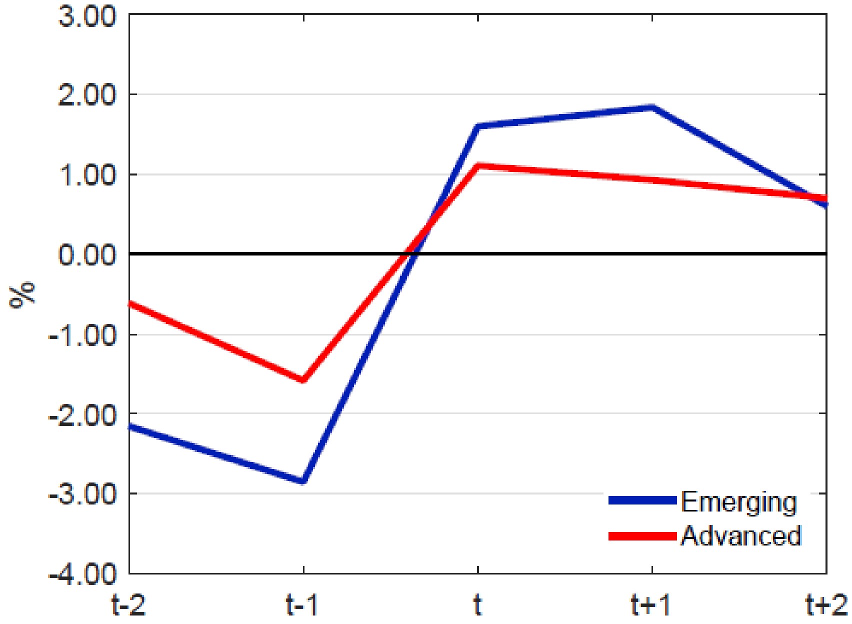
C. Engel

“Sudden Stops” refers to the situation experienced by many emerging markets at times. They were receiving capital inflows, and then those stop.

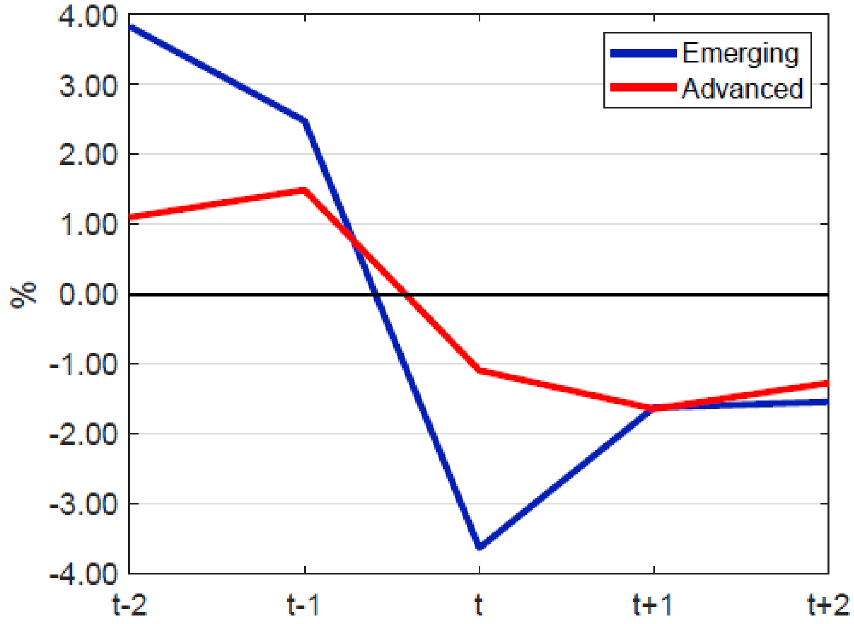
For 35 emerging market countries and 23 advanced countries, 1979-2016

1. A typical SS involves a current account reversal of 3.7% of GDP (4.4 for EMS, and 2.7 for AEs).
2. They are infrequent – only 51 during this time period (36 in Ems and 15 in AEs)
3. Sudden stops are clustered together across countries
4. GDP and consumption fall sharply during sudden stops (2.5% and 1.6% below trend.) Effects are larger for EMs (3.6 and 1.5) than for AEs (1.1 and 1.6)
5. Equity prices fall and there are real depreciations

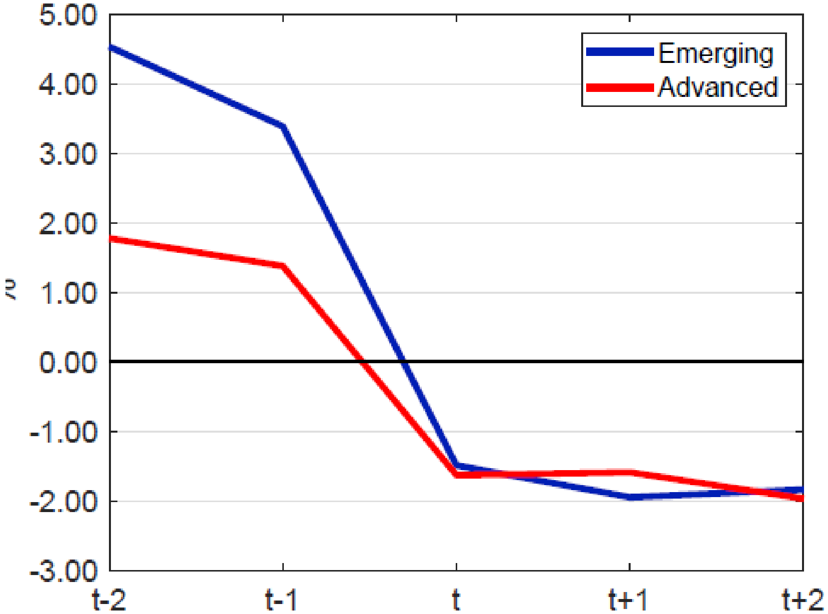
(a) Current Account



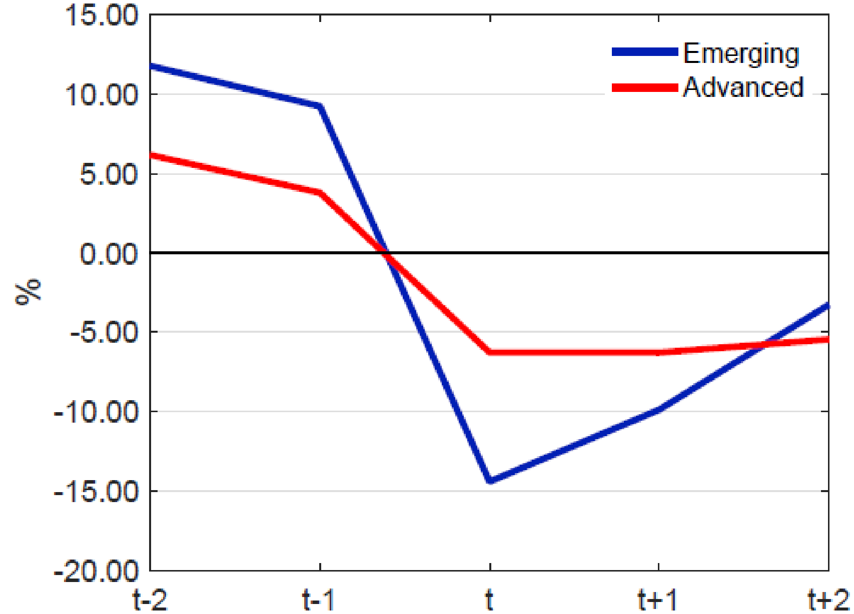
(b) GDP



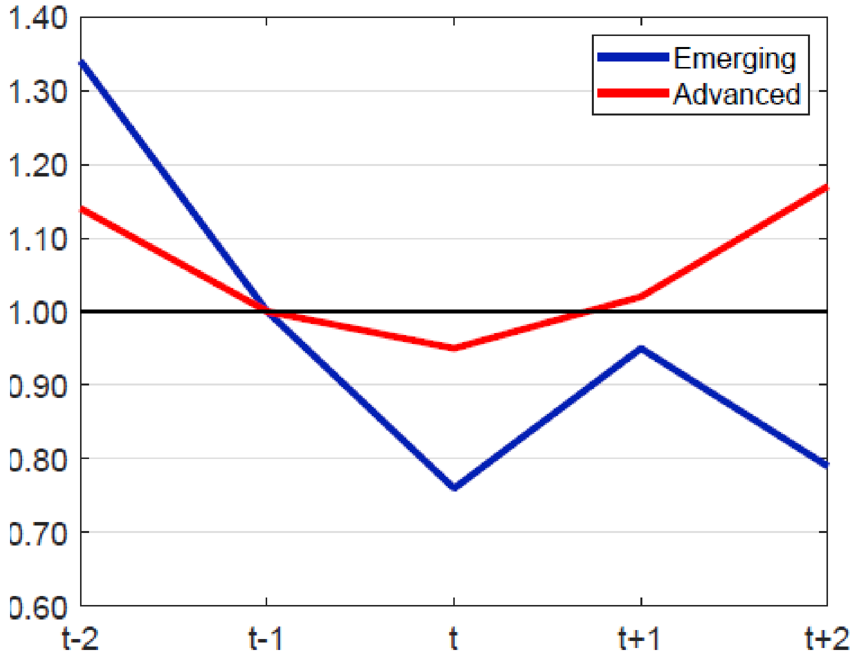
(c) Consumption



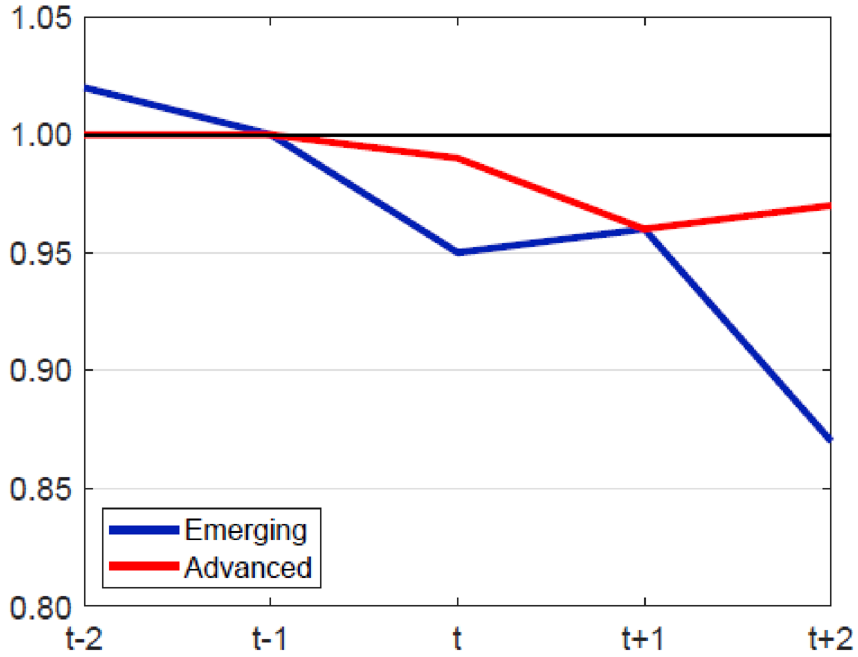
(d) Investment



(e) Equity Prices



(f) Real Exchange Rate



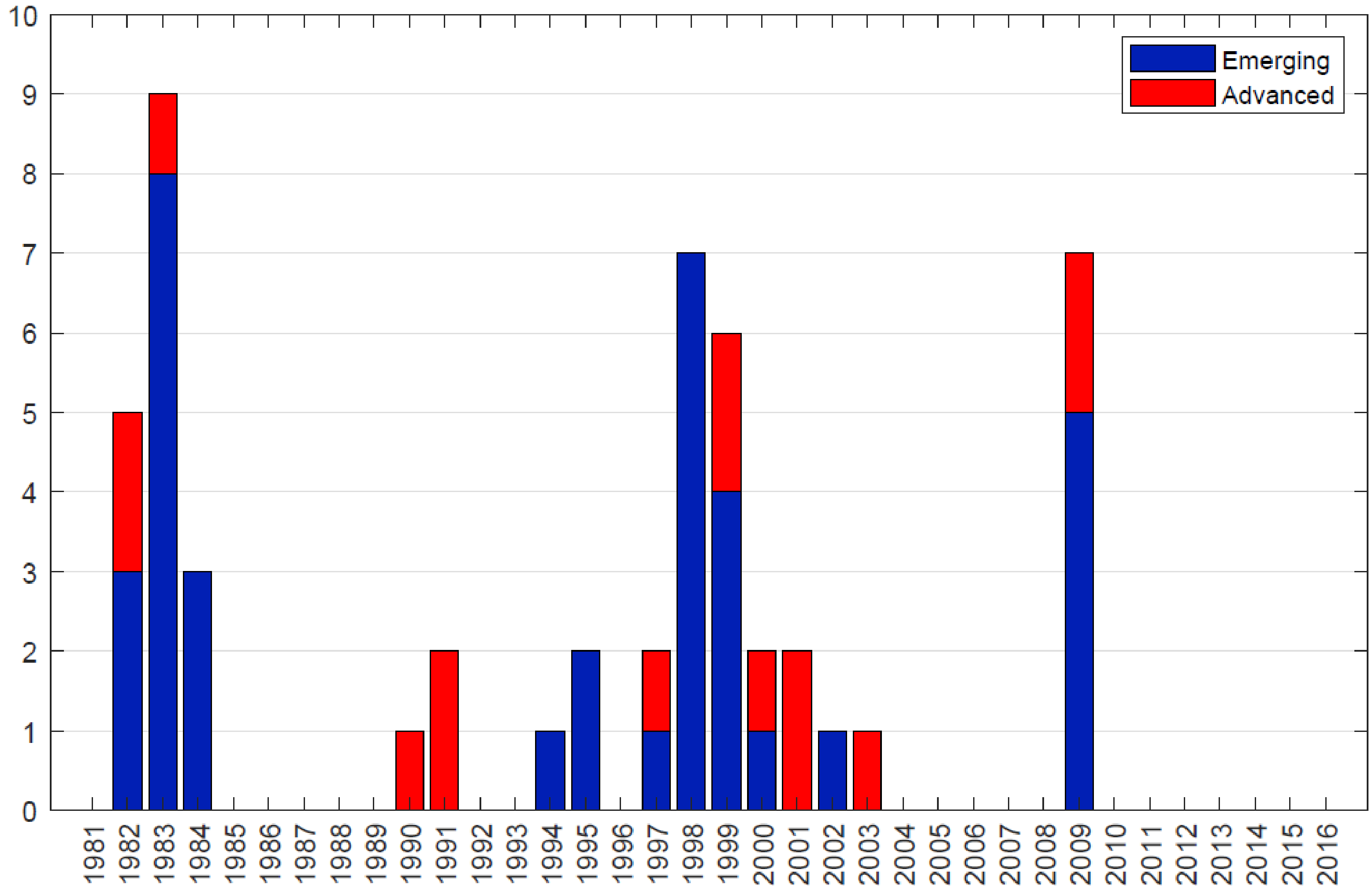


Figure 2: Number of Sudden Stops per Year

The basic mechanism to model these is sometimes called Fisherian deflation.

Borrowers are credit-constrained.

They face a collateral constraint. Their borrowing might be limited to be a multiple of income (flow constraint), or a fraction of the value of assets they hold (stock constraint.)

In Bianchi (2011) the constraint is that the amount of borrowing cannot exceed some multiple of GDP (including non-traded output.) Alternatively, the constraint might be that borrowing cannot exceed some fraction of the *value* of assets held by the borrower.

Suppose a bad shock hits tradeable GDP in Bianchi (2011). The borrowing constraint begins to be more nearly binding. Households consume less, save more.

As each household reduces its consumption, the relative price of nontraded goods falls. (If it were a stock constraint, they might sell some assets, which lowers the price of the asset, as all households sell.)

No household takes into account its effect on the prices, because each household is small.

But the aggregate effect is to make the prices fall, which lowers the value of collateral.

Now there is a spiral – as collateral constraints tighten for more households, more reduce their borrowing or sell assets, leading to further declines in the value of collateral.

There is a *pecuniary externality*. People do not take into account the effect their actions have on prices, which affect others. But in a frictionless economy, pecuniary externalities are not true externalities. They are, however, in models with asset-market frictions.

Bianchi (2011) describes this as a situation of *overborrowing*.

In a world without collateral constraints, households would be borrowing more. So why is this called “overborrowing”?

From the *constrained* planner’s standpoint, households borrow too much. That is, a planner that maximizes social welfare, but respects the collateral constraint imposed by lenders, would prefer that households borrow less than they do in the decentralized economy.

Why? Because households do not take into account the effects of their actions on the collateral constraints of others as the borrowing limit nears.

Bianchi suggests *macroprudential* policy – taxing borrowing during good times.

Consider a simple perfect-foresight model (which therefore does not capture the role of precautionary saving.)

Households in a small open economy consumer traded and non-traded goods. They maximize discounted utility over an infinite lifetime.

The CES aggregator over consumption is given by

$$c_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0,1)$$

The within-period 1st-order condition is:

$$p_t^N = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1}$$

The intertemporal 1st-order condition is:

$$u_T(t) = \beta R\left[u_T(t+1)\right] + \mu_t$$

R is the (constant) world real interest rate (in units of traded goods.) μ_t is the Lagrange multiplier on the collateral constraint:

$$q^b b_{t+1} \geq -\kappa\left[y_t^T + p_t^N \overline{y}^N\right]$$

If the collateral constraint is binding, $\mu_t > 0$. This means that $u_T(t)$ is greater than it would be with no constraint, which implies traded consumption is lower.

Equilibrium in non-traded market: $c_t^N = \overline{y}^N$

(Endowment of the non-traded good is fixed.)

Budget constraint: $c_t^T = y_t^T - q^b b_{t+1} + b_t$

Then, from the intra-temporal f.o.c., and using the fact that C_t^N is a fixed constant, we can solve that the price is a function of C_t^T: $p^N\left(C_t^T\right)$.

Then, when the credit constraint holds with equality, we can get:

$$C_t^T = (1+\kappa) y_t^T + \kappa p^N (C_t^T) \overline{y}^N + b_t$$

The lifetime budget constraint is:

$$\sum_{t=0}^{\infty} R^{-t} c_t^T = \sum_{t=0}^{\infty} R^{*-t} y_t^T + b_0$$

Define $\sum_{t=0}^{\infty} R^{*-t} y_t^T \equiv W_0$

Now make some assumptions that simplify and assure that the household is a debtor in period 1:

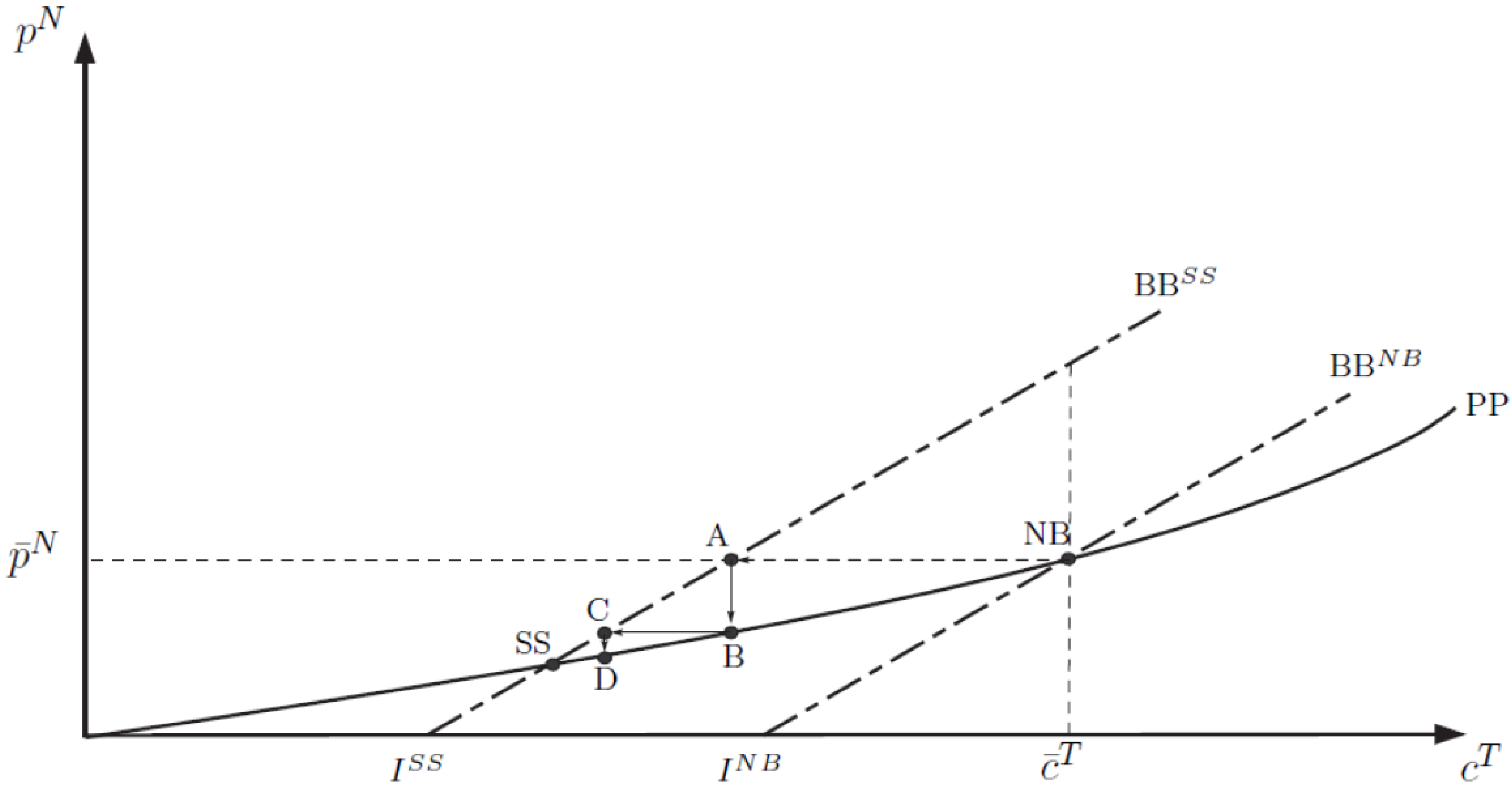
$\beta R = 1$
$b_0 < 0$
y_0^T is lower than in the future, so agents want $b_1 < 0$
We consider shocks that lower y_0^T but keep W_0 constant

When y_0^T falls below a threshold, $\hat{y}_0^T$, the collateral constraint binds

If $y_0^T \geq \hat{y}_0^T$, the collateral constraint does not bind, and we have simply:

$$\overline{c}^T = (1 - \beta)(W_0 + b_0)$$

When $y_0^T < \hat{y}_0^T$, c_0^T falls below $\overline{c}^T$. Then p_0^N falls to clear the goods market, which triggers the Fisherian deflation.



The PP curve is $p^N\left(C_t^T\right)$

The BB curves solve $C_t^T = (1+\kappa)y_t^T + \kappa p^N(C_t^T)\overline{y}^N + b_t$

BB^{SS} is when the constraint binds $y_0^T < \hat{y}_0^T$, BB^{NB} is for $y_0^T = \hat{y}_0^T$

Suppose we start at point NB, and then income falls.

To stay on the budget constraint, each person reduces their consumption until we hit point A.

But at point A, prices of the nontraded good must fall, to point B.

That further tightens the budget constraint, and we move to C, …

The new equilibrium is point SS.

Now consider the optimal policy. Here we will look at the problem in Bianchi (2011), which is identical to the one above, except that the weights in the collateral constraint on traded and nontrade output might be different. The social planner's problem is:

$$V(b, \mathbf{y}) = \max_{b', c^T} u(c(c^T, y^N)) + \beta \mathbb{E}_{\mathbf{y}'|\mathbf{y}} V(b', \mathbf{y}')$$

subject to

$$b' + c^T = y^T + b(1 + r)$$

$$b' \geq -\left(\kappa^N \frac{1 - \omega}{\omega} \left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + \kappa^T y^T\right).$$

In essence, the social planner's problem is the same as the individual's, but the social planner chooses aggregate consumption, and the social planner does not take the price as given but substitutes in the equilibrium value.

Compare the f.o.c. for the household (when the constraint is not binding) to the social planner (again, when the constraint does not bind):

$$u_T(t) = \beta(1 + r)\mathbb{E}_t u_T(t + 1)$$

$$u_T(t) = \beta(1 + r)\mathbb{E}_t\left[u_T(t + 1) + \mu_{t+1}^{sp}\Psi_{t+1}\right]$$

where $\Psi_t \equiv \kappa^N\left(p_t^N c_t^N\right)/\left(c_t^T\right)(1 + \eta) > 0$ is the amount that the value of collateral changes when tradeable consumption changes. That is, the planner takes into account the effects of borrowing on the aggregate value of collateral.

The efficient allocation can be achieved by imposing a tax on debt. Let τ_t be a tax on debt issued at time *t*. The private agent's f.o.c. becomes:

$$u_T(t) = \beta(1 + r)(1 + \tau_t)\mathbb{E}_t u_T(t + 1) + \mu_t$$

where μ_t is the Lagrange multiplier on the credit constraint.

With the following tax, the private agent's f.o.c. aligns with the planner's:

$$\tau_t^* = (\mathbb{E}_t \mu_{t+1}^{sp} \Psi_{t+1})/(\mathbb{E}_t u_T(t+1))$$

is the tax when the constraint is not binding.

When the constraint is binding, the tax does not influence the amount of debt (since the amount of debt is determined by the collateral constraint.) So the planner can set the tax to zero when the collateral constraint binds.

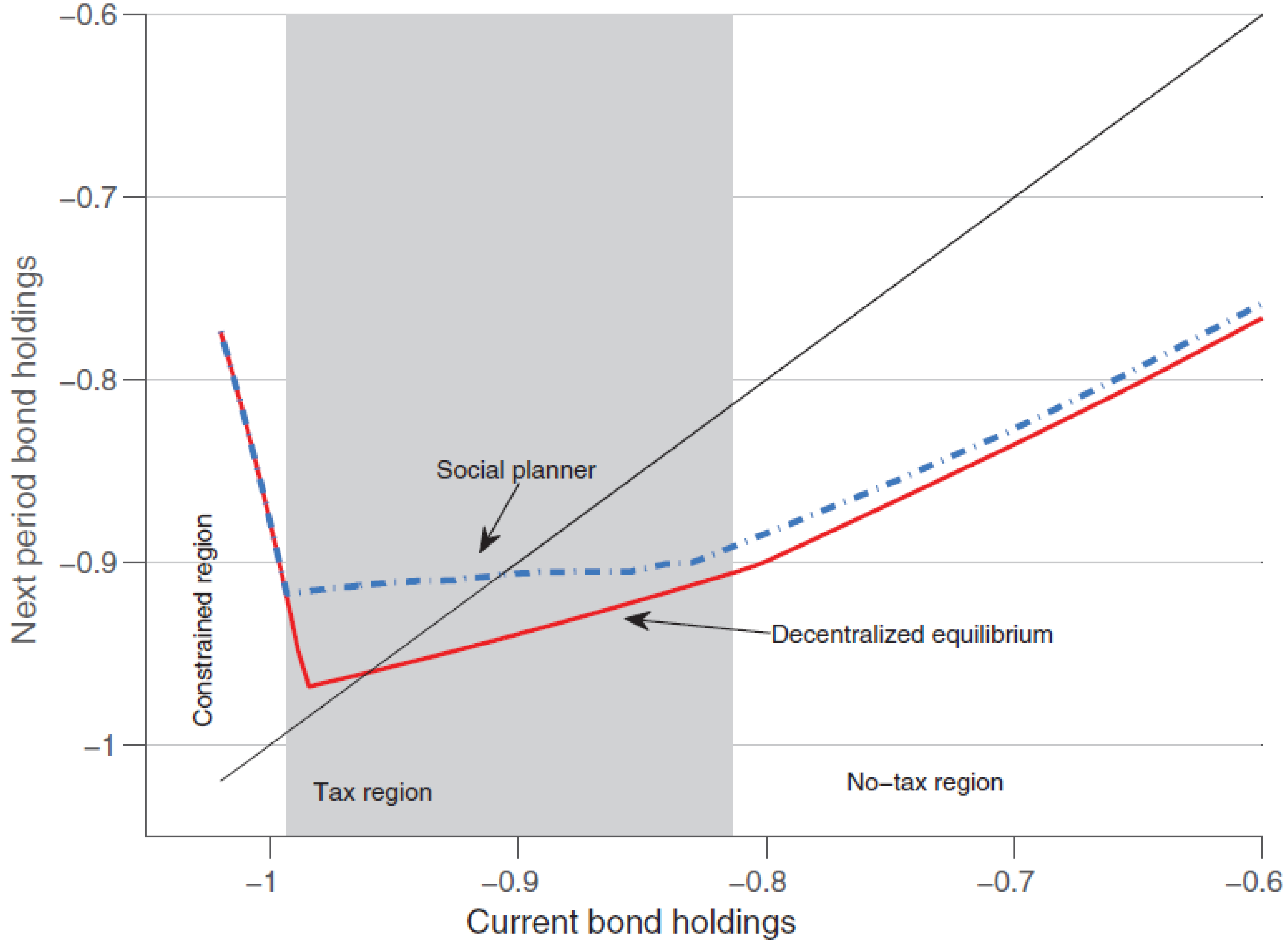
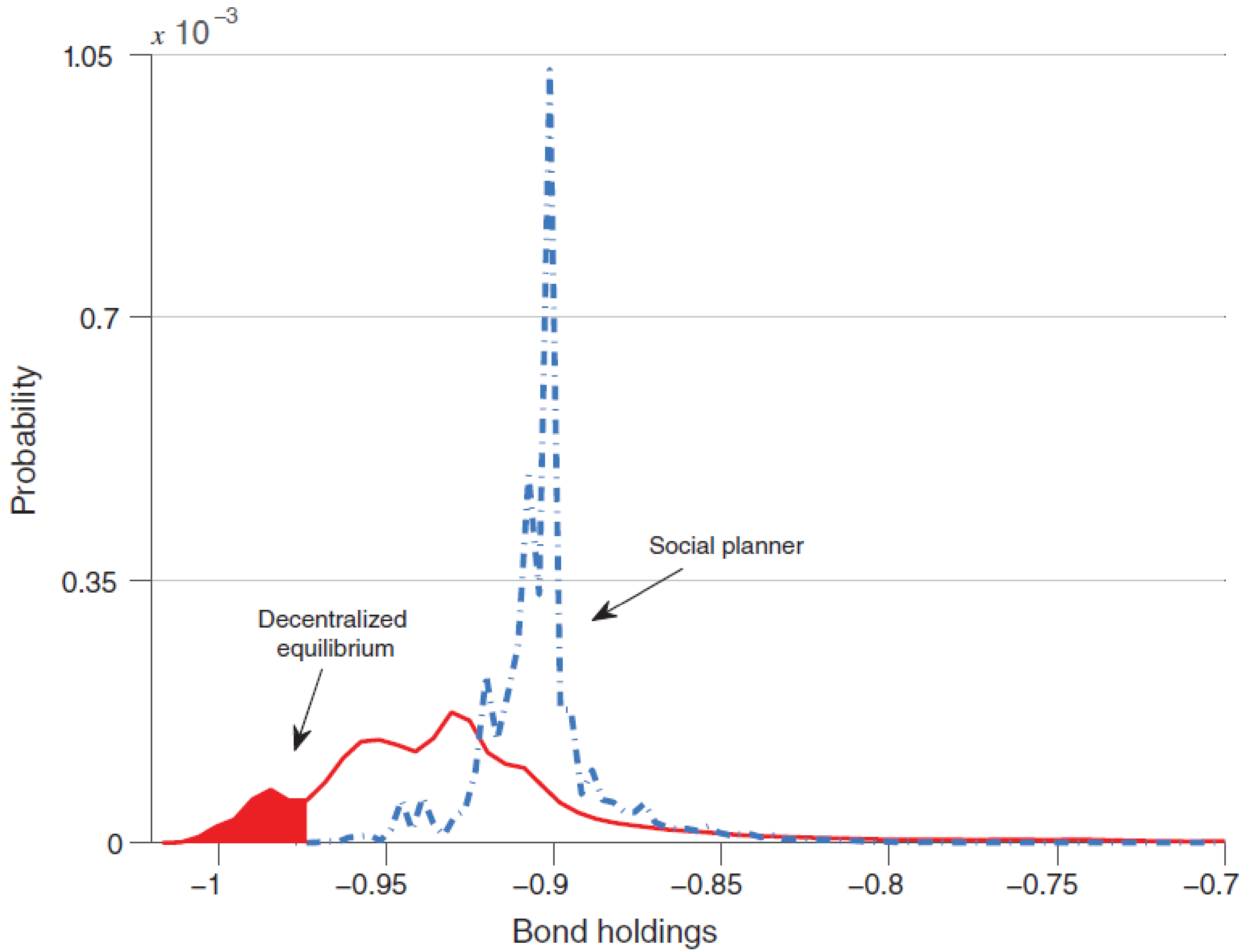


Figure 1. Bond Decision Rules for Negative One–Standard Deviation Shocks


x 10^-3
1.05
0.7
0.35
0
Probability
Social planner
Decentralized equilibrium
–1
–0.95
–0.9
–0.85
–0.8
–0.75
–0.7
Bond holdings

Bianchi goes on to show that the optimal policy greatly reduces the chances of a crisis (in numerical simulations), and when a crisis does occur (defined as the credit constraint binding, and capital flows in excess of one standard deviation of net capital outflows in the ergodic distribution), the crisis is less severe.

This tax is a macroprudential policy. What about other policies?

1. An ex post policy (meaning after the constraint binds) increases the relative price of nontraded goods (for example, by reallocating labor to the traded sector, in a model where labor produces output) can raise the value of collateral (Benigno et al.)
2. Simple, constant tax rates on debt may actually be welfare reducing.
3. When investment is introduce into the model, there is a tradeoff, because taxing debt may reduce investment and therefore reduce output.

Gabaix - Maggiori

C. Engel

Gabaix, Xavier and Matteo Maggiori. 2015. "International Liquidity and Exchange Rate Dynamics." Quarterly Journal of Economics 130, 1369-1420.

This paper looks at a model in which a financial institution that makes the market for foreign exchange faces portfolio constraints.

In the model, if there are large flows of capital in one direction, it alters the ex ante returns on interest-bearing assets in one country relative to another.

The paper offers a model of the UIP puzzle, the exchange-rate disconnect puzzle, and the excess volatility of exchange rates.

Demand for foreign exchange in this model is channeled through an intermediary – the financial institution. Its willingness to take advantage of differences in expected returns is constrained by equity holders in the bank. Hence, UIP deviations can persist.

Mundell-Fleming with Imperfect Capital Mobililty

The idea of the simple model in the paper harks back to pre-1970s models of the exchange rate. The flow demand for foreign exchange determines the exchange rate, rather than modeling an asset-market equilibrium.

The older literature does not consider the effects of changes in the demand for foreign exchange arising because of its effects on the value of assets in the portfolio. There was no “asset market equilibrium” condition – that agents are satisfied with their portfolio in equilibrium.

In the M-F model, there was a flow demand for foreign exchange arising from two sources – demand for foreign exchange to pay for trade in goods, and a flow demand for assets arising from expected returns.

When initial asset positions are zero, the first-period current account balance equals exports minus imports.

But the current account balance also equals the saving decision. The model for import demand and export supply must be consistent with the saving decision.

In GM, the saving decision is a bit “hidden” under the assumptions. The paper directly derives export supply and import demand.

Demand for the home currency, first, arises (in period 0) from net exports: $\xi_0 e_0 - \iota_0$. e_0 is the home currency price of foreign currency exchange rate. ξ_0 is the value of home exports in foreign currency terms, so that $\xi_0 e_0$ is the home currency value of home exports.

When foreigner want home goods, they demand $\xi_0 e_0$ units of home currency in order to buy the goods.

ι_0 is home's demand for imports, expressed in units of the home currency. Home demands foreign currency, and supplies home currency, in order to import.

Then $\xi_0 e_0 - \iota_0$ is net demand for home currency coming from trade.

We will see this is also equal consistent with the home saving decision in the initial period.

Flow Demand for Foreign Exchange from Asset Demand

M-F assumes there is a *flow* demand for home currency based on the difference in expected returns on home and foreign bonds. This equation is also derived in GM:

$$Q_0 = \frac{1}{\Gamma} E\left[e_0 - e_1 \frac{R^*}{R} \right]$$

Equilibrium in the foreign exchange market then requires:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0$$

where $-Q_0$ is the net demand for foreign assets.

We can then solve for e_0 to find:

$$e_0 = \frac{\Gamma}{1+\xi_0\Gamma}\left[\iota_0 + \frac{R^*}{R} E e_1\right]$$

In M-F, Ee_1 is taken as exogenous. In G-M, it is determined by the second-period equilibrium condition:

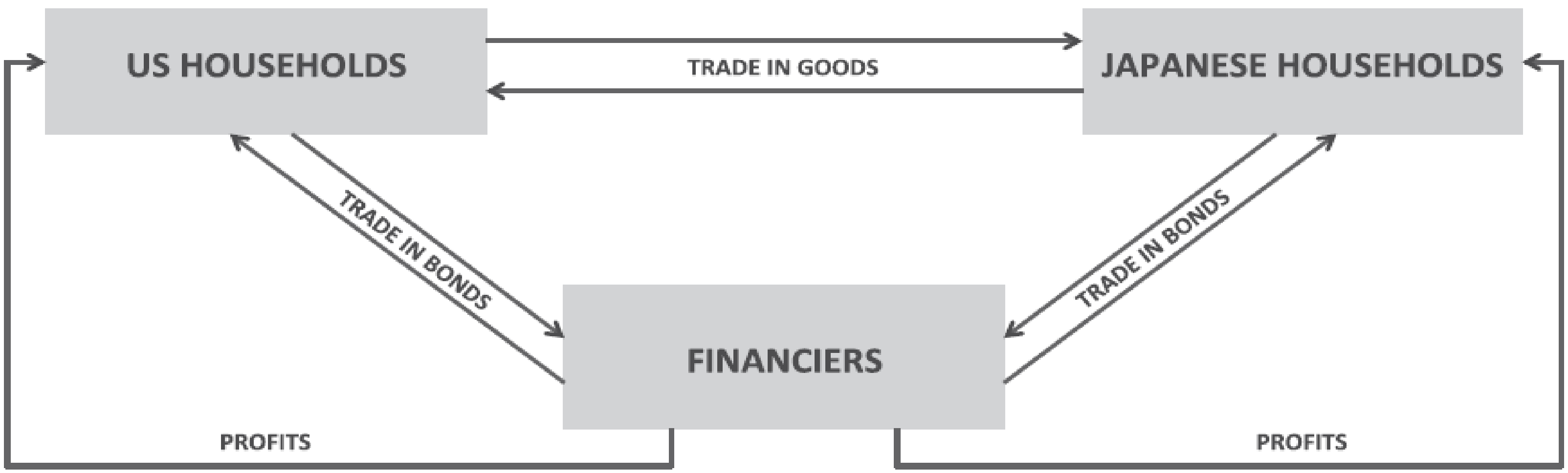
$$\xi_1 e_1 - \iota_1 - R Q_0 = 0$$

G-M Model Set-Up

Two countries (U.S. and Japan). Two periods ($t = 0,1$). Continuum of households in each country.

Households produce and trade. They also hold a risk-free bond denominated in their own currency. (There is a non-traded good in each country with a nominal price of one.)

Households hold no claims on foreigners. There is a financier that holds a portfolio and remits profits to households of both countries.



Households

Home households maximize utility given by:

$$\theta_0 \ln\left(C_0\right) + \beta E\left[\theta_1 \ln\left(C_1\right)\right]$$

where:

$$C_t = \left[\left(C_{NT,t}\right)^{\chi_t}\left(C_{H,t}\right)^{a_t}\left(C_{F,t}\right)^{\iota_t}\right]^{\frac{1}{\theta_t}}$$

$C_{N,t}$ is U.S. consumption of the non-tradable good, $C_{H,t}$ is U.S. consumption of the U.S. tradable good, and $C_{F,t}$ is U.S. consumption of the Japanese tradable good.

Assume

$$\theta_t = \chi_t + a_t + \iota_t$$

U.S. households face the intertemporal budget constraint:

$$\sum_{t=0}^{1} R^{-t}\left(Y_{NT,t} + p_{H,t}Y_{H,t}\right) = \sum_{t=0}^{1} R^{-t}\left(C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t}\right)$$

R is the real return on a bond denominated in units of the nontraded good (or, equivalently in units of home currency, since the dollar price of the nontraded good is set at 1. We can think of monetary policy as targeting a nontraded price of 1.) It is riskless, and traded only among U.S. residents.

The nontraded good is supplied through an endowment. The model, at this point, does not make any assumption about where the output of the traded good comes from.

The static optimization problem for the U.S. household is

$$\max_{C_{NT,t},C_{H,t},C_{F,t}} \left\{ \chi_t \ln\left(C_{NT,t}\right) + a_t \ln\left(C_{H,t}\right) + \iota_t \ln\left(C_{F,t}\right)\right\}$$
$$+\lambda_t \left(CE_t - C_{NT,t} - p_{H,t}C_{H,t} - p_{F,t}C_{F,t}\right)$$

where CE_t is the aggregate consumption expenditure, solved from the intertemporal problem.

Two of the first-order conditions are:

$$\frac{\chi_t}{C_{NT,t}} = \lambda_t \qquad\qquad \frac{\iota_t}{C_{F,t}} = \lambda_t p_{F,t}$$

Now, G-M make the following kind of weird assumption about the endowment process:

$$Y_{NT,t} = \chi_t$$

This assumption is made because when we then impose equilibrium in the non-traded goods market and use the 1st-order condition above, we get $\lambda_t = 1$. In words, as people desire to spend a greater share on the non-traded good, the supply of the non-traded good coincidentally increases.

This then gives us the convenient equation,

$$p_{F,t}C_{F,t} = \iota_t$$

where ι_t is an exogenously given random variable that determines the expenditure share (and, expenditure total) on the imported good.

Foreign country (Japan)

Maximizes

$$\theta_0^* \ln C_0^* + \beta^* \mathbb{E}[\theta_1^* \ln C_1^*]$$

where

$$C_t^* \equiv \left[(C_{NT,t}^*)^{\chi_t^*} (C_{H,t}^*)^{\xi_t} (C_{F,t}^*)^{a_t^*} \right]^{\frac{1}{\theta_t^*}}$$

$$\theta_t^* \equiv \chi_t^* + a_t^* + \xi_t$$

Assume

$$Y_{NT,t}^* = \chi_t^*$$

So we get Japanese demand for U.S. exports (in Japanese currency):

$$p_{H,t}^* C_{H,t}^* = \xi_t$$

Net exports are given by:

$$NX_t = e_t p^*_{H,t} C^*_{H,t} - p_{F,t} C_{F,t} = \xi_t e_t - \iota_t$$

LEMMA 1. (Net Exports). Expressed in dollars, U.S. exports to Japan are $\xi_t e_t$; U.S. imports from Japan are ι_t; so that U.S. net exports are $NX_t = \xi_t e_t - \iota_t$.

Intertemporal optimization pins down *R*:

$$1 = \mathbb{E}\left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\right] = \mathbb{E}\left[\beta R \frac{\left(\frac{\chi_1}{C_{NT,1}}\right)}{\left(\frac{\chi_0}{C_{NT,0}}\right)}\right] = \beta R$$

Saving/Borrowing Decision

How is the current account balance is determined by static demand?

The first-order conditions give us

$$P_{F,0}C_{F,0} = \frac{\iota_0}{\chi_0}C_{N,0} \text{ and } P_{H,0}C_{H,0} = \frac{a_0}{\chi_0}C_{N,0}$$

But because of the assumption that $Y_{N,0} = \chi_0$, these reduce to:

$P_{F,0}C_{F,0} = \iota_0$ and $P_{H,0}C_{H,0} = a_0$, so nominal consumption decisions are constant, or at least exogenous random variables.

National saving is then given by:

$$P_{H,0}Y_{H,0} - P_{H,0}C_{H,0} - P_{F,0}C_{F,0} = P_{H,0}Y_{H,0} - a_0 - i_0$$

Though the paper does not note this, it means that home exports in period 0 must equal:

$$P_{H,0}Y_{H,0} - a_0$$

Since these must equal Japanese imports, $\xi_0 e_0$, we must have:

$$P_{H,0}Y_{H,0} - a_0 = \xi_0 e_0$$

If there were no capital flows, and trade was balanced, $e_0 = \iota_0 / \xi_0$.

Then $P_{H,0}Y_{H,0} = a_0 + \iota_0$. So that is just saying that the nominal value of home output would equal the demand for goods by home households.

Financiers

The financier clears the demand for foreign exchange, but also holds foreign exchange on its own behalf to make a profit. It doesn't charge for its service of exchanging foreign currency but can make a profit off its monopoly position as the only entity that can hold bonds from both countries.

They start period 0 with no net wealth, so if they want to go long in the home bond, they must borrow in the foreign bond. They maximize their expected value:

$$V_0 = \mathbb{E}\left[\beta\left(R - R^* \frac{e_1}{e_0}\right)\right] q_0 = \Omega_0 q_0$$

q_0 is the size of the position they take. It is not limited by risk aversion, but by an exogenously given constraint. This is sort of like a constraint in a Gertler-Kiyotaki model.

Constraint

Investors are worried that the financiers might run away with the money, and not pay back investors. So there is a limit on how large they allow the value of the firm to be. Specifically:

$$\underbrace{\frac{V_0}{e_0}}_{\substack{\text{Intermediary Value}\\ \text{in yen}}} \geq \underbrace{\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Total}\\ \text{Claims}}} \underbrace{\Gamma\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Diverted}\\ \text{Portion}}} = \underbrace{\Gamma\left(\frac{q_0}{e_0}\right)^2}_{\substack{\text{Total divertable}\\ \text{Funds}}}.$$

Here, the total absolute value of debt, in yen terms, is given by $\left|\frac{q_0}{e_0}\right|$.

The financiers can abscond with a fraction of it given by $\Gamma\left|\frac{q_0}{e_0}\right|$.

Lenders insist that the value of the intermediary be greater than or equal to the value of the funds that the financier can abscond with, so he won't be attempted to abscond and fold the firm.

The *fraction* of $\left|\frac{q_0}{e_0}\right|$ that they can run away with is increasing in $\left|\frac{q_0}{e_0}\right|$. This is crucial. If, for example, the fraction were just a constant, Γ, then as q increased, the constraint would just increase proportionately. We'll see why that matters in a moment.

In addition, G-M assume in an ad hoc way that

$$\Gamma = \gamma\, var(e_1)^{\alpha}$$

The firm could abscond with more funds if the variance of the exchange rate were greater.

We can write the firm's optimization as:

$$\max_{q_0} V_0 = \mathbb{E}\left[\beta\left(R - R^* \frac{e_1}{e_0}\right)\right] q_0, \qquad \text{subject to} \quad V_0 \geq \Gamma \frac{q_0^2}{e_0}$$

The constraint will always be binding. That's because the intermediary would like to take as large a position as possible, but the larger position he takes, the more he could abscond with. The rhs of the constraint goes up with the square of q_0. So he takes as large a position as he can, subject to the constraint, which makes the constraint bind.

We have then, using $\beta = 1/R$ that

$$q_0 = \frac{1}{\Gamma} \mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$$

and aggregating, we get:

$$Q_0 = \frac{1}{\Gamma} \mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$$

Lemma 2. (Financiers' downward sloping demand for dollars). The financiers' constrained optimization problem implies that the aggregate financial sector's optimal demand for Dollar bonds versus Yen bonds follows:

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right], \tag{10}$$

where

$$\Gamma = \gamma(var(e_1))^{\alpha}. \tag{11}$$

Note the role that is played by the assumption that the fraction that can be absconded with is increasing in $\left|\frac{q_0}{e_0}\right|$.

Also, note that this is just the same as the *ad hoc* flow demand for home currency that I described in the Mundell-Fleming model.

<u>Equilibrium</u>

We have:

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$$

$$\xi_0 e_0 - \iota_0 + Q_0 = 0$$

$$\xi_1 e_1 - \iota_1 - RQ_0 = 0$$

What if there were one more period?

$$\xi_1 e_1 - \iota_1 - rQ_0 = Q_1 - Q_0$$

We don't normally see a flow constraint. The constraint on the financier is set up like a constraint on its gross asset position, not on the flow. If the model were more than two periods, the logical think would be for the constraint to be on the stock, Q_1, not the flow, $Q_1 - Q_0$.

It seems a bit misleading to say that flows matter in this model, unless for some reason the financier faces constraints on flows, not stocks, of assets.

For simplicity, assume

$$\beta = \beta^* = 1, \text{ which implies } R = R^* = 1$$

and

$$\xi_t = 1 \text{ for } t = 0, 1$$

Solutions for Exchange Rate:

$$e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}$$

If this were a model without the intermediary, we could find an equilibrium by imposing goods market equilibrium for each good in each period, and the intertemporal budget constraint for each country.

In addition, we would impose UIP. The solution is the same as above, but with $\Gamma = 0$. That is the same as assuming an unconstrained intermediary.

Here, an increase in import demand in the U.S. depreciates the dollar. But also,

$$\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0$$

This arises from rearranging the condition that the sum of export supply over the two periods must equal the sum of import demand (in dollars). An increase in ι_0 leads directly to an increase in Ee_1, but that effect on Ee_1 is more than offset by an increase in e_0. The net effect of an increase in ι_0 is to have Ee_1 / e_0 fall, which means the foreign currency is expected to depreciate, which reduces demand for the foreign currency.

On net, then, the currency changes less than one for one with ι_0.

Also, defining the innovation in X as $\{X\} = X - E_0 X$, we find:

$$e_1 = \iota_0 + \iota_1 - e_0 = \iota_0 + \mathbb{E}[\iota_1] + \{\iota_1\} - e_0$$
$$= \{\iota_1\} + \iota_0 + \mathbb{E}[\iota_1] - \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1]}{2+\Gamma}$$

It follows that

$$var(e_1) = var(\iota_1)$$

so

$$\Gamma = \gamma\, var(\iota_1)^{\alpha}$$

Proposition 1. (Basic gamma equilibrium exchange rate). Assume that $\xi_t = 1$ for $t=0,1$, and that interest rates are zero in both countries. The exchange rate follows:

$$e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}, \tag{16}$$

$$e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1]}{2+\Gamma},$$

where $\{\iota_1\}$ is the time 1 import shock. The expected dollar appreciation is: $\mathbb{E}\left[\frac{e_0 - e_1}{e_0}\right] = \frac{\Gamma(\iota_0 - \mathbb{E}[\iota_1])}{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}$. Furthermore, $\Gamma = \gamma\, var(\iota_1)^{\alpha}$.

That last part says:

$$\mathbb{E}\left[\frac{e_0 - e_1}{e_0}\right] = \frac{\Gamma(\iota_0 - \mathbb{E}[\iota_1])}{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}$$

NFA Position and Excess Returns

At the end of period 0, the NFA position of the U.S. is given by:

$$N_{0^+} = \xi_0 e_0 - \iota_0 = \frac{\mathbb{E}[\iota_1]-\iota_0}{2+\Gamma}$$

So the U.S. has a positive NFA if

$$\iota_0 < \mathbb{E}[\iota_1]$$

That is, the U.S. has a current account surplus if it expects to need to finance imports next period.

In this case, the U.S. has lent to the financier, who is then short in dollars and long in yen. In order to be compensated for going long in yen, it must receive an excess return on yen.

That means the yen must be expected to appreciate, which means it is weak in period 0 (that is, e_0 is low.) The weakness of the yen is greater when the financier is more constrained:

$$\frac{\partial e_0}{\partial \Gamma} = \frac{\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} = \frac{-N_{0^+}}{2+\Gamma}$$

We can then infer the effects of financial shocks, which are an increase in γ:

Proposition 2. (Effect of financial disruptions on the exchange rate). In the basic gamma model, we have $\frac{\gamma}{\Gamma}\frac{de_0}{d\gamma} = \frac{\partial e_0}{\partial \Gamma} = \frac{-N_{0^+}}{2+\Gamma}$, where $N_{0^+} = \frac{\mathbb{E}[\iota_1]-\iota_0}{2+\Gamma}$ is the U.S. net foreign asset (NFA) position. When there is a financial disruption ($\uparrow \gamma, \uparrow \Gamma$), countries that are net external debtors ($N_{0^+} < 0$) experience a currency depreciation ($\uparrow e$), while the opposite is true for net creditor countries.

Because there are only two periods and initial positions are zero, there is no difference between the net flow demand in the first period and the net asset position at the end of the first period.

The general model is summarized in the following proposition:

Proposition 3. With general trade shocks and interest rates (ι_t, ξ_t, R, R^*), the values of exchange rate at times t = *0,1* are:

$$e_0 = \frac{\mathbb{E}\left[\frac{\iota_0 + \frac{\iota_1}{R}}{\xi_1}\right] + \frac{\Gamma \iota_0}{R^*}}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma \xi_0}{R^*}}; \qquad e_1 = \mathbb{E}[e_1] + \{e_1\}, \tag{17}$$

where we again denote by $\{X\} \equiv X - \mathbb{E}[X]$ the innovation to a random variable X, and

$$\mathbb{E}[e_1] = \frac{R}{R^*} \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma \xi_0 \mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma \xi_0}, \tag{18}$$

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R \frac{\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma \xi_0} \left\{\frac{1}{\xi_1}\right\}. \tag{19}$$

When ξ_1 is deterministic, $\Gamma = \gamma var(\frac{\iota_1}{\xi_1})^{\alpha}$. The proof of this

Exchange Rate Flows Matter

The next section of G-M is devoted to showing that exchange rate flows really do matter for the exchange rate determination. I think that is obvious, given the set-up, but perhaps it is confusing since the previous section referred to the flow in the first period as the NFA position at the end of the first period.

So, to emphasize their point, they imagine that in addition to the demand for dollars coming from their demand for U.S. exports, the Japanese household has an exogenous flow demand for U.S. bonds, given by f^*, funded by an offsetting position in $-f^*/e_0$ yen bonds.

Now the flow conditions for demand for U.S. dollars are given by:

$$\xi_0 e_0 - \iota_0 + Q_0 + f^* = 0 \qquad \xi_1 e_1 - \iota_1 - RQ_0 - Rf^* = 0$$

Going back to the case in which we assume $\xi_t = R = R^* = 1$, we find:

$$e_0 = \frac{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma} \qquad e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}$$

We can see that the flows matter for the exchange rate.

They then note that flows would not matter if bonds are traded but UIP holds (that is, if we did not have a financier with constraints.) It also would not matter if markets were complete. Indeed, in the latter case, the real exchange rate is just a constant because the marginal utility of nontraded consumption in each country is a constant (because of the special assumptions that $Y_{NT,t} = \chi_t$ and $Y^*_{NT,t} = \chi^*_t$.)

Exchange-Rate Disconnect

The paper mentions disconnect in two places. In the first mention, they note that empirical models generally do not consider the role of $\Gamma = \gamma \operatorname{var}(e_1)^{\alpha}$ nor the role of financial flows, so maybe there is disconnect because empirical work has neglected key fundamentals.

Then later, they discuss how output may be disconnected from exchange rates if there is local currency pricing, LCP. They consider a super-simple set-up where nominal prices are rigid downward. If the economy is at full employment, then $Y = L$ because labor supply is exogenous and fixed. But if demand is insufficient, prices cannot fall to equilibrate the market. The home country sets a price for consumers in each country. Then we have:

$$Y_{H,t} = \min\left(\frac{a_t}{\bar{p}_H} + \frac{\xi_t}{\bar{p}_H^*}, L\right)$$

So output does not depend on the exchange rate. It almost goes without saying that in a more serious model, even with LCP, there are other channels through which the exchange rate could affect output.

Carry Trade and UIP Puzzle

The model can account for the UIP puzzle in the following way: Suppose that the interest rates are different in the two countries: $R = \beta^{-1}$ and $R^* = \beta^{*-1}$ and $R > R^*$. This means the home country is more impatient than the foreign country.

Ceteris paribus, this means that the home country has a relatively high demand for imports compared to the foreign country in time 0 – that is, the U.S. has a high demand for yen, relative to Japanese demand for dollars.

Hence, the financier will need to hold more dollars in equilibrium. In equilibrium, he will need to have a higher expected return on dollars in order to hold them. That is, there will be an excess return on the U.S. asset, and the financier will take as large a position as he can.

We have as equilibrium conditions (setting all of the import demands equal to one):

$$\Gamma Q_0 = e_0 - e_1 \frac{R^*}{R} \qquad e_0 - 1 + Q_0 = 0 \qquad e_1 - 1 - RQ_0 = 0$$

Start from $R^* = R = 1$. Then initially, $Q_0 = 0$ and $e_0 = e_1 = 1$. Now increase R^*. *Ceteris paribus*, $e_0 - e_1 \frac{R^*}{R}$ falls, so the excess return on the foreign bond increases. But could e_1 / e_0 fall far enough that $e_0 - e_1 \frac{R^*}{R}$ does not increase?

If Q_0 did not change, then we still have $e_0 = e_1 = 1$, but with $R^* > 1$, this is a contradiction. If Q_0 rose, the second equation would imply e_1 / e_0 goes up, which is also a contradiction. So Q_0 must fall. There must be an decrease in demand for dollars by the financier.

Conclusions

The model has a lot of special features. I think the main idea is that there can be an excess return on one of the assets, and credit constraints keep people from fully exploiting that. In a sense, there is not sufficient liquidity in the markets for these short-term bonds, or in the market for foreign exchange, to exploit the expected profit difference.

It is difficult to pin down exactly where G-M believe the illiquidity lies. Is it because the foreign exchange market is illiquid, or the short-term bond market? In the model, there is only one agent that has a portfolio choice.

It would be useful to write out a full model, and then test it empirically.

Perhaps my main concern is this:

It is not obvious why current account imbalances necessarily imply something about portfolio composition.

That is, suppose the home country wants to run a current account deficit because expenditures exceed income. It needs to borrow.

But what is it that says they must borrow in home currency or foreign currency? In this model, it is assumed that each country must borrow in its own currency, and there is an intermediary that has limits on its positions.

If one goes beyond simply assuming it, the question is why, even if a country has balance sheet limits that restrict its new borrowing, does this affect the composition of the country's portfolio?

Exchange Rate Disconnect

Itskhoki, Oleg, and Dmitry Mukhin. 2021. "Exchange Rate Disconnect in Equlibrium. **Journal of Political Economy** 129, 2183-2232.

This paper builds a model to solve all of the following puzzles:

1. Exchange rate disconnect – exchange rate is not correlated with fundamentals, and is more volatile.
2. PPP puzzle – real exchange rate closely tracks nominal exchange rate, but mean reversion is very slow
3. Terms of trade puzzle – t.o.t. weakly correlated with real exchange rate, but less volatile
4. Backus-Smith puzzle – relative consumption not highly correlated with real exchange rate
5. Forward premium puzzle - $i_t^* + s_{t+1} - s_t - i_t$ correlated with $i_t - i_t^*$

The key innovation is to add noise traders in foreign exchange markets, and a financial intermediary that is risk averse.

Two-country, flexible price model, no nontraded goods, extreme home bias in consumption.

Households maximize: $U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi} \right)$

subject to: $P_t C_t + \frac{B_{t+1}}{R_t} = W_t L_t + B_t + \Pi_t$

R_t is gross nominal interest rate. Confusingly, they call B_{t+1} the home bond. It is actually home lending.

How does a model with non-state-contingent bonds differ from a model with state-contingent bonds?

Ultimately, in this flexible-price model, IM assume monetary policy in each country completely stabilizes the consumer price level (P_t for home.)
What does this imply about the denomination of bonds? Does that matter?
(In this model, households in different countries do not trade bonds.)

1st-order Conditions

$$C_t^{\sigma} L_t^{1/\nu} = W_t / P_t$$
$$1 = \beta R_t E_t \left\{ \left(C_{t+1} / C_t \right)^{-\sigma} P_t / P_{t+1} \right\}$$

Households trade only local currency bonds and own home firms.

Foreign firms are symmetric, and they trade only foreign-currency bonds B_{t+1}^{*} that pay nominal interest R_t^{*}, and foreign firms pay Π_t^{*} dividends.

There is a continuum of goods produced in Home and Foreign. Home consumption is given by:

$$C_t = \left(\int_0^1 \left[(1-\gamma)^{\frac{1}{\theta}} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{Ft}(i)^{\frac{\theta-1}{\theta}} \right] \right)^{\frac{\theta}{\theta-1}}$$

We get for the home country:

$$C_{Ht}(i) = (1-\gamma)\left(\frac{P_{Ht}(i)}{P_t} \right)^{-\theta} C_t \qquad C_{Ft}(i) = \gamma \left(\frac{P_{Ft}(i)}{P_t} \right)^{-\theta} C_t .$$

Analogously for the foreign country:

$$C^*_{Ht}(i) = \gamma \left(\frac{P^*_{Ht}(i)}{P^*_t} \right)^{-\theta} C^*_t \qquad C^*_{Ft}(i) = (1-\gamma)\left(\frac{P^*_{Ft}(i)}{P^*_t} \right)^{-\theta} C^*_t .$$

Producers

$$Y_t = e^{a_t} L_t \qquad a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a$$

Profits:

$$\Pi_t(i) = \left(P_{Hi}(t) - MC_t\right) C_{Ht}(i) + \left(P_{Ht}^*(i)\mathcal{E}_t - MC_t\right) C_{Ht}^*(i)$$

where $MC_t = e^{-a_t} W_t$ and $Y_t(i) = C_{Ht}(i) + C_{Ht}^*(i)$

Prices are flexible and set according to:

$$P_{Ht}(i) = P_{Ht} = \frac{\theta}{\theta - 1} e^{-a_t} W_t \quad \text{and} \quad P_{Ht}^*(i) = P_{Ht}^* = P_{Ht} / \mathcal{E}_t$$

$$P_{Ft}^*(i) = P_{Ft}^* = \frac{\theta}{\theta - 1} e^{-a_t^*} W_t^* \quad \text{and} \quad P_{Ft}(i) = P_{Ft} = P_{Ft}^* \mathcal{E}_t$$

Equilibrium

$$Y_t(i) = C_{Ht} + C_{Ht}^* = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\theta} C_t + \gamma\left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\theta} C_t^*$$

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{where} \qquad NX_t = \mathcal{E}_t P_{Ht}^* C_{Ht}^* - P_{Ft} C_{Ft}$$

Define the terms of trade: $S_t = P_{Ft} / \mathcal{E}_t P_{Ht}^*$

and real exchange rate: $Q_t = \mathcal{E}_t P_t^* / P_t$

Monetary Policy:

Each country simply completely stabilizes consumer price levels:

$P_t = 1$ and $P_t^* = 1$, so we can think of this as a real model.

Segmented Financial Markets:

The key innovation of this paper is to combine the Gabaix-Maggiori style model of segmented financial markets with "noise traders".

Note that the financial intermediary does not face balance sheet constraints here, but rather is risk averse (but not with the SDF of either home or foreign households.)

The noise traders hold stocks of foreign bonds or home bonds. All trade must go through the financial intermediary, and the intermediary has limited risk-bearing capacity.

Households in Home and Foreign only trade assets with the intermediary.

The noise traders are people that live outside the model – their utility is not considered.

Noise Traders

Have a zero net position each period: $\frac{N_{t+1}}{R_t} = -\frac{\mathcal{E}_t N^*_{t+1}}{R^*_t}$.

The amount held each period is random:

$$\frac{N^*_{t+1}}{R^*_t} = n\left(e^{\psi_t} - 1\right)$$

$\psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon^\psi_t$, and n is the mass of noise traders.

Financial Intermediaries

The financial intermediary buys or sells bonds to Home and Foreign households, as well as with the noise trader. They have zero capital.

They follow a carry trade strategy that involves a long position in the foreign bond and a short position in the home bond (if $D^*_{t+1} > 0$ and $D_{t+1} < 0$), or vice-versa.

The size of an individual intermediary's position is chosen to maximize the CARA utility in units of the foreign good:

$$\max_{d^*_{t+1}} E_t\left\{-\frac{1}{\omega}\exp\left(-\omega\frac{\hat{R}^*_{t+1}}{P^*_{t+1}}\frac{d^*_{t+1}}{R^*_t}\right)\right\}, \text{ where } \hat{R}^*_{t+1} = R^*_t - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}.$$

We have $D^*_{t+1} = md^*_{t+1}$, and $\dfrac{D_{t+1}}{R_t} = -\dfrac{\mathcal{E}_t D^*_{t+1}}{R^*_t}$

Equilibrium in Financial Markets

$$B_{t+1} + N_{t+1} + D_{t+1} = 0 \qquad \text{and} \qquad B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0$$

Deviation from U.I.P.

Log-linearize model around steady-state where $\bar{B} = \bar{B}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{Q} = 1$:

$$i_t - i^*_t - E_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}$$

where $i_t - i^*_t = \ln\left(R_t / R^*_t\right)$ and $b_t = B_t / \bar{Y}$, $\chi_1 = \dfrac{n\omega\sigma_e^2}{\beta m}$, $\chi_2 = \bar{Y}\dfrac{\omega\sigma_e^2}{m}$

An increase in ψ_t raises the return on home bonds as noise traders demand more of the foreign bond. If Home households demand more, the excess return on home bonds goes down.

n is the number of noise traders, *m* is the number of intermediaries

Solving the model

First, we can solve for consumption in home relative to foreign:

$$\tilde{c}_t = \kappa_a \tilde{a}_t - \gamma \kappa_q q_t$$

Note that when γ is very small, so there is a lot of home bias, then relative consumption is not very responsive to real exchange rate changes.

From the Euler equations, we have:

$$E_t \left\{ \sigma \Delta c_{t+1} - \Delta q_{t+1} \right\} = i_t - i_t^* - E_t \Delta e_{t+1}$$

Combining with the equation above for the U.I.P. deviation, we get:

$$\left(1 + \gamma \sigma \kappa_q\right) E_t \Delta q_{t+1} = \chi_2 b_{t+1} - \chi_1 \psi_t + \sigma \kappa_a E_t \Delta \tilde{a}_{t+1}$$

We can rewrite the above equation by dividing through by $1+\gamma\sigma\kappa_q$:

$$E_t \Delta q_{t+1} = \hat{\chi}_2 \hat{b}_{t+1} - \hat{\chi}_1 \psi_t + \left(1-\rho\right) k\tilde{a}_{t+1},$$

where $\hat{b}_t = \dfrac{\beta}{\gamma\lambda_q} b_t$, $\hat{\chi}_2 = \dfrac{\gamma\lambda_q}{\beta\left(1+\gamma\sigma\kappa_1\right)} \chi_2$, etc.

We also have:

$$\hat{b}_{t+1} = q_t + \left(1/\beta\right)\hat{b}_t + \hat{a}_t.$$

This latter equation follows with some algebra from the condition that the bond accumulation equals the trade balance.

We can write the system as:

$$\begin{bmatrix} Eq_{t+1} \\ \hat{b}_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\hat{\chi}_2 & \hat{\chi}_2 / \beta \\ 1 & 1/\beta \end{bmatrix} \begin{bmatrix} q_t \\ \hat{b}_t \end{bmatrix} - \begin{bmatrix} \hat{\chi}_1 & (1-\rho)k - \hat{\chi}_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_t \\ \tilde{a}_t \end{bmatrix}.$$

The eigenvalues of the system are given by:

$$\mu_{1,2} = \frac{1}{2}\left[1+\tilde{\chi}_2 + 1/\beta \mp \sqrt{\left(1+\tilde{\chi}_2 + 1/\beta\right) - 4/\beta}\right], \text{ with}$$

$$0 < \mu_1 \le 1 < \frac{1}{\beta} < \mu_2$$

$\wp_t = q_t + \left(1/\beta - \mu_1\right)\hat{b}_t$ is the linear combination that is solved forward using the root μ_2. That is $\wp_t$ is a present discounted value of expected future values of the stochastic driving processes.

Now, we can use a result from Engel and West (2005): If $\wp_t$ is a present value of stochastic processes that have a unit root, then as the discount factor goes to one, $\wp_t$ goes to a random walk.

But Engel and West (2005) show that the result also applies to stationary stochastic processes in the limit as the largest root goes to one.

So, if $1/\mu_2 \to 1$ and $\rho \to 1$, then $\wp_t$ goes to a random walk.

As $\gamma \to 0$, $\mu_2 \to 1/\beta$. If we then let $\beta \to 1$, we have $1/\mu_2 \to 1$, which then gives us that $\wp_t$ goes to a random walk.

But $\wp_t = q_t + (1/\beta - \mu_1)\hat{b}_t$. As $\gamma \to 0$, $\mu_1 \to 1$, and since $\beta \to 1$, we have $\wp_t \to q_t$. Therefore, $q_t = e_t$ goes to a random walk.

Now, note that under these assumptions, $E_t \Delta q_{t+1} = \chi_2 b_{t+1} - \chi_1 \psi_t$. The expected change in the exchange rate is not zero, so how can it be going to a random walk? Because the innovations in the exchange rate are getting very large!

Proposition 1 is exactly the proposition that the exchange rate follows a random walk with a large variance. What lies behind this?

In particular, it is the noise trader shocks driving the exchange rate, so what is going on?

Suppose noise traders want to borrow more from Home, so Home ends up lending more.

Then Home households must reduce consumption. If home consumes less, the price of home goods must fall, so there is a home depreciation, as we will see. Because consumption is very insensitive to the real exchange rate when the economy is nearly closed, there must be a large depreciation.

Hence, a surprise shock from noise trader bond demand leads to a large exchange rate change.

Why is an increase in home consumption relative to foreign consumption associated with an appreciation – a rise in the price of home goods?

Suppose home output were fixed because labor supply was fixed and there were no productivity shocks.

An increase in the home consumption would raise the relative price of home output.

In fact, an increase in the price of the home good causes the wage to rise, since the real product wage is unchanged (since productivity did not change.) But the real consumption wage rises since the consumer price level rises less than the price of the home good. This leads to greater labor supply.

Hence, output rises. An increase in output is met with an increase in consumption in both countries, but more so in the Home country because of home bias.

$$\sigma c_t + \varphi y_t = (1+\varphi) a_t - \frac{\gamma}{1-2\gamma} q_t$$

Proposition 1:

The equilibrium nominal exchange rate follows a volatile near random walk process; in particular, when both shock persistence and discount factor ≈ 1, then $corr\left(\Delta e_t \Delta e_{t-1}\right) \approx 0$.

Since the nominal price level is stabilized in both countries, the real exchange rate and nominal exchange rate have a correlation of one.

That gives us Proposition 2, which takes care of the PPP puzzle: real and nominal exchange rates follow near random walks, and the real and nominal exchange rates are very persistent, and highly correlated.

Proposition 5 is the exchange-rate disconnect, which we have also already covered. The exchange rate is volatile – much more volatile than real output, for example. But there is "disconnect" because there is so much home bias.

The Backus-Smith puzzle:

In standard models, when markets are complete: $\sigma\left(\Delta c_{t+1} - \Delta c_{t+1}^{*}\right) = \Delta q_{t+1}$

But in the data for advanced countries, this correlation is slightly negative.

Even in a model with incomplete markets, $\sigma E_t\left(\Delta c_{t+1} - \Delta c_{t+1}^{*}\right) = E_t \Delta q_{t+1}$. This type of model still tends to make the wrong prediction about the correlation of relative consumption changes and changes in the real exchange rate. Why?

$\sigma\left(\Delta c_{t+1} - \Delta c_{t+1}^{*}\right) = \sigma E_t\left(\Delta c_{t+1} - \Delta c_{t+1}^{*}\right) + \sigma S_{t+1}\left(\Delta c_{t+1} - \Delta c_{t+1}^{*}\right)$ and

$\Delta q_{t+1} = E_t \Delta q_{t+1} + S_{t+1} \Delta q_{t+1}$

where $S_{t+1} x_{t+1} = x_{t+1} - E_t x_{t+1}$ -- the "surprise" in x_{t+1}.

Unless the covariances of the surprises is large and the opposite sign from the expected change, the covariance of the actual changes will equal the covariance of the expected changes.

This model can produce the opposite correlation when real exchange rates are primarily driven by noise trader shocks. We have already said why there is a real appreciation when home consumption rises.

Let's see it in equations:

$\sigma c_t + \varphi\left(y_t - a_t\right) = a_t - \frac{\gamma}{1-2\gamma} q_t$. The rhs is the MRS between consumption and leisure. The RHS is the MPL.

$y_t = \left(1-\gamma\right) c_t + \gamma c_t^* + 2\theta \frac{\gamma\left(1-\gamma\right)}{1-2\gamma} q_t$ - home goods market clearing for.

Then take home relative to foreign (using analogous equations for foreign):

$$c_t - c_t^* = \kappa_a\left(a_t - a_t^*\right) - \gamma \kappa_q q_t$$

When productivity shocks are relative unimportant, and the real exchange rate is volatile, then we get the negative correlation.

U.I.P. puzzle

Using the Euler equation and the equation above for relative consumption, we find:

$i_t - i_t^* = \sigma E_t\left(\Delta c_{t+1} - \Delta c_{t+1}^*\right) = -\gamma\sigma\kappa_q E_t \Delta e_{t+1}$, where I am ignoring productivity shocks. This equation alone shows how the UIP puzzle is solved.

In words: Suppose there is a positive shock to ψ_t so noise traders want to sell home bonds (borrow more from households.) On the one hand, some of this demand spills over to the financial intermediary, who insists on a higher expected return on home bonds.

On the other hand, as we have seen, some of this also spills over to home households. They lend more to noise traders, thereby reducing their consumption. This reduction in consumption leads to a real depreciation for the reasons we discussed above.

As consumption is expected to increase over time, the currency is expected to appreciate.

Note that if noise trader shocks dominate, we should see a tendency for the currency to depreciate on impact when the home interest rate increases.

Then from above, we had the U.I.P. equation when $\chi_2 \approx 0$:

$i_t - i_t^* - E_t \Delta e_{t+1} = \chi_1 \psi_t$, which gives us

$$i_t - i_t^* = \frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\chi_1\psi_t.$$

and

$$E_t \Delta e_{t+1} = -\frac{1}{1+\gamma\sigma\kappa_q}\chi_1\psi_t$$

The key here is breaking the relationship between relative consumption and the real exchange rate.

Terms of Trade Puzzle:

This one is not solved because $q_t = -(1-\gamma)s_t$. But when γ is close to zero, the terms of trade and the real exchange rate are roughly equally volatile.

Is this the right model?

1. The model implies a nearly perfect correlation between the trade balance and the real exchange rate, but in the data, it is close to zero.

This is not coming from an expenditure switching effect. Again, it is the mechanism described above: Noise traders want to borrow more. Home consumption drops, leading to a trade surplus, and the currency depreciates.

Indeed, these papers, which were the original “disconnect” papers, put special emphasis on exactly that correlation:

Obstfeld, Maurice, and Kenneth Rogoff. 2001. “The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?” **NBER Macroeconomics Annual** 15, 339-390.

Baxter, Marianne, and Alan C. Stockman. 1989. “Business Cycles and the Exchange Rate Regime: Some International Evidence.” **Journal of Monetary Economics** 23, 377-400.

2. The model has the Euler equation holding, $i_t - i_t^* = \sigma E_t\left(\Delta c_{t+1} - \Delta c_{t+1}^*\right)$, but this is strongly rejected in the data.
3. The model predicts an increase in the home interest rate is associated with a depreciation.
4. There are no wealth effects of exchange rate changes in the model because it is linearized around $\bar{B} = 0$.
5. Is home bias the right way to model low levels of international trade? Is it equivalent to trade costs?
6. Non-traded goods? Model assumes home bias encompasses nontraded goods, but that doesn't seem right.
7. Pricing to market?
8. Persistence (near random walk behavior) arises purely because of shock persistence. Why are noise trader shocks so persistent?
9. While it is generally agreed that standard preferences don't generate large risk premiums, there the intermediaries must be extremely risk averse. With CARA preferences, the coefficient of relative risk aversion depends on their wealth or consumption level. When they are small, they are more risk averse.

One final note

It is difficult to reconcile the Gourinchas-Rey paper from earlier in the semester and this paper.

Gourinchas and Rey say that during times of global stress, the U.S. makes a transfer to the rest of the world. It's consumption falls relative to the rest of the world (hence, its relative marginal utility of consumption rises) when its real exchange rate appreciates. (This, by the way, is the correlation that Maggiori finds puzzling that generates his "reserve currency paradox".)

In other words, Gourinchas and Rey say that $\sigma\left(\Delta c_{t+1} - \Delta c^{*}_{t+1}\right)$ and Δq_{t+1} are positively correlated.

Itskhoki and Mukhin say that $\sigma\left(\Delta c_{t+1} - \Delta c^{*}_{t+1}\right)$ and Δq_{t+1} are positively correlated.

It's hard to resolve puzzles in the field if we cannot agree on the facts.

Collateral Advantage: Exchange Rates, Capital Flows, and Global Cycles

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March 31, 2023

Purdue University

- A large recent literature has focused on the liquidity yield or "**convenience yield"** of short-term U.S. government bonds.
- The expected return on U.S. government bonds is **lower** than corresponding rates for government bonds from other advanced countries
  - **Price risk** does **not** entirely account for this difference
  - Nor does **default risk**
- This liquidity yield has the potential to help explain a large number of interesting empirical facts
- Many models of the convenience yield are not strongly **microfounded** (e.g., bonds in the utility function.)
  - But microfoundations can **matter** for implications of the relationship between the convenience yield and macro variables

Objectives:

- Build a DSGE model with an **endogenous convenience yield** arising from demand for liquid dollar assets and examine global economy.
- What drives the **dollar exchange rate** over the global cycle?
- How are **asset returns** influenced by global financial shocks?
- Why is the **U.S. dollar** special?
- What determines international **financial flows** during times of global stress?

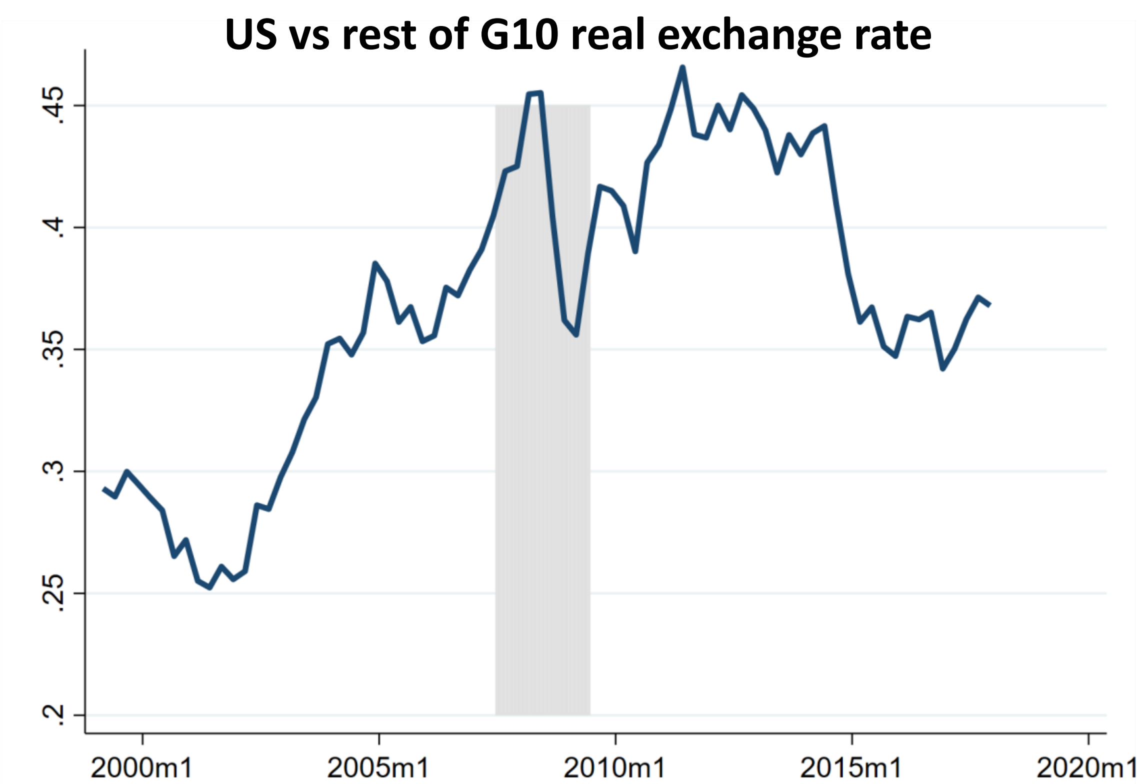
What Does our Framework Say about Policy?

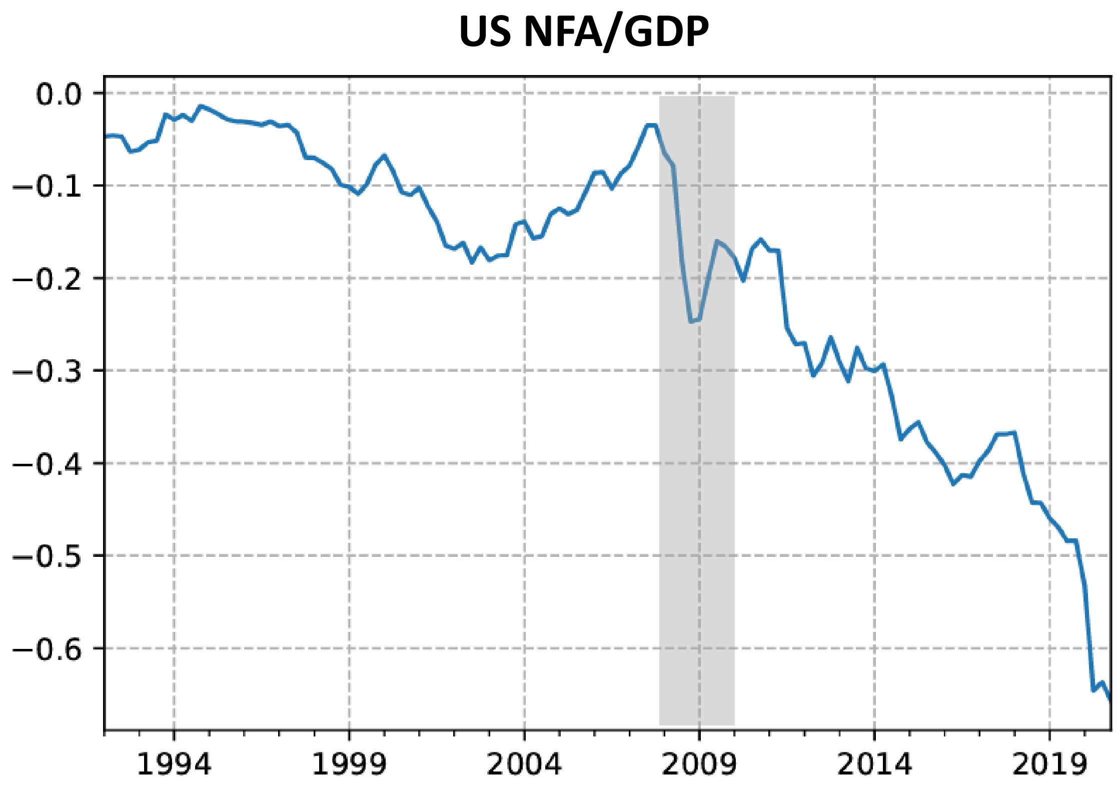
- How does a U.S. monetary policy shock **spill over** to the rest of the world?
- What are the **differential effects** across countries of a **global monetary contraction**, such as the current one?
- How do conventional monetary policy and **quantitative easing/ti**ghtening differ and how might they interact?
- Is there a role for **sterilized intervention** in foreign exchange markets, and what is the mechanism?

External positions + exchange rates

- Exchange rate appreciation in global financial crisis (GFC)
- Commonly labelled as “Flight to safety/quality”

→ wealth transfer to the Rest of the World





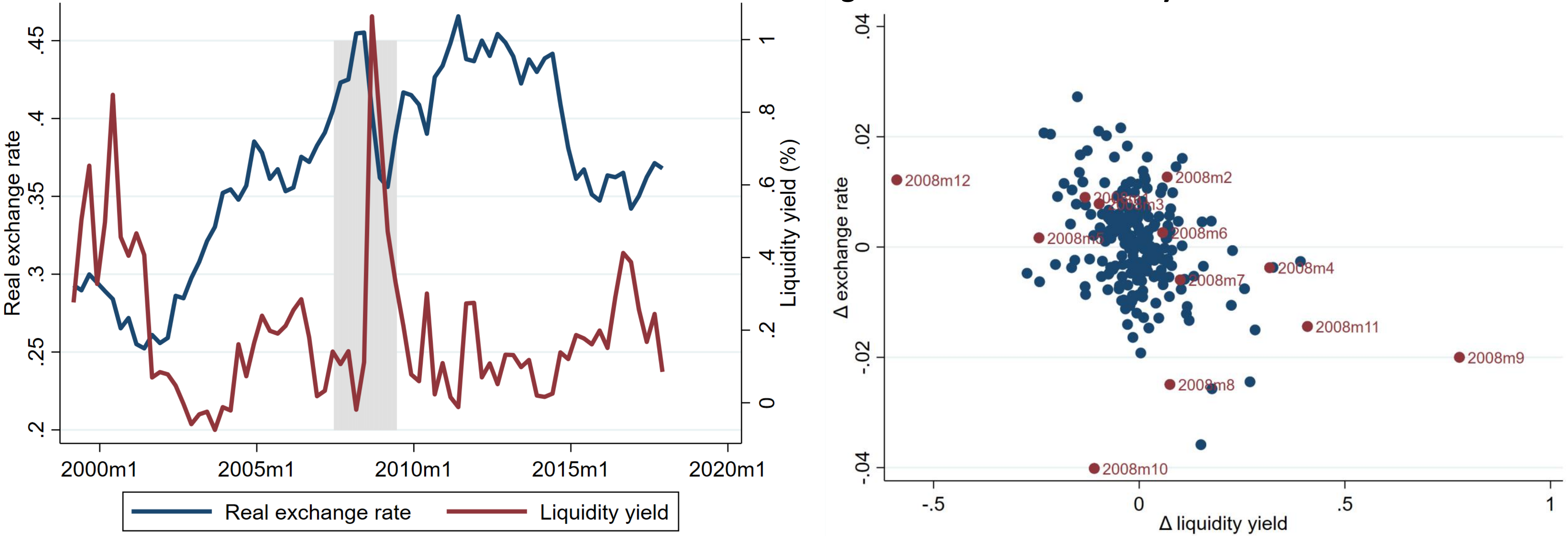
- Referred as “Exorbitant duty” of the US

Source: Atkeson, Heathcote, Perri 2022

Exchange rates and liquidity yield

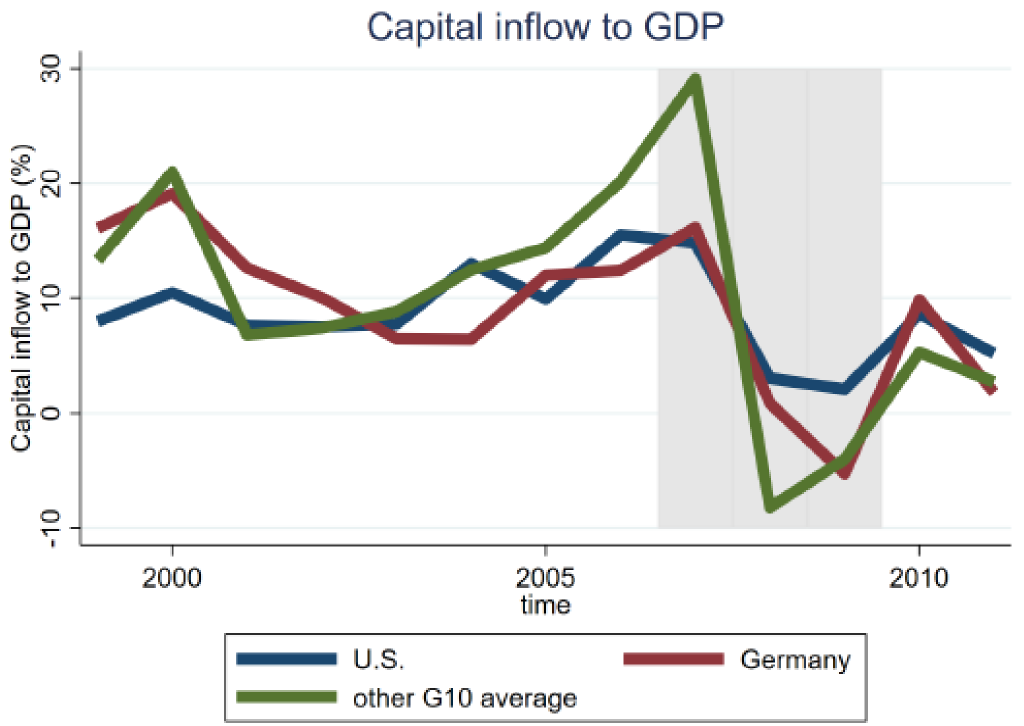
- Exchange rate is notoriously hard to explain (exchange rate disconnect)
- A recent literature focus on liquidity/convenience yield of Treasury securities
  - Engel and Wu 2022, Jiang, Krishnamurthy, Lustig 2021

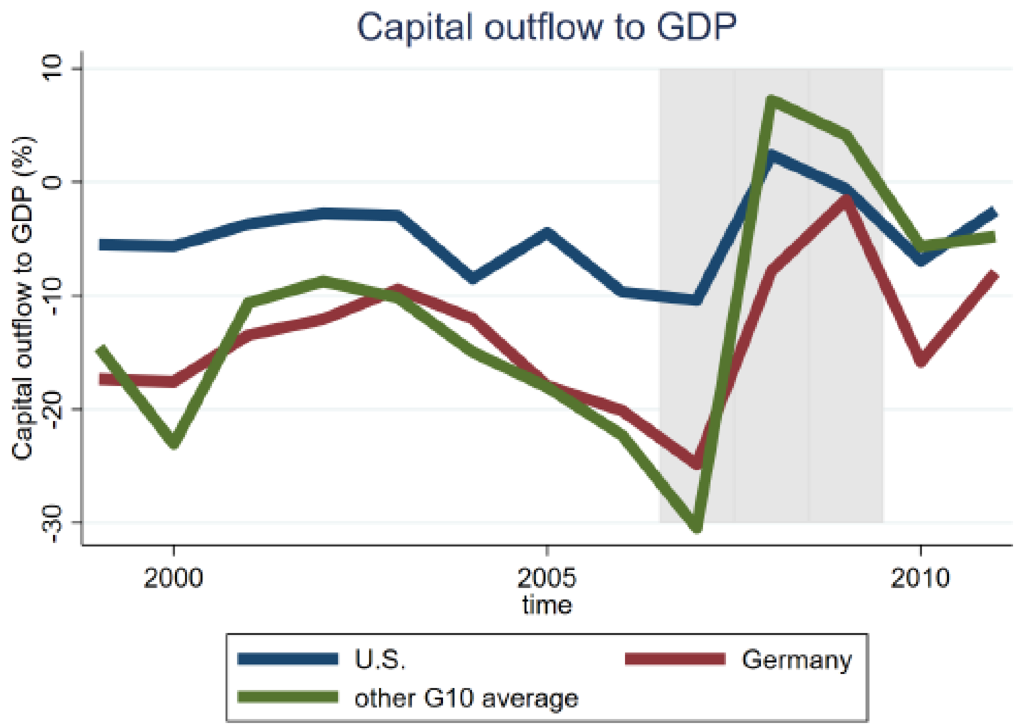
US vs rest of G10 real exchange rate and convenience yield



Data: Engel and Wu 2022

Retrenchment





Recent literature raises puzzles

- **Risk story** Gourinchas and Rey: US as global insurance provider
- Insurance premium → excess return on NFA (risk premium)
- Insurance payout by USD appreciation in bad times
- USD appreciation driven by strong "flight to insurance" purchase of US assets

1) Can a country **buy insurance** when the bad event is happening?
2) **Reserve currency paradox** – Maggiori 2017
If there is a wealth transfer (insurance payout) from the US to the rest of the world (RoW), US net worth falls relative to the RoW. Home demand for goods falls (relatively).
If the real exchange rate is driven by the terms of trade and there is home bias in preferences, the US dollar should **depreciate** rather than appreciate.

Recent literature: exchange rates

- **Convenience yield story**
- Treasury bonds are attractive not just for the monetary return but for some liquidity or convenience services (non-pecuniary return)
- High convenience demand results in USD appreciation
  (like unobserved dividends)

Issues:

1) Almost all models **assume exogenous or ad hoc model** of convenience yield

Engel and Wu 2022, Jiang, Krishnamurthy, Lustig 2021, Kekre and Lenel 2021

2) Models of **prices but not flows of assets**
3) By endogenizing the convenience yield, we find plausible predictions about variables such as asset flows and investment that arise from balance sheet constraints.

What we do in this paper

- A NK DSGE model with banks to generate endogenous convenience yield

- Banks as in Gertler Karadi 2011, Gertler Kiyotaki 2010 who face collateral constraint on their asset holding

- Symmetric 2-country model (US and foreign) with one asymmetry

US bond is assumed to be better collateral

- Why? Easier to assess value, widely traded in deep markets (liquidity)

What we find

- Upon a uniform global financial shock

1. Banks has tight balance sheet constraints→ run to least constrained assets (US bonds)
2. Demand for US bond appreciates the currency
3. Wealth transfer from the US to RoW → no reserve currency paradox (role of LCP)
4. Retrenchment for both countries
5. Investment falls in both countries

- Exchange rates

1. Uncovered interest parity deviation (driven by convenience yield)
2. Endogenous convenience yield

Literature review

- **Exorbitant Privilege and global imbalances:**

Caballero, Farhi, and Gourinchas (2008, 2015), Gourinchas and Rey (2007a, 2007b, 2022)

+ Reserve Currency Paradox: Maggiori (2017); Farhi and Maggiori (2018); Gourinchas, Rey and Sauzet (2019)

- **Global financial intermediation and exchange rates:**

Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021, 2022), Bruno and Shin (2015), Du, Tepper, and Verdelhan (2018), Fang and Liu (2021), Banerjee, Devereux, and Lombardo (2016), Devereux and Yu (2020), Gopinath and Stein (2020)

- **Convenience yields:**

Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), Du, Im and Schreger (2018)

+ exchange rates: Engel (2016), Valchev (2018), Jiang, Krishnamurthy, and Lustig (2021a, 2022), Engel and Wu (2021), Bianchi, Bigio, and Engel (2022)

+ global imbalances: Kekre and Lenel (2021), Jiang, Krishnamurthy, and Lustig (2021)

Two Key Differences with Recent Related Literature

1. Exogenous vs. endogenous
   a) Itskhoki-Mukhin (2022, 2023) introduce a shock to relative expected returns that arises from *exogenous* changes in demand for home versus foreign bonds from **noise traders**.
   b) Kekre and Lenel (2022) and Jiang, Krishnamurthy, and Lustig (2022a,b,c) have *exogenous* changes in demand for U.S. bonds from investors.
   c) In our model, there is a **neutral** global financial tightening, but that leads to an *endogenous* change in demand toward U.S. bonds.
2. The source of the shock matters for macro variables. In our paper, we spotlight financial stress
   a) Directly, investment falls **as balance sheets tighten**
   b) In our framework, there is a magnifying effect as financial intermediaries **rebalance** portfolios toward government-backed liquid bonds.

Road map

1. A preliminary empirical result

2. Model

3. Quantitative calibration (impuse responses. We’re working on simulations!)

4. Impulse Response Functions

Convenience yield and value of the dollar

- Some preliminary results extend the findings of Jiang et al. (J. of Finance, 2021) and Engel and Wu (ReStud, 2023) on the **convenience yield** and exchange rates.
- The **covered dollar return** on a foreign 1-year government bond is compared to the return on a U.S. government bond.
- Both investments are **riskless** (when including the cost of CDSs.) When this return rises, we say the **convenience yield** on U.S. government bonds rises – perhaps the liquidity return on U.S. bonds increases.
- Here, we **instrument** the convenience yield using financial shocks constructed by Ottonello and Song (2022), which are high-frequency changes in the market value of US intermediaries' net worth in a narrow 60-min window around their earnings announcements. January 2001 – July 2014.

$$\Delta s_{j,t} = \alpha_j + \beta_1 s_{j,t-1} + \beta_2 \Delta\eta_{j,t} + \beta_3 \Delta(i - i^*)_{j,t} + \beta_4 \eta_{j,t-1} + \beta_5 (i - i^*)_{j,t-1} + u_{j,t}$$

Table 1: Daily exchange rate regression with financial shocks as instrumental variables

	LHS: $\Delta s_{j,t}$
$\Delta\hat{\eta}_{j,t}$	-9.07**
	(4.53)
$\Delta i - i^*_{j,t}$	-8.58**
	(3.86)
$\eta_{j,t-1}$	-0.002
	(0.002)
$i - i^*_{j,t-1}$	-0.040
	(0.03)
$s_{j,t-1}$	-0.003
	(0.002)
N	1746

A two-country New Keynesian model with Treasury liquidity

- Goods market

- Home (US) and foreign (Eurozone) goods

- Nominal price stickiness with pricing to market (i.e., local currency pricing – LCP)

- Banking sector

- Gertler Karadi / Gertler Kiyotaki type of Home and Foreign banks

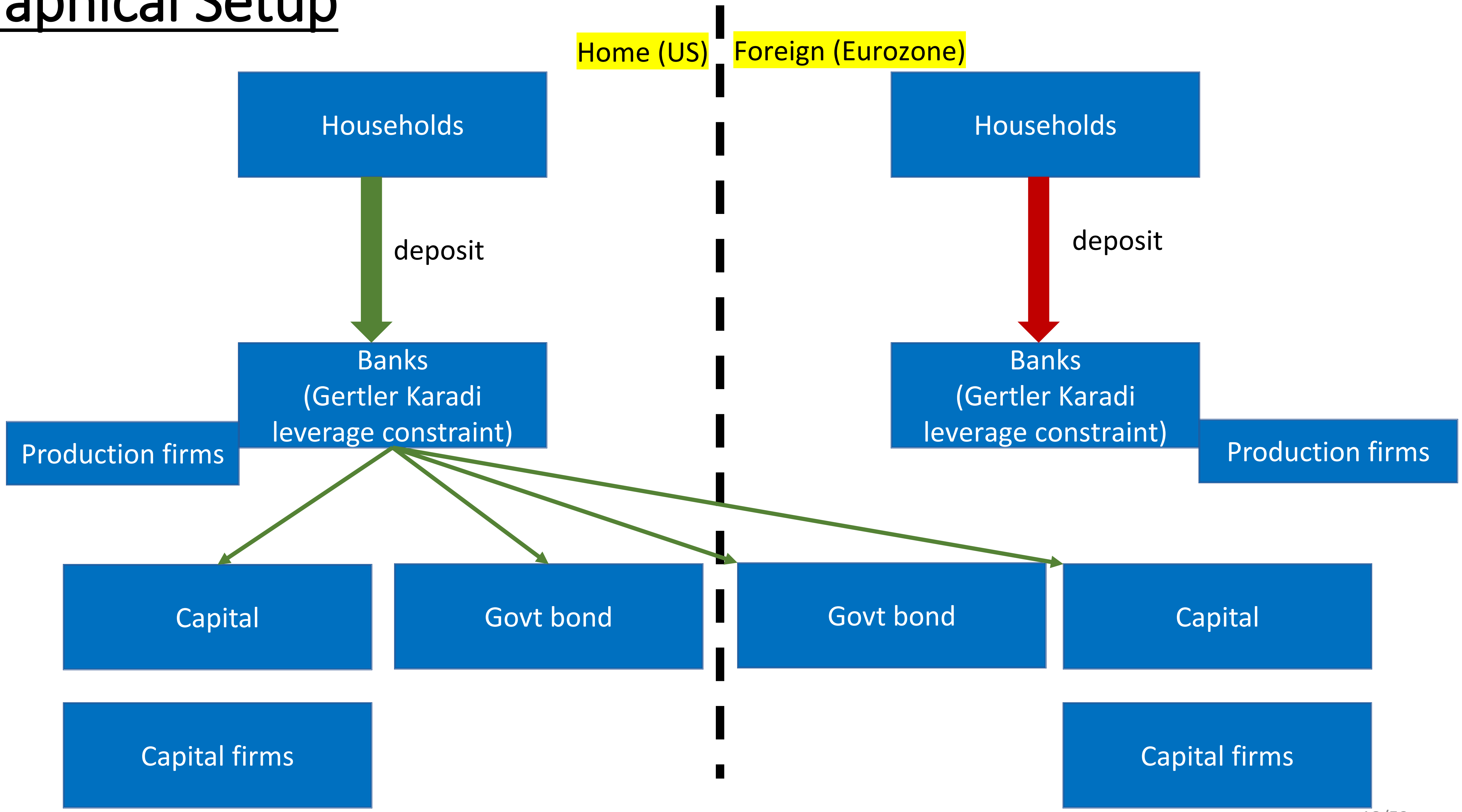
- Moral hazard problem → Incentive constraint on asset holding

- Assets market

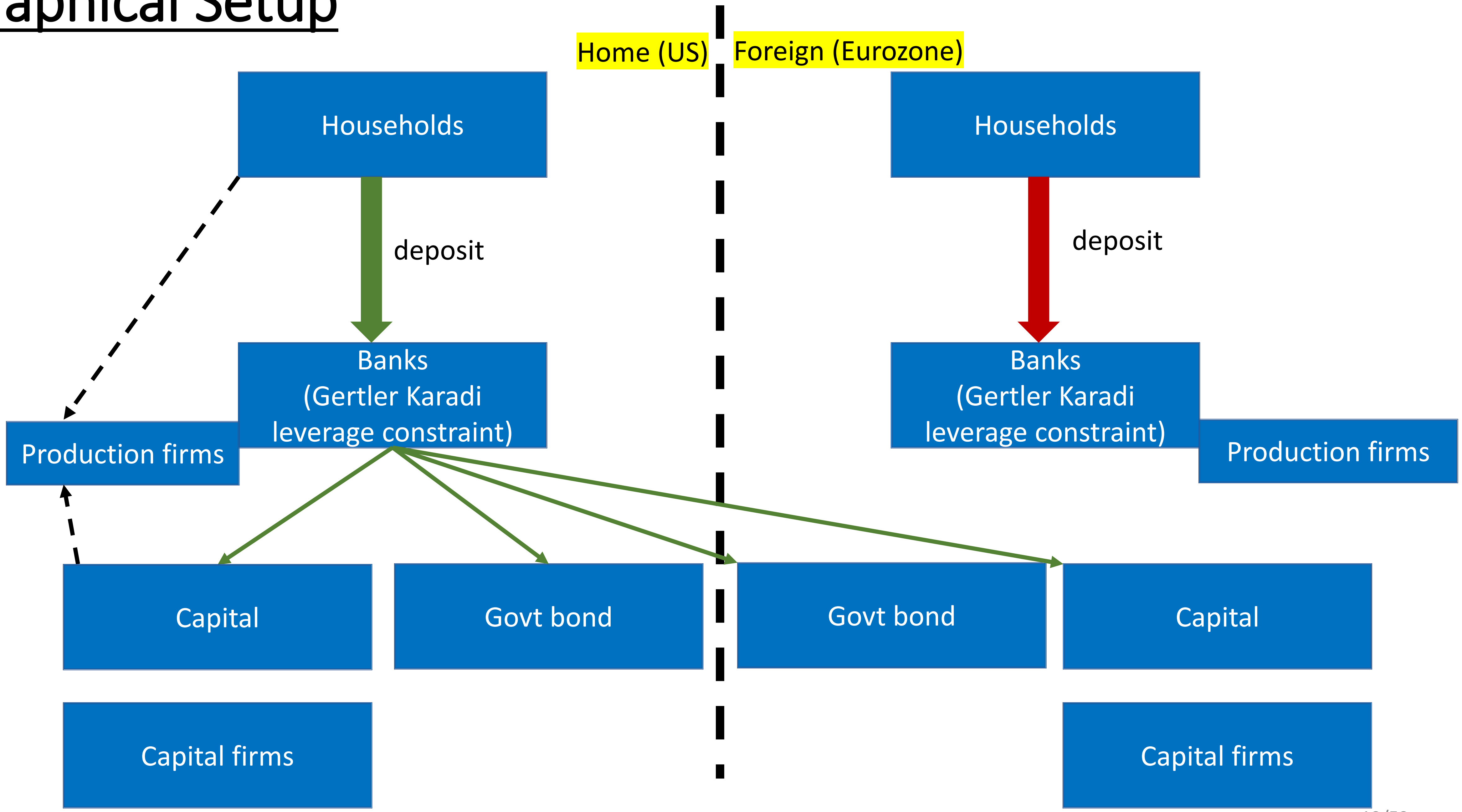
- Home bond, foreign bond, home capital, foreign capital

- Key is that home bond is a better collateral

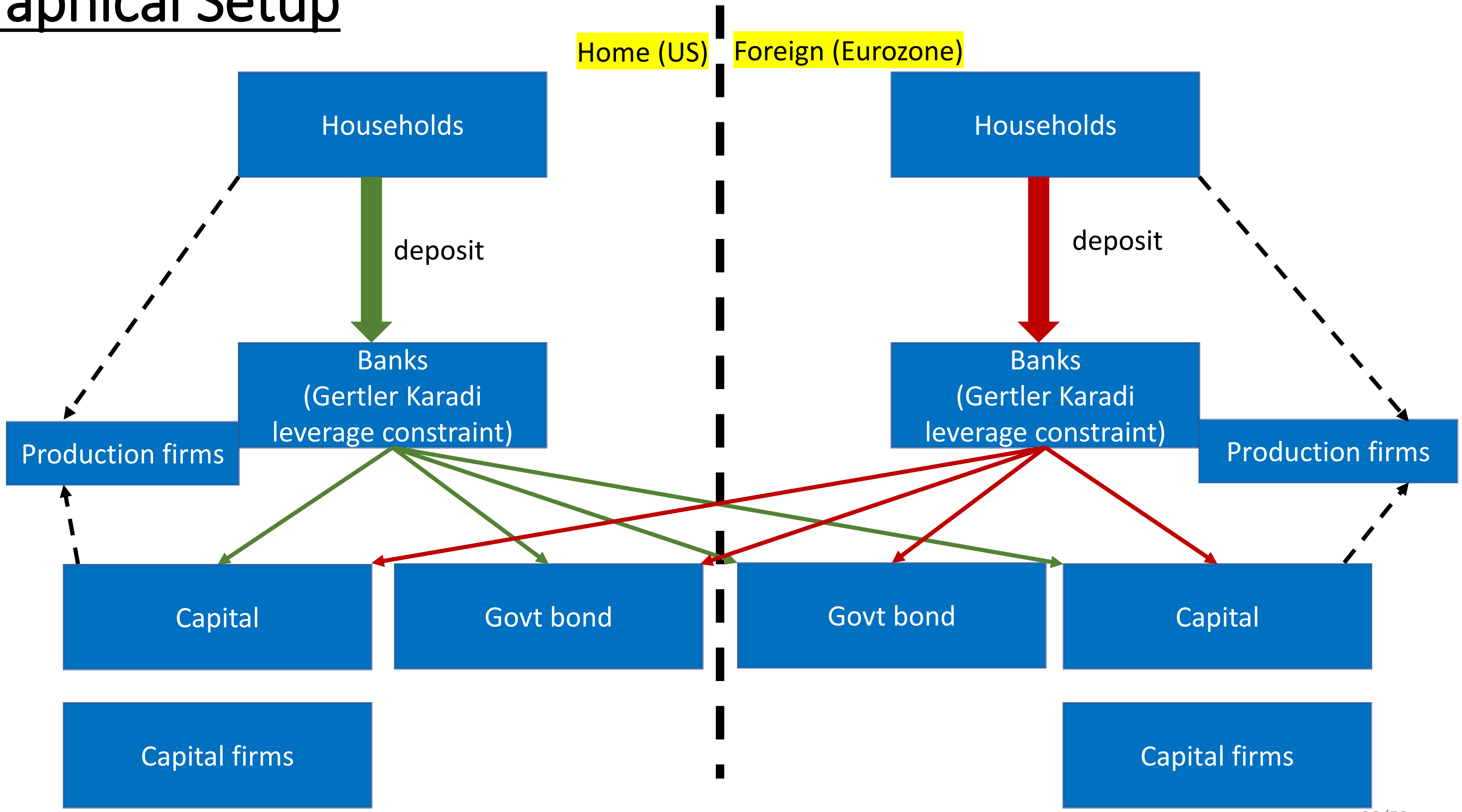
Graphical Setup



Graphical Setup



Graphical Setup



Households

$$U = E_t \sum_t \beta^t [\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\psi} L_t^{1+\psi}]$$

subject to

$$C_t = \left(\omega^{\frac{1}{\lambda}} C_{h,t}^{1-\frac{1}{\lambda}} + (1-\omega)^{\frac{1}{\lambda}} C_{f,t}^{1-\frac{1}{\lambda}}\right)^{\frac{1}{1-\frac{1}{\lambda}}}$$

and

$$P_t C_t + \underbrace{B_t}_{\text{Deposits}} = W_t L_t + R_t B_{t-1} + \underbrace{\Pi_t}_{\text{Profits}} + \underbrace{TR_t}_{\text{Govt transfer}} - \underbrace{T_{s,t}}_{\text{Injection for new banks}}$$

Firms

- In each country, a continuum of firms produce differentiated goods

- Production function is

$$Y_t = A_t(L_t^{1-\alpha}K_t^{\alpha})$$

- Profit of a firm:

$$\Pi_t = \begin{pmatrix} (1+sub)\left(P_{h,t}Y_{h,t} + S_t P^*_{h,t} Y^*_{h,t}\right) \\ -MC_t\left(Y_{h,t} + Y^*_{h,t}\right) - \xi\left(\frac{P_{h,t}}{P_{h,t-1}}\right) P_{h,t}Y_{h,t} - \xi\left(\frac{P^*_{h,t}}{P^*_{h,t-1}}\right) S_t P^*_{h,t} Y^*_{h,t} \end{pmatrix}$$

- Local Currency Pricing ($P_{h,t}, P^*_{h,t}$) pricing allows exchange rate to be delinked from terms of trade and relative consumption
- ξ is Rotemberg price adjustment
- Two Phillips Curve for each country

Capital producers

- In each country, capital goods producers buy undepreciated capital from banks and use it to produce investment goods. They sell the new capital back to banks.

- Profit is

$$Q_t I_t - P_t(I_t + K_t \psi(\frac{I_t}{K_t}))$$

where ψ is a quadratic function

Banks

- Follows the Gertler and Karadi framework

- A fraction θ of each household becomes a banker each period, and continues with probability θ, and reverts to being a consumer with probability $1-\theta$

- Balance sheet of bank (omitted *i* subscript):

$$\left[Q_t K_{h,t+1} + D_{h,t}\right] + S_t\left[Q_t^* K_{f,t+1} + D_{f,t}\right] = N_t + B_t$$

[investment in Home asset] + [investment in Foreign asset] = Net worth + deposit

where Q_t is the home capital price, S_t is the home price of a foreign currency

K_h is the home bank holding of <u>home capital</u>
K_f is the home bank holding of <u>foreign capital</u>
D_h is the home bank holding of <u>home bond</u>
D_f is the home bank holding of <u>foreign bond</u>

Banks' net worth dynamics

$$N_{t+1} =$$

$$\tilde{R}_{K,t+1} Q_t K_{h,t+1} + R_{h+1} D_{h,t}$$ [Return in investment in Home asset]

$$+ \left(\frac{S_{t+1}}{S_t}\right) \left[S_t \tilde{R}^*_{K,t+1} Q^*_t K_{f,t+1} + R_{f,t+1} S_t D_{f,t} \right]$$ [Return in investment in Foreign asset]

$$- R_{t+1} B_t$$ [Payment to Depositors]

Banks' problem

- Banks' value function is

$$V_t = E_t \Omega_{t+1}[(1-\theta)N_{t+1} + \theta V_{t+1}]$$

- Maximize value function by choosing the four assets (K_h, K_f, D_h, D_f)
- Subject to Gertler-Kiyotaki, Gertler-Karadi type of incentive constraint
- Banker can abscond κ amount of the assets so

value of the bank $\geq \kappa$(value of the assets)

value if staying in business

value if running away

Banks' problem

- Banks' value function is

$$V_t = E_t \Omega_{t+1}[(1-\theta)N_{t+1} + \theta V_{t+1}]$$

- Maximize value function by choosing the four assets (K_h, K_f, D_h, D_f)
- Subject to Gertler-Kiyotaki, Gertler-Karadi type of incentive constraint
- Banker can abscond κ amount of the assets so

$$V_t \geq \vartheta\left[\left(\kappa_{K,h} Q_t K_{h,t+1} + \kappa_h D_{h,t}\right) + \left(\kappa_{K,f} S_t Q_t^* K_{f,t+1} + \kappa_f S_t D_{f,t}\right)\right]$$

- The lower the parameter κ, the less it is divertible, or the more it is pledgeable
- **Key assumption:**

Home bond is the best collateral $\boldsymbol{\kappa_h < \kappa_f \leq \kappa_{K,h} \leq \kappa_{K,f}}$

The same for the foreign banks $\boldsymbol{\kappa_h^* < \kappa_f^* \leq \kappa_{K,f}^* \leq \kappa_{K,h}^*}$

Convexity of Constraint

$$\kappa_h = \kappa_{h1} + \kappa_{h2} D_{h,t} \qquad \kappa_{Kh} = \kappa_{Kh1} + \kappa_{Kh2} Q_t K_{h,t+1}$$

$$\kappa_f = \kappa_{f1} + \kappa_{f2} S_t D_{f,t} \qquad \kappa_{Kf} = \kappa_{Kf1} + \kappa_{Kf2} S_t Q_t^* K_{f,t+1}$$

- This assumption is needed to give us a non-stochastic steady state
  - Which is only needed because we solve by linearizing
  - It does give us the ability to calibrate a plausible steady state
    - home bias in equities, long-run positions
    - Assumptions here similar to recent papers that assume "preferred habitats"
- The "fancy" kappas are very small and play essentially no role in the dynamics
  - It is only the relative fancy kappas that give us the s.s. portfolio

First-order conditions

Bank SDF:

$$\Lambda_{t+1} = \Omega_{t+1}((1-\theta) + \theta\nu_{t+1})$$

These are zeros in frictionless models

$$FOC[D_h]: \quad E_t\Lambda_{t+1}\left(R_{h,t+1} - R_{t+1}\right) = \eta_t\vartheta(\kappa_{h,t})$$

$$FOC\left[D_f\right]: \quad E_t\Lambda_{t+1}\left(\frac{S_{t+1}}{S_t}R_{f,t+1} - R_{t+1}\right) = \eta_t\vartheta(\kappa_{f,t})$$

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$$FOC[K_h]: \; E_t \Lambda_{t+1}\left(\tilde{R}_{K,t+1} - R_{t+1}\right) = \eta_t \vartheta\left(\kappa_{K,h,t}\right)$$

$$FOC\left[K_f\right]: \; E_t \Lambda_{t+1}\left(\frac{S_{t+1}}{S_t} \tilde{R}^*_{K,t+1} - R_{t+1}\right) = \eta_t \vartheta(\kappa_{K,f,t})$$

- Combining $FOC[D_h]$ and $FOC\left[D_f\right]$ gives

UIP wedge

$$E_t \Lambda_{t+1}\left(\frac{S_{t+1}}{S_t} R_{f,t+1} - R_{h,t+1}\right) = \eta_t \vartheta(\kappa_{f,t} - \kappa_{h,t})$$

- As the constraint tightens, η_t rises

Modified UIP and exchange rates

- Combining $FOC[D_h]$ and $FOC[D_f]$ gives

$$E_t\Lambda_{t+1}\left(\frac{S_{t+1}}{S_t}R_{f,t+1} - R_{h,t+1}\right) = \eta_t\vartheta(\kappa_{f,t} - \kappa_{h,t})$$

- Log linearized gives modified UIP

$$E_tS_{t+1} - S_t = R_{h,t} - R_{f,t} + \tilde{\eta}_t$$

- Forward iterating gives

$$S_t = -E_t\left\{\sum_{t=1}^{\infty}\left(R_{h,t} - R_{f,t}\right) + \sum_{t=1}^{\infty}(\tilde{\eta}_t)\right\} + \lim_{k\to\infty} E_t s_{t+k} - k\bar{s}$$

Modified UIP and exchange rates

- Combining $FOC[D_h]$ and $FOC[D_f]$ gives

$$E_t \Lambda_{t+1} \left(\frac{S_{t+1}}{S_t} R_{f,t+1} - R_{h,t+1} \right) = \eta_t \vartheta (\kappa_{f,t} - \kappa_{h,t})$$

- Log linearized gives modified UIP

$$E_t s_{t+1} - s_t = R_{h,t} - R_{f,t} + \tilde{\eta}_t$$

- Forward iterating gives

$$s_t = -E_t \left\{ \sum_{t=1}^{\infty} \left(R_{h,t} - R_{f,t} \right) + \sum_{t=1}^{\infty} (\tilde{\eta}_t) \right\} + \lim_{k \to \infty} E_t s_{t+k} - k\bar{s}$$

- Convenience yield in the model is u.i.p. deviation

$$E_t s_{t+1} - s_t - \left(R_{h,t} - R^{*}_{f,t} \right) \equiv \tilde{\eta}_t$$

LCP and Deviations from LOOP

- Without deviation of LOOP

$$RER_t = \left(\frac{P_{ft}}{P_{ht}}\right)^{2\omega-1}$$

- With LCP, there is deviation of LOOP (D_t)

$$RER_t = \left(\frac{P_{ft}}{P_{ht}}\right)^{2\omega-1} \times D_t$$

where $D_t = \frac{S_t P^*_{h,t}}{P_{h,t}} \backsimeq \frac{S_t P^*_{f,t}}{P_{f,t}} = \frac{Price\ of\ Home\ goods\ selling\ abroad}{Price\ of\ Home\ goods\ selling\ locally}$

Policies

- Monetary policy – CPI inflation targeting

$$R_{h,t} = \frac{1}{\beta}\left(\frac{P_t}{P_{t-1}}\right)^{\eta\rho} R_{h,t-1}^{1-\rho}$$

- Fiscal policy – bond payment + monopoly subsidy

$$\overline{D}_h = R_{h,t}\overline{D}_h + sub\left(P_{h,t}Y_{h,t} + S_t P_{h,t}^{*} Y_{h,t}^{*}\right) + TR_t$$

where $\overline{D}_h$ is exogenous supply of govt bond

Market clearing

- Home and foreign goods market clear, labor market clear

- Bond market clear

$$\overline{D}_h = D_{h,t} + D^*_{h,t}, \overline{D}_f = D_{f,t} + D^*_{f,t}$$

- Capital market clear

$$K_t = K_{h,t} + K^*_{h,t}, K^*_t = K_{f,t} + K^*_{f,t}$$

Home Balance of payments

$$\overbrace{P_{h,t}Y_{h,t}\left(1-\xi\left(\frac{P_{h,t}}{P_{h,t-1}}\right)\right)+S_tP^*_{h,t}Y^*_{h,t}\left(1-\xi\left(\frac{P^*_{h,t}}{P^*_{h,t-1}}\right)\right)-P_t\left(C_t+X_t+I_t+K_t\psi\left(\frac{I_t}{K_t}\right)\right)}^{\textbf{Y-C-I-X}}$$

$$=$$

New assets

$$\left(D_{h,t}-\overline{D}_{h,t}\right)+Q_tK_{h,t+1}+S_tD_{f,t}+S_tQ^*_tK_{f,t+1}$$

$$-R_h\left(D_{h,t-1}-\overline{D}_{h,t-1}\right)-S_tR_{f,t}D_{f,t-1}-Q_{t-1}R_{K,t}\left(K_{h,t}-K_t\right)-S_tR^*_{K,t}Q^*_{t-1}K_{f,t}$$

Return on existing assets

Road map

1. Model
2. **Quantitative calibration**
3. IRFs to mimic GFC
4. Preliminary empirical comparison

Calibration

- Annual frequency
- Solved by log-linearization
- Added a small quadratic cost to pin down the portfolio (similar to SGU 2003)

- The households’ and firms’ side of the model are standard
- E.g. $\delta = 0.1$, CRRA = 5, $\beta = 0.97$ etc

Calibration table

Symbol	Meaning	Value	target
$\overline{D_h} = \overline{D_f}$	Total govt debt	0.4	Debt to GDP of 77%
θ	Bank survival prob.	0.878	Leverage of ~ 6
κ_h	Home constraint cost of holding home bond	0.075	Convenience yield = 1%
κ_h^*	Foreign constraint cost of holding home bond	0.085	Net foreign income / GDP = 0.0013
κ_f	Home constraint cost of holding foreign bond	0.414	Foreign holding of US Treasury of 27%
κ_f^*	Foreign constraint cost of holding foreign bond	0.41	NFA/GDP -0.011
$\kappa_{Kh}^* = \kappa_{Kf}$	Constraint cost of holding external capital	0.65	Equity premium of 5%
$\kappa_{Kh} = \kappa_{Kf}^*$	Constraint cost of holding own capital	0.48	Home bias of equity of ~ 85%

Steady state

Symbol	Steady state
NFA/GDP	-1.11%
$r_f - r_h$	4.4 - 3.4% = 1%
Net income from abroad / GDP	0.13%

Exorbitant privilege:
Net income from abroad > 0 because of convenience yield despite NFA < 0

Symbol	Steady state
C, C*	0.3985 , 0.3980
L, L*	0.329 , 0.331
Y, Y*	0.516 , 0.518
Home, Foreign bank's leverage (asset/equity)	5.92 , 5.49

US has slightly higher consumption, despite lower L and Y
US bank is more leveraged

Road map

1. Model

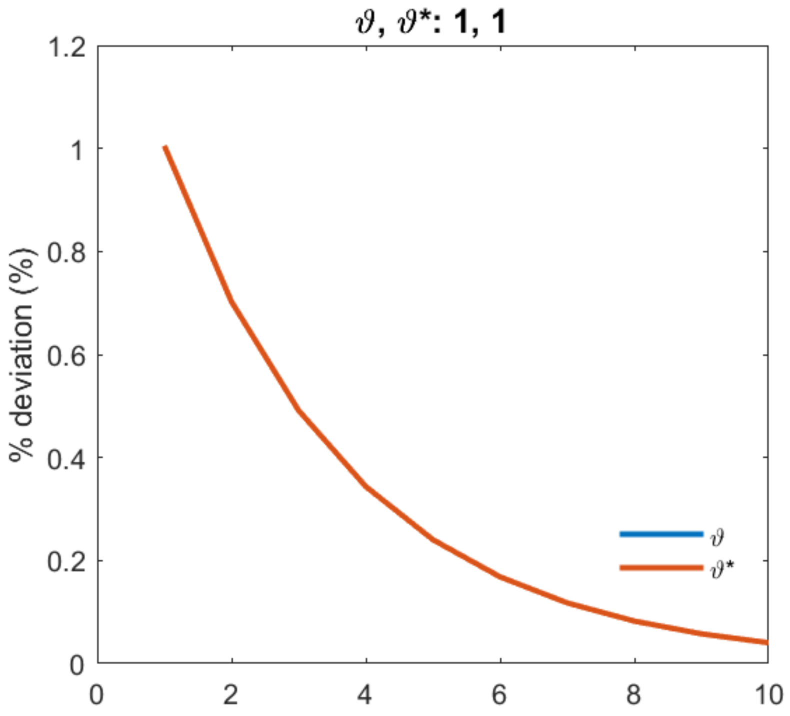
2. Quantitative calibration

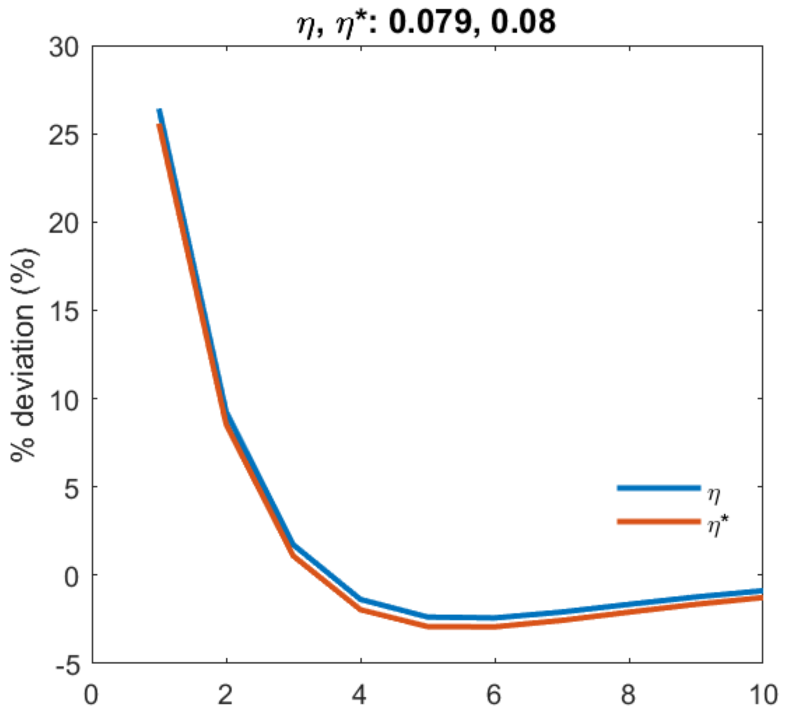
3. **IRFs**

4. Preliminary empirical comparison

Experiment

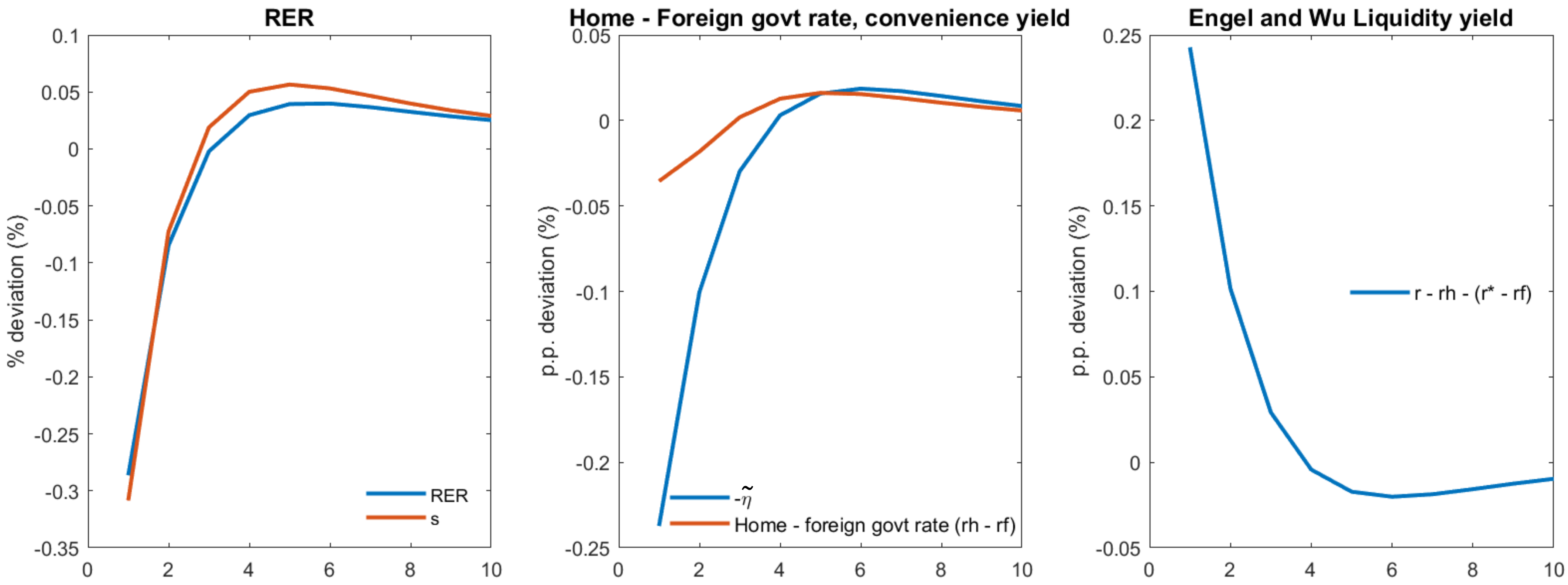
- Look at the no cross-country capital trade for sharper illustration
- A 1% shock to ϑ and ϑ^* (1% tightening to all assets on incentive constraint)
- The shock is AR1, with persistence of 0.7





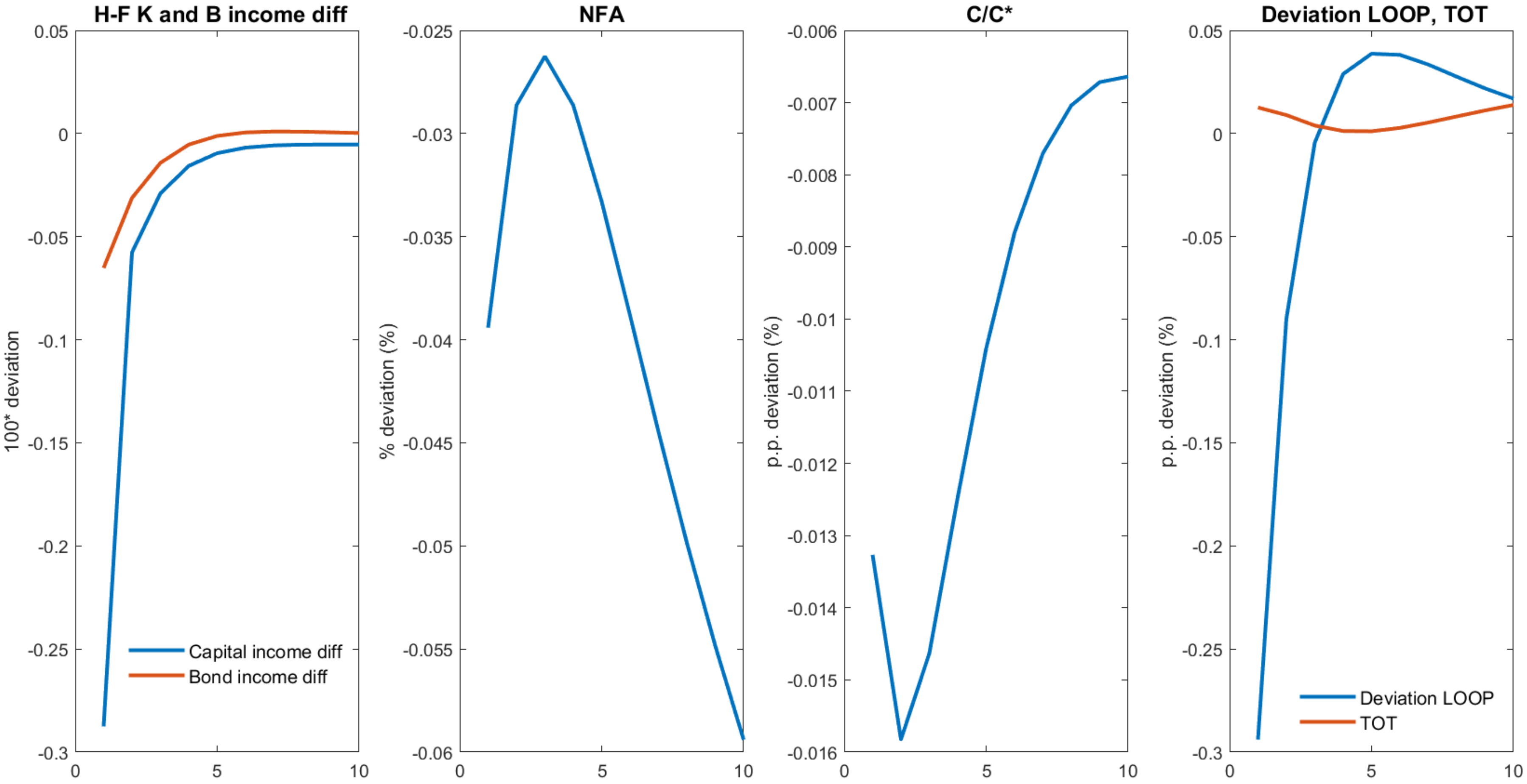
IRF of $\boldsymbol{\vartheta}$ shock – exchange rate

- Convenience yield/UIP: $E_t RER_{t+1} - RER_t - \left(r_{h,t} - r_{f,t}\right) \equiv \tilde{\eta}_t$
- Real interest rates move a little as monetary policy reacts
- The increase in the liquidity yield of the dollar leads to increase in convenience yield and appreciation of dollar

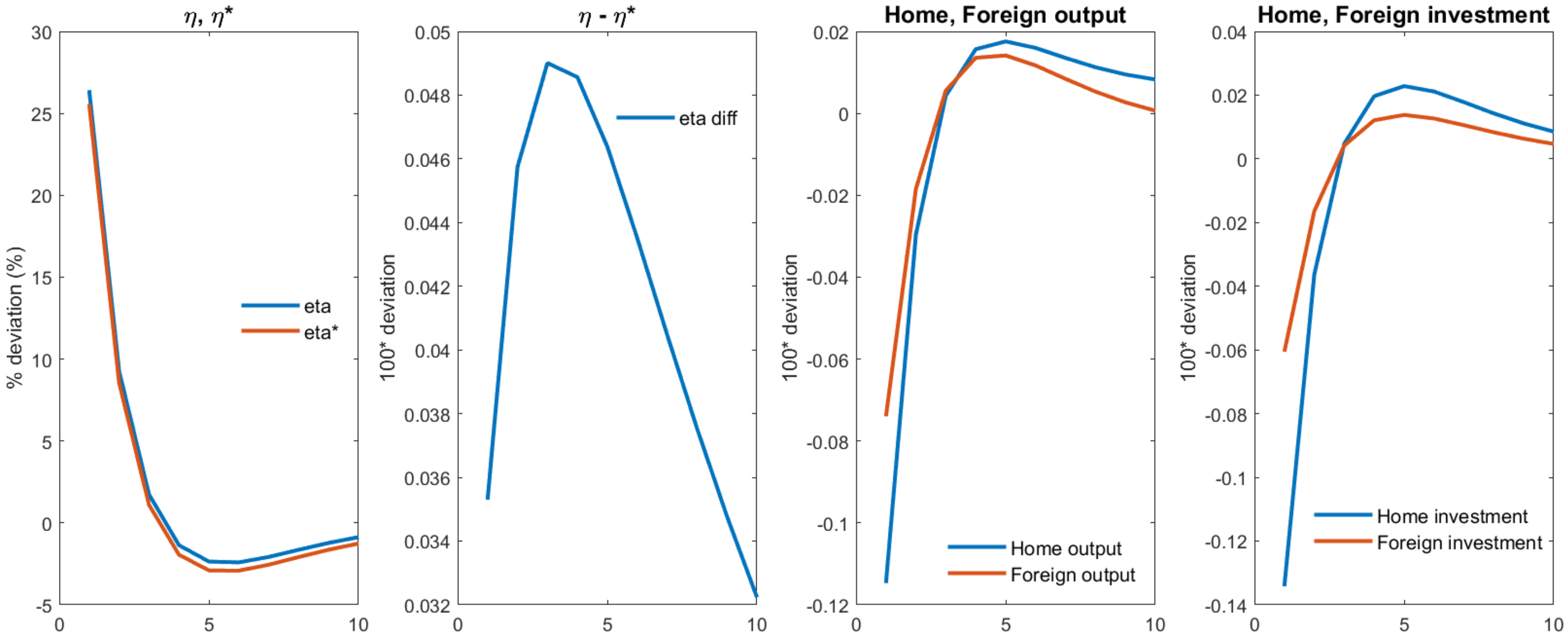


IRF of ϑ shock – reserve currency paradox

- Recall that $RER_t = TOT_t^{2\omega-1} \times D_t$



IRF of $\boldsymbol{\vartheta}$ shock – real outcomes

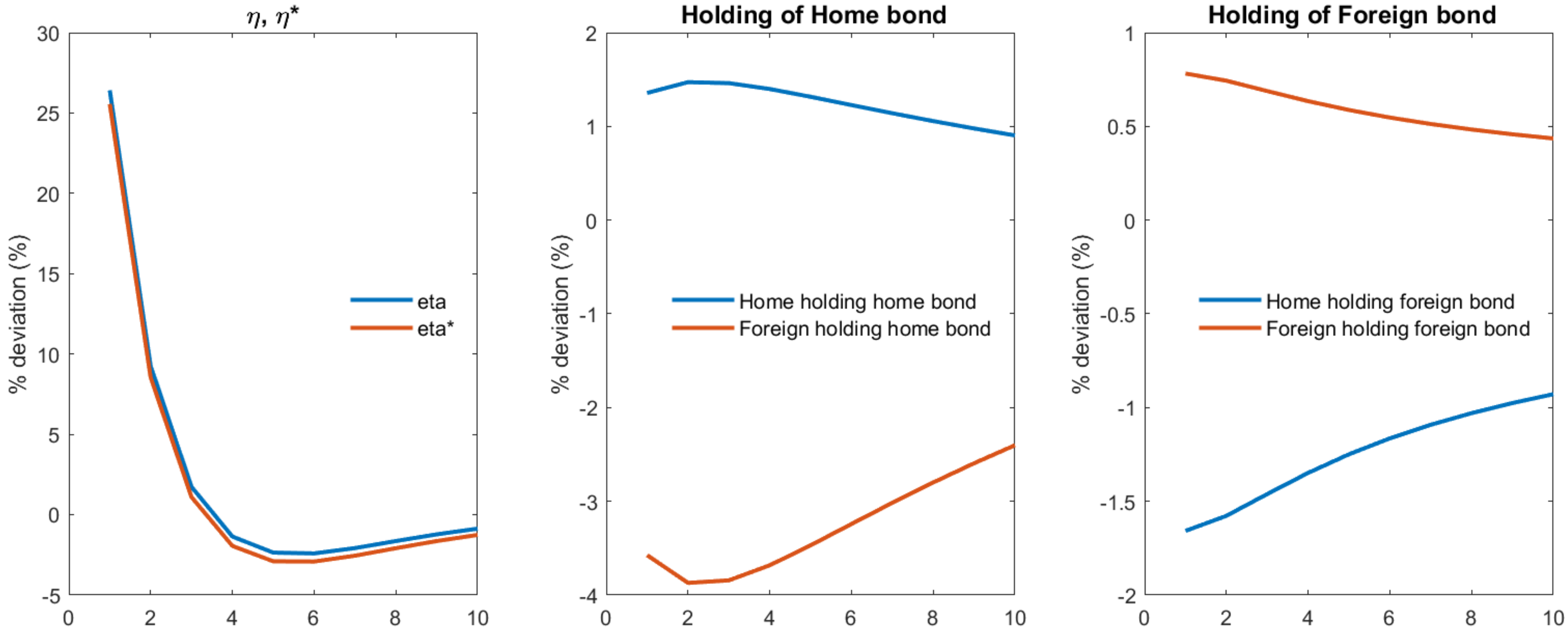


- $E_t\Lambda_{t+1}\left(\widetilde{R}_{K,t+1} - R_{t+1}\right) = \eta_t\vartheta(\kappa_{K,h,t})$ and $E_t\Lambda_{t+1}(R_{h,t+1} - R_{t+1}) = \eta_t\vartheta(\kappa_{h,t})$
- Relatively → $E_t\Lambda_{t+1}\left(\widetilde{R}_{K,t+1} - R_{h,t+1}\right) = \eta_t\vartheta(\kappa_{K,h,t} - \kappa_{h,t})$

Foreign analogy is: $E_t\Lambda^*_{t+1}\left(\widetilde{R}^*_{K,t+1} - R_{f,t+1}\right) = \eta^*_t\vartheta(\kappa^*_{K,h,t} - \kappa_{f,t})$

- Intuition: Home bond is great → Home banks shift out from investment more during a crisis

IRF of ϑ shock – capital flows



→ Home banks demand more of the U.S. bond

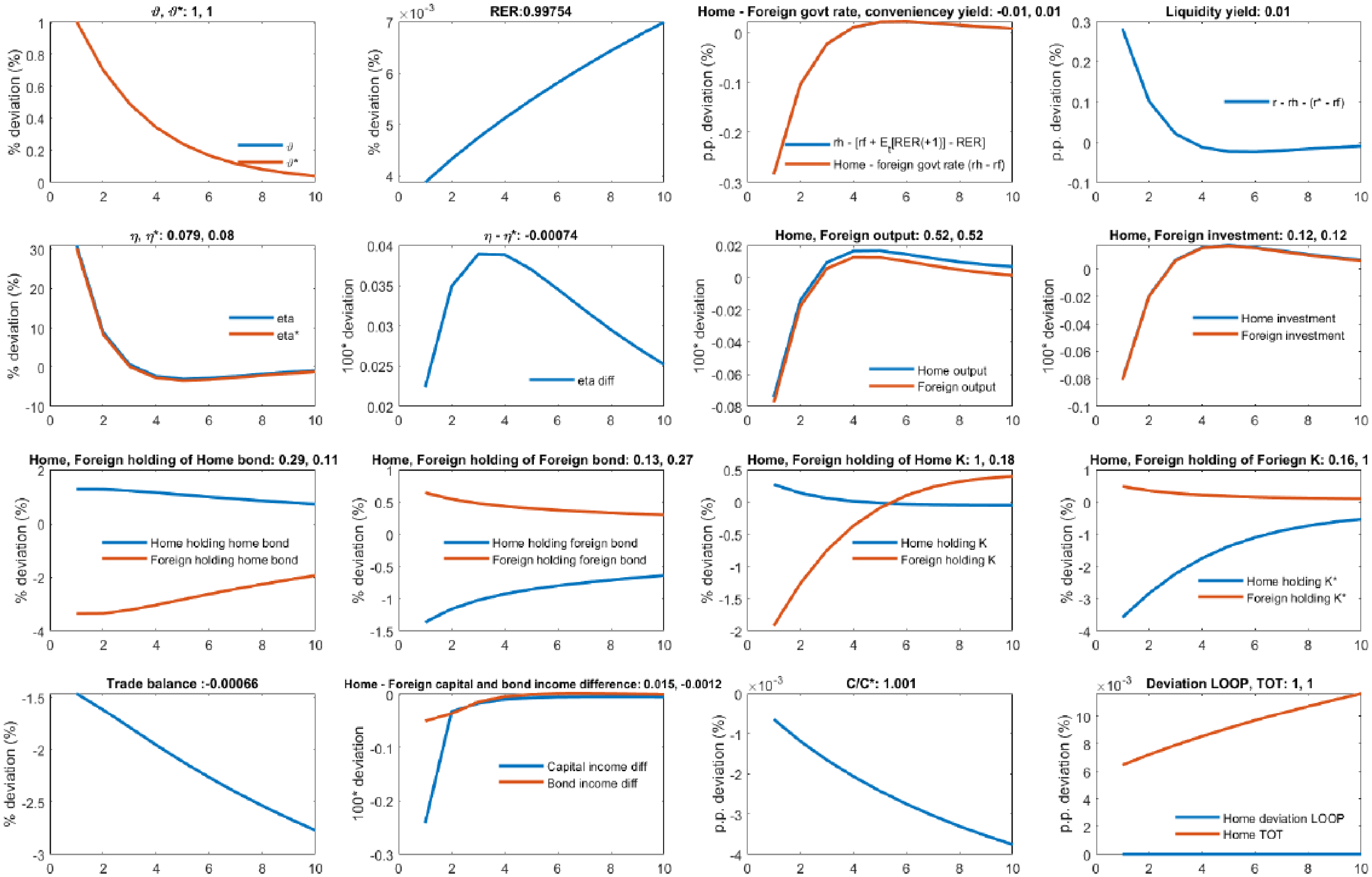
→ Foreign selling home bonds despite they also demand more of the liquid bond

→ Retrenchment of capital flows

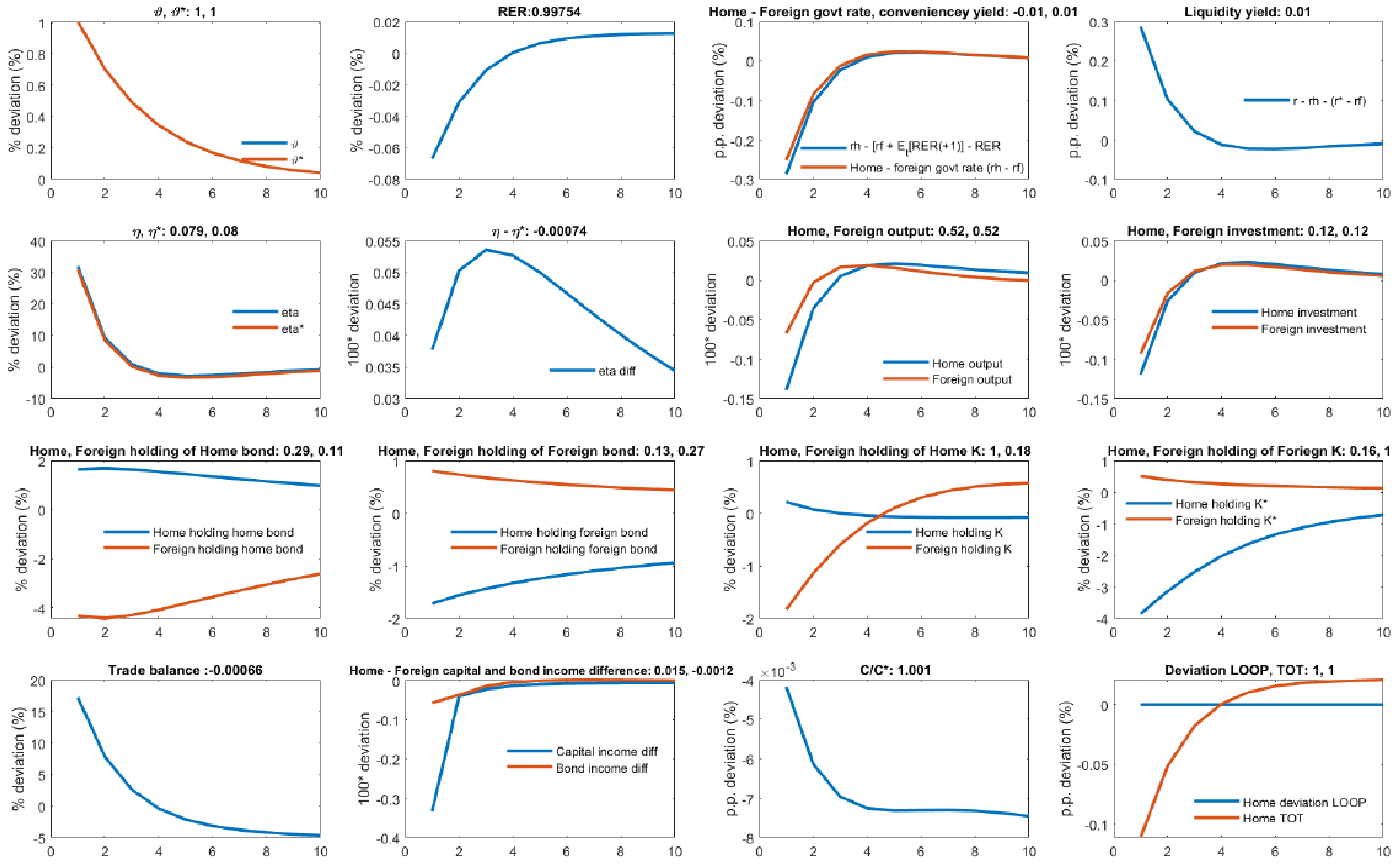
→ In equilibrium, changes in asset prices and expected returns matter

Note: direction of capital flows $\neq$ demand revelation

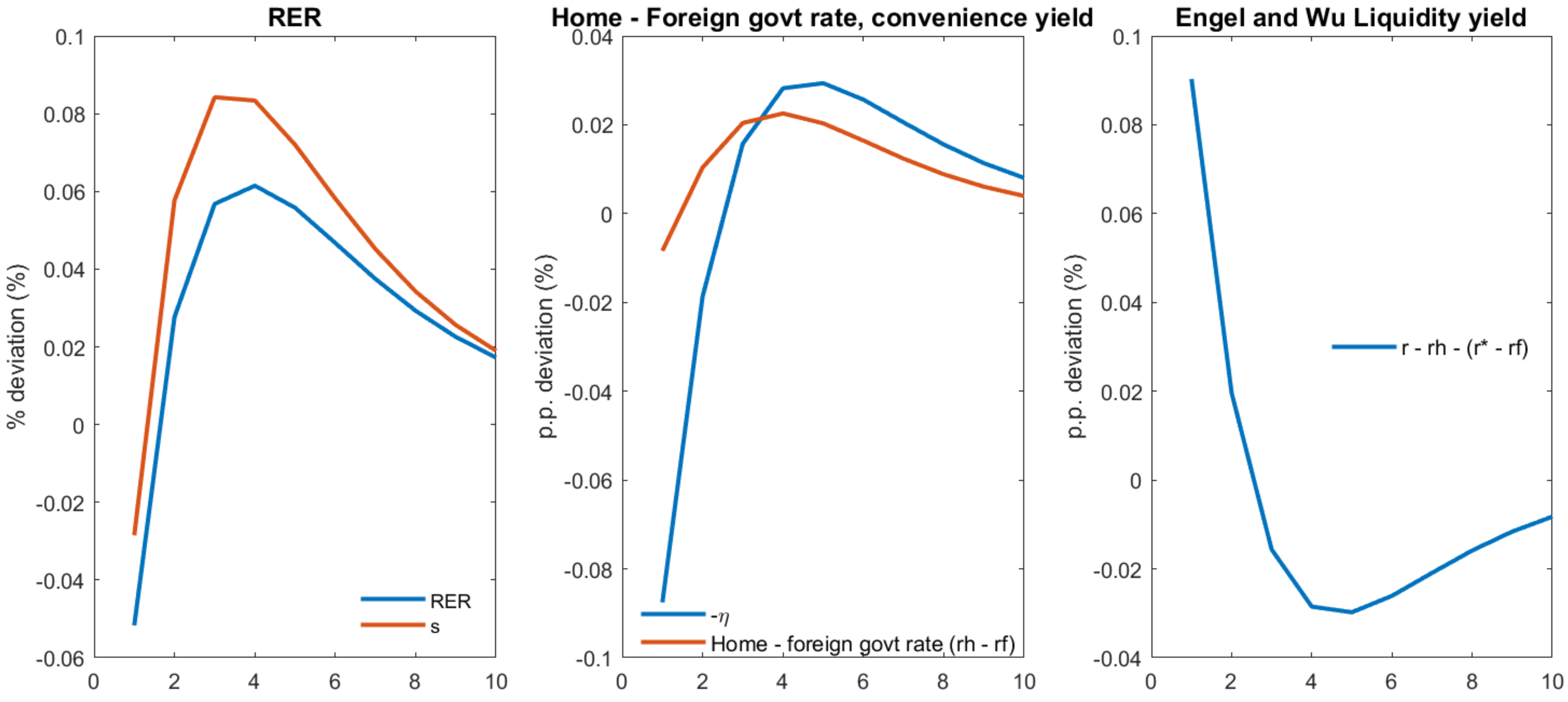
IRF of ϑ shock under flexible prices



IRF of ϑ shock under PCP

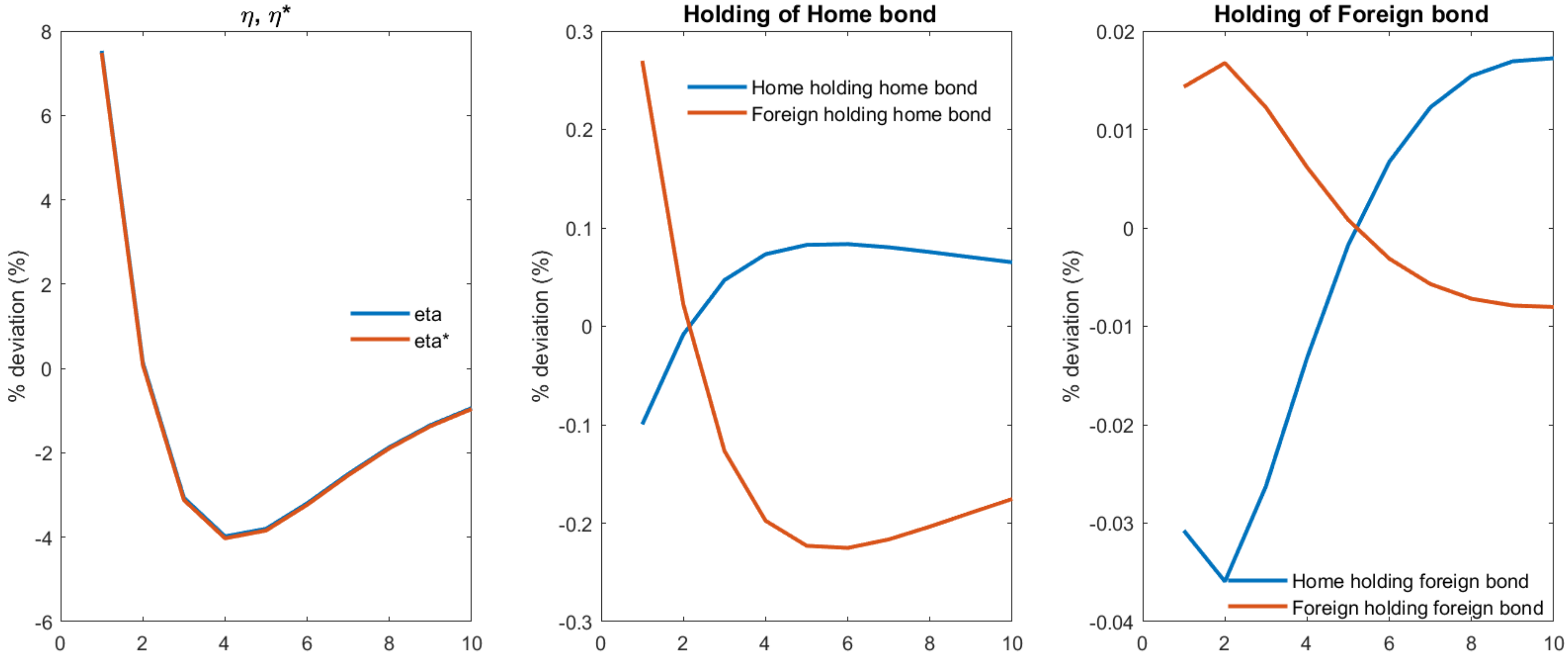


IRF of Productivity Shock



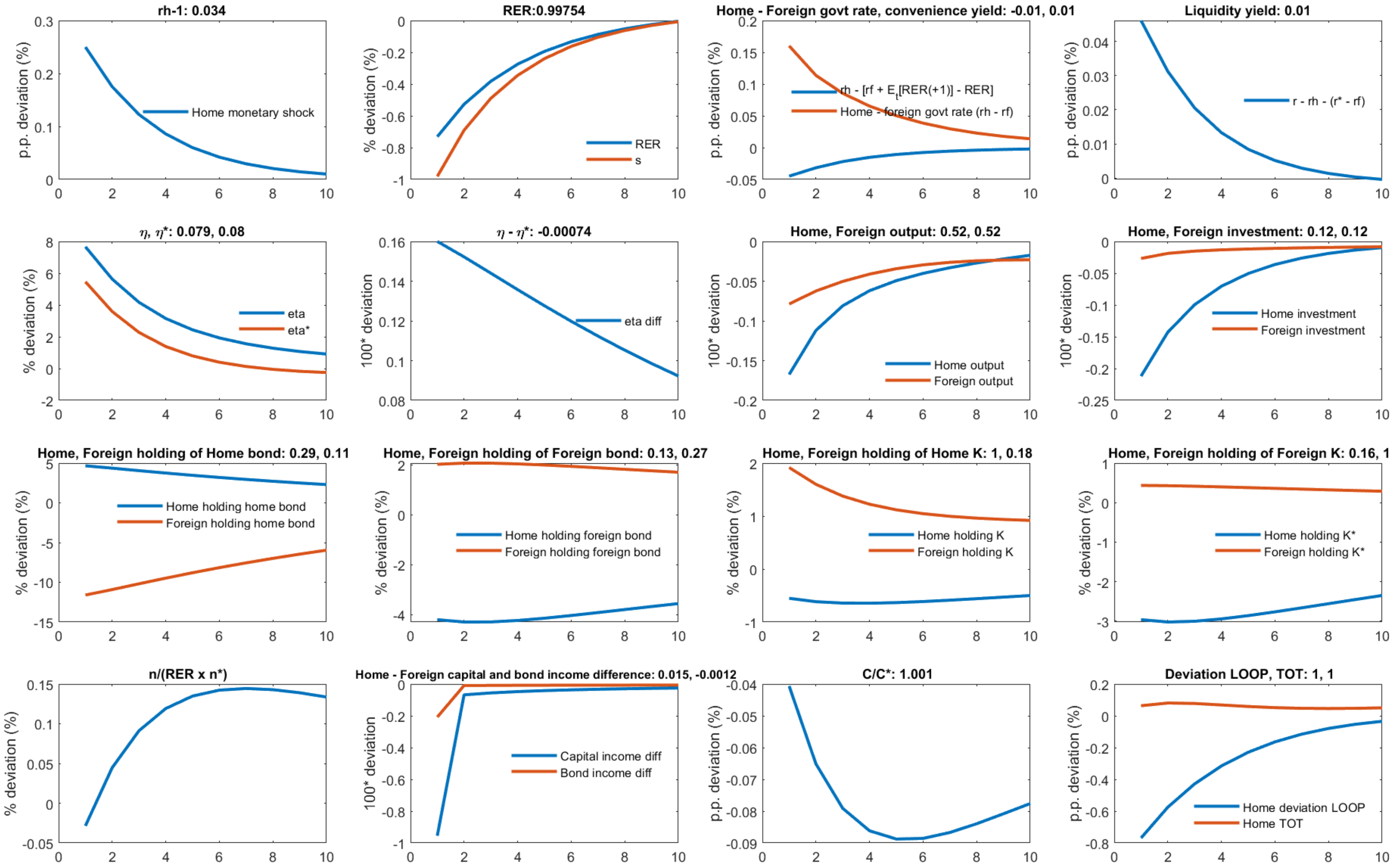
- The effects on real exchange rates and convenience yields similar to case of financial tightening

IRF of Productivity Shock

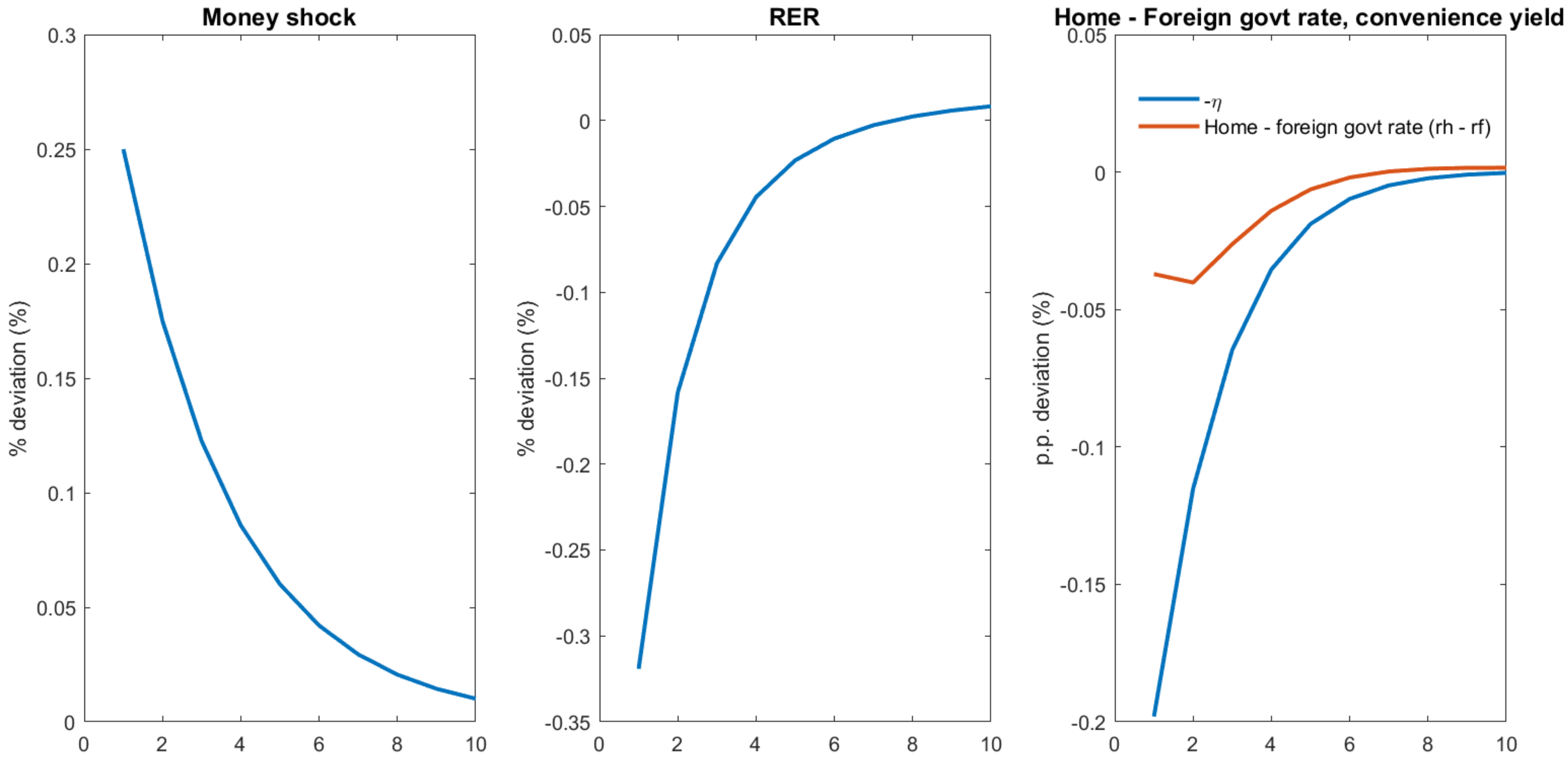


- Financial flows are different

IRF of U.S. money shock (Rey)



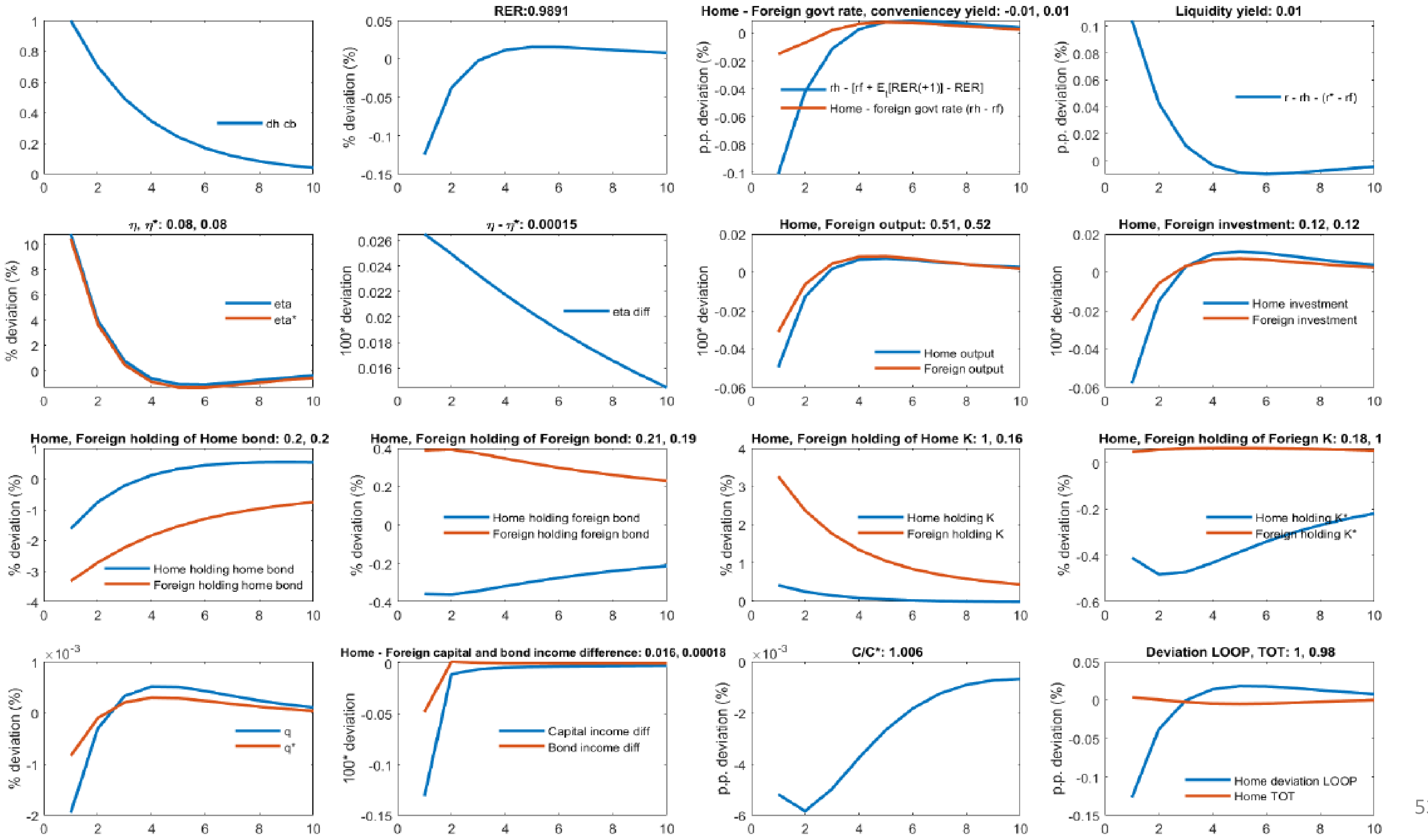
IRF of symmetric money shock



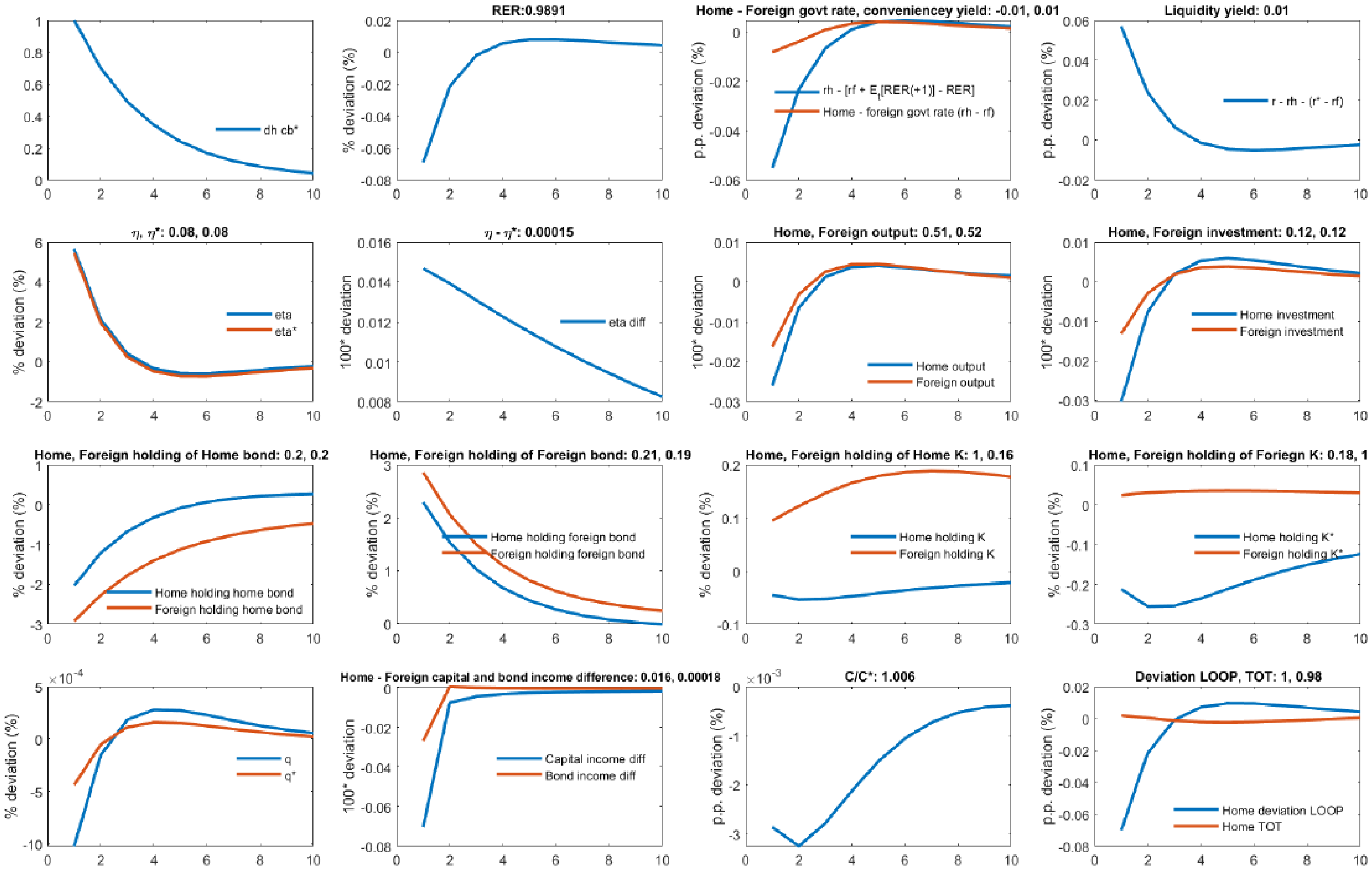
- Same size of global tightening results in USD RER appreciation
- Convenience yield demand drives most of the RER appreciation
- In eqm, the US interest rate is lower than the Foreign

→ Home inflation pressure is less than the Foreign

IRF of quantitative tightening



IRF of sterilized intervention



Conclusion

- A DSGE model of endogenous convenience yield
- Convenience yield links to banking friction – no exogenous yield / noise trader
- **An asymmetry – US bond is a better collateral – but not asymmetry in shocks**
- Matches US external positions and exchange rate dynamics well