# The Exact Distribution of the White t-Ratio: Supplemental Appendix

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June, 2021

*Research support from the NSF and the Phipps Chair are gratefully acknowledged.
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# 1 Size

In the paper, Tables 1-4 and 6 present the actual size of two-sided t-tests constructed with the HC1 covariance matrix estimator, and Table 7 presents one set of results for t-tests constructed with the HC3 covariance matrix estimator. Here, we present the same results for the HC1, HC2, and HC3 covariance matrix estimators. All results are calculated with 20,000 simulation replications and 1000 bootstrap replications. In addition, in the paper, Table 5 presented the computation cost of the proposed distribution calculation for the case of homoskedastic models. Here, we present the costs for the heteroskedastic model as well.

The Tables reported here use different numbering than in the paper.

From the full tables reported here we observe the following:

Tests with the student t critical values are greatly improved using HC2 and HC3, but are still greatly oversized. A similar observation applies to the wild bootstrap tests.

The tests with the Bell-McCaffrey critical values are increasingly conservative as we move from HC1 to HC2, and from HC2 to HC3. Tests with HC3 are very conservative for the designs with homoskedastic errors and small samples.

The four-parameter distribution approximation has near exact size in all configurations.

The size of the feasible tests (Table 8) are similar across HC1, HC2, and HC3, but are somewhat less distorted for HC2 than HC1, and for HC3 than HC2. For all three methods, size is excellent for $n \geq 120$ or when the errors are homoskedastic.

Table 1: Actual size using 5% $t_{n-k}$ critical values

| | Homoskedastic Error | | | | Heteroskedastic Error | | | |
|---|---|---|---|---|---|---|---|---|
| | $n=30$ | $n=60$ | $n=120$ | $n=500$ | $n=30$ | $n=60$ | $n=120$ | $n=500$ |
| | HC1 Covariance Matrix | | | | | | | |
| Dummy | 0.17 | 0.20 | 0.22 | 0.25 | 0.19 | 0.22 | 0.23 | 0.25 |
| Pareto(2) | 0.13 | 0.12 | 0.10 | 0.08 | 0.24 | 0.21 | 0.19 | 0.15 |
| Gamma(1/4,1) | 0.13 | 0.10 | 0.08 | 0.06 | 0.21 | 0.17 | 0.12 | 0.08 |
| logNormal(0,1) | 0.10 | 0.09 | 0.08 | 0.06 | 0.20 | 0.17 | 0.13 | 0.09 |
| logNormal(0,4) | 0.20 | 0.19 | 0.16 | 0.12 | 0.49 | 0.41 | 0.34 | 0.24 |
| | HC2 Covariance Matrix | | | | | | | |
| Dummy | 0.13 | 0.15 | 0.17 | 0.19 | 0.15 | 0.17 | 0.18 | 0.19 |
| Pareto(2) | 0.10 | 0.10 | 0.09 | 0.07 | 0.18 | 0.16 | 0.14 | 0.12 |
| Gamma(1/4,1) | 0.10 | 0.09 | 0.07 | 0.06 | 0.16 | 0.13 | 0.10 | 0.07 |
| logNormal(0,1) | 0.08 | 0.08 | 0.07 | 0.06 | 0.16 | 0.13 | 0.11 | 0.08 |
| logNormal(0,4) | 0.14 | 0.14 | 0.13 | 0.10 | 0.32 | 0.27 | 0.23 | 0.17 |
| | HC3 Covariance Matrix | | | | | | | |
| Dummy | 0.10 | 0.11 | 0.13 | 0.14 | 0.11 | 0.12 | 0.13 | 0.14 |
| Pareto(2) | 0.07 | 0.07 | 0.07 | 0.06 | 0.12 | 0.12 | 0.11 | 0.09 |
| Gamma(1/4,1) | 0.07 | 0.07 | 0.06 | 0.06 | 0.11 | 0.10 | 0.08 | 0.06 |
| logNormal(0,1) | 0.06 | 0.06 | 0.06 | 0.06 | 0.11 | 0.10 | 0.08 | 0.07 |
| logNormal(0,4) | 0.08 | 0.09 | 0.09 | 0.09 | 0.17 | 0.16 | 0.15 | 0.12 |

# 2 Distribution Approximations

In the paper, Figure 1 displayed the cumulative distribution function of the exact distribution of the HC1 t-ratio along with the student t and Bell-McCraffrey distributions, for the homoskedastic and heteroskedastic logNormal(0,4) regressor design and $n = 30$. Here, we display similar figures for all designs: $n = 30$, $n = 120$, and $n = 500$; HC1, HC2, and HC3; all five regressor designs; and homoskedastic and heteroskedastic errors. This amounts to 90 graphs. We collect the graphs in ten figures. Figure 1 displays the dummy regressor design with homoskedastic errors in a $3 \times 3$ grid for the three sample sizes and three covariance matrix estimators. Figure 2 similarly displays the Pareto regressor design with homoskedastic errors. The remaining figures are similar, for the remaining sampling designs.

Each plot displays three lines: the CDF of the exact distribution (solid line), the student t approximation (dash-dotted), and the Bell-McCaffrey approximation (dashed line). In all displays, the CDF of the student t distribution lies on top of the exact dis-

Table 2: Actual size using 5% wild bootstrap critical values

| | Homoskedastic Error | | | | Heteroskedastic Error | | | |
|---|---|---|---|---|---|---|---|---|
| | $n = 30$ | $n = 60$ | $n = 120$ | $n = 500$ | $n = 30$ | $n = 60$ | $n = 120$ | $n = 500$ |
| | HC1 Covariance Matrix | | | | | | | |
| Dummy | 0.10 | 0.10 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| Pareto(2) | 0.15 | 0.12 | 0.10 | 0.08 | 0.19 | 0.14 | 0.11 | 0.07 |
| Gamma(1/4,1) | 0.14 | 0.10 | 0.07 | 0.05 | 0.17 | 0.11 | 0.07 | 0.05 |
| logNormal(0,1) | 0.11 | 0.09 | 0.07 | 0.05 | 0.14 | 0.09 | 0.07 | 0.05 |
| logNormal(0,4) | 0.24 | 0.21 | 0.17 | 0.11 | 0.35 | 0.25 | 0.19 | 0.11 |
| | HC2 Covariance Matrix | | | | | | | |
| Dummy | 0.08 | 0.08 | 0.08 | 0.07 | 0.08 | 0.08 | 0.08 | 0.07 |
| Pareto(2) | 0.13 | 0.11 | 0.09 | 0.07 | 0.16 | 0.12 | 0.09 | 0.06 |
| Gamma(1/4,1) | 0.12 | 0.08 | 0.06 | 0.05 | 0.15 | 0.09 | 0.06 | 0.05 |
| logNormal(0,1) | 0.10 | 0.08 | 0.06 | 0.05 | 0.12 | 0.08 | 0.06 | 0.05 |
| logNormal(0,4) | 0.19 | 0.17 | 0.14 | 0.10 | 0.25 | 0.20 | 0.15 | 0.09 |
| | HC3 Covariance Matrix | | | | | | | |
| Dummy | 0.06 | 0.07 | 0.06 | 0.05 | 0.06 | 0.06 | 0.06 | 0.05 |
| Pareto(2) | 0.10 | 0.09 | 0.07 | 0.06 | 0.13 | 0.10 | 0.07 | 0.05 |
| Gamma(1/4,1) | 0.09 | 0.07 | 0.05 | 0.05 | 0.12 | 0.07 | 0.05 | 0.04 |
| logNormal(0,1) | 0.08 | 0.07 | 0.05 | 0.05 | 0.09 | 0.07 | 0.05 | 0.04 |
| logNormal(0,4) | 0.13 | 0.13 | 0.11 | 0.08 | 0.17 | 0.15 | 0.11 | 0.07 |

tribution, indicating global under-sized test performance. In all homoskedastic error cases, the Bell-McCaffrey approximation lies either below the exact distribution, indicating conservative size distortion, or lies close to the exact CDF. In the heteroskedastic error cases the performance of Bell-McCaffrey is more complicated; in some cases it lies above the exact CDF, in some cases below, and some cases the distributions cross, meaning that the test could be over-rejecting or under-rejecting depending on the selected level of the test.

In the paper, Figure 2 displayed the cumulative distribution function of the exact distribution of the HC1 t-ratio along with our two proposed distribution approximations: the three-parameter and four-parameter distributions. Figure 2 displayed the CDFs for the homoskedastic and heteroskedastic logNormal(0,4) regressor design and $n = 120$. Here, we display similar figures for all designs.

Each plot displays three lines: the CDF of the exact distribution (solid line), the three-parameter distribution (dash-dotted), and the four-parameter distribution (dashed line). In most designs the four-parameter distribution is nearly identical to the exact distri-

Table 3: Actual size using 5% Bell-McCaffrey critical values

| | Homoskedastic Error | | | | Heteroskedastic Error | | | |
|---|---|---|---|---|---|---|---|---|
| | $n = 30$ | $n = 60$ | $n = 120$ | $n = 500$ | $n = 30$ | $n = 60$ | $n = 120$ | $n = 500$ |
| | HC1 Covariance Matrix | | | | | | | |
| Dummy | 0.03 | 0.03 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.03 | 0.04 | 0.04 | 0.04 | 0.10 | 0.10 | 0.09 | 0.09 |
| Gamma(1/4,1) | 0.03 | 0.04 | 0.04 | 0.05 | 0.09 | 0.08 | 0.07 | 0.06 |
| logNormal(0,1) | 0.04 | 0.04 | 0.04 | 0.05 | 0.11 | 0.09 | 0.08 | 0.07 |
| logNormal(0,4) | 0.02 | 0.02 | 0.03 | 0.04 | 0.18 | 0.15 | 0.13 | 0.11 |
| | HC2 Covariance Matrix | | | | | | | |
| Dummy | 0.03 | 0.03 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.02 | 0.03 | 0.03 | 0.04 | 0.06 | 0.07 | 0.07 | 0.07 |
| Gamma(1/4,1) | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.07 | 0.06 |
| logNormal(0,1) | 0.03 | 0.04 | 0.04 | 0.05 | 0.08 | 0.08 | 0.07 | 0.07 |
| logNormal(0,4) | 0.01 | 0.01 | 0.02 | 0.03 | 0.08 | 0.07 | 0.07 | 0.07 |
| | HC3 Covariance Matrix | | | | | | | |
| Dummy | 0.03 | 0.04 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.01 | 0.02 | 0.03 | 0.04 | 0.03 | 0.05 | 0.05 | 0.06 |
| Gamma(1/4,1) | 0.01 | 0.03 | 0.04 | 0.05 | 0.04 | 0.05 | 0.06 | 0.06 |
| logNormal(0,1) | 0.02 | 0.03 | 0.04 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 |
| logNormal(0,4) | 0.00 | 0.00 | 0.01 | 0.02 | 0.04 | 0.03 | 0.03 | 0.04 |

bution, but in some designs it lies slightly above or below the exact distribution. The three-parameter distribution has similar behavior, but in many designs is further away from the exact distribution, and exclusively lies above the exact distribution, implying that tests based on the three-parameter distribution will be over-sized.

Table 4: Actual size using 5% critical values. HC1 covariance matrix, infeasible three-parameter and four-parameter distributions

| | $n = 30$ | $n = 60$ | $n = 120$ | $n = 250$ | $n = 500$ |
|---|---|---|---|---|---|
| | Three-Parameter Distribution, Homoskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 |
| | Four-Parameter Distribution, Homoskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | Three-Parameter Distribution, Heteroskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| Gamma(1/4,1) | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.06 | 0.06 | 0.06 | 0.05 | 0.06 |
| logNormal(0,4) | 0.07 | 0.07 | 0.07 | 0.06 | 0.06 |
| | Four-Parameter Distribution, Heteroskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 |

Table 5: Actual size using 5% critical values. HC2 covariance matrix, infeasible three-parameter and four-parameter distributions

| | $n = 30$ | $n = 60$ | $n = 120$ | $n = 250$ | $n = 500$ |
|---|---|---|---|---|---|
| | Three-Parameter Distribution, Homoskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.06 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 |
| | Four-Parameter Distribution, Homoskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | Three-Parameter Distribution, Heteroskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| Gamma(1/4,1) | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 |
| logNormal(0,1) | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| logNormal(0,4) | 0.07 | 0.06 | 0.07 | 0.06 | 0.07 |
| | Four-Parameter Distribution, Heteroskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 |

Table 6: Actual size using 5% critical values. HC3 covariance matrix, infeasible three-parameter and four-parameter distributions

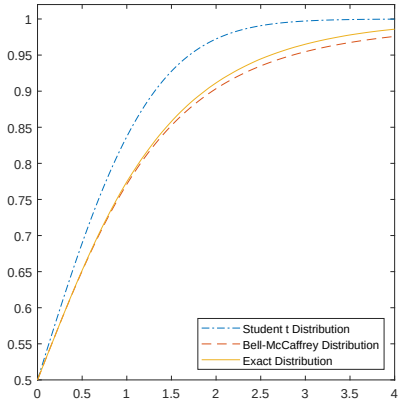
| | $n = 30$ | $n = 60$ | $n = 120$ | $n = 250$ | $n = 500$ |
|---|---|---|---|---|---|
| | Three-Parameter Distribution, Homoskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.06 | 0.06 | 0.06 | 0.05 | 0.06 |
| | Four-Parameter Distribution, Homoskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | Three-Parameter Distribution, Heteroskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| Gamma(1/4,1) | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 |
| logNormal(0,1) | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| logNormal(0,4) | 0.06 | 0.06 | 0.07 | 0.06 | 0.07 |
| | Four-Parameter Distribution, Heteroskedastic Case | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 |

Table 7: Computation Cost

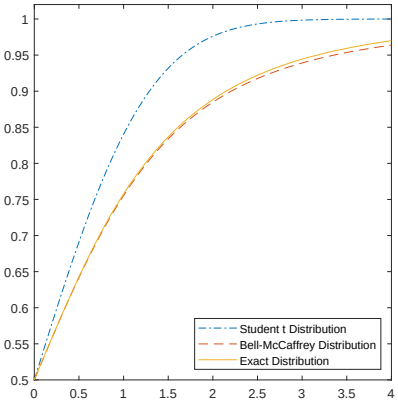
| | $n=30$ | $n=60$ | $n=120$ | $n=250$ | $n=500$ | $n=1000$ | $n=2000$ |
|---|---|---|---|---|---|---|---|
| Homoskedastic Case | | | | | | | |
| | Three-Parameter Distribution, Computation Time (Seconds) | | | | | | |
| Dummy | 0.001 | 0.002 | 0.002 | 0.002 | 0.012 | 0.067 | 0.352 |
| Pareto(2) | 0.001 | 0.001 | 0.003 | 0.003 | 0.011 | 0.061 | 0.356 |
| Gamma(1/4,1) | 0.001 | 0.001 | 0.002 | 0.003 | 0.011 | 0.060 | 0.357 |
| logNormal(0,1) | 0.001 | 0.001 | 0.003 | 0.004 | 0.010 | 0.063 | 0.362 |
| logNormal(0,4) | 0.001 | 0.003 | 0.002 | 0.003 | 0.012 | 0.061 | 0.365 |
| | Four-Parameter Distribution, Computation Time (Seconds) | | | | | | |
| Dummy | 0.001 | 0.003 | 0.004 | 0.010 | 0.056 | 0.267 | 1.067 |
| Pareto(2) | 0.001 | 0.002 | 0.002 | 0.003 | 0.013 | 0.044 | 0.249 |
| Gamma(1/4,1) | 0.001 | 0.001 | 0.001 | 0.002 | 0.010 | 0.045 | 0.247 |
| logNormal(0,1) | 0.001 | 0.002 | 0.002 | 0.002 | 0.010 | 0.050 | 0.246 |
| logNormal(0,4) | 0.001 | 0.002 | 0.001 | 0.003 | 0.012 | 0.046 | 0.244 |
| | Four-Parameter Distribution, Number of series terms ($M$) | | | | | | |
| Dummy | 740 | 3319 | 14,003 | 62,429 | 252,776 | 1,017,415 | 4,076,592 |
| Pareto(2) | 65 | 65 | 64 | 66 | 71 | 77 | 85 |
| Gamma(1/4,1) | 63 | 57 | 57 | 61 | 69 | 81 | 98 |
| logNormal(0,1) | 55 | 61 | 65 | 71 | 79 | 90 | 104 |
| logNormal(0,4) | 90 | 69 | 58 | 55 | 56 | 59 | 63 |
| Heteroskedastic Case | | | | | | | |
| | Three-Parameter Distribution, Computation Time (Seconds) | | | | | | |
| Dummy | 0.001 | 0.001 | 0.002 | 0.003 | 0.012 | 0.061 | 0.361 |
| Pareto(2) | 0.001 | 0.002 | 0.003 | 0.003 | 0.012 | 0.062 | 0.360 |
| Gamma(1/4,1) | 0.002 | 0.001 | 0.002 | 0.003 | 0.011 | 0.060 | 0.362 |
| logNormal(0,1) | 0.001 | 0.003 | 0.003 | 0.003 | 0.010 | 0.066 | 0.367 |
| logNormal(0,4) | 0.001 | 0.001 | 0.003 | 0.003 | 0.011 | 0.059 | 0.364 |
| | Four-Parameter Distribution, Computation Time (Seconds) | | | | | | |
| Dummy | 0.002 | 0.004 | 0.010 | 0.053 | 0.213 | 0.880 | * |
| Pareto(2) | 0.001 | 0.002 | 0.002 | 0.002 | 0.009 | 0.047 | 0.249 |
| Gamma(1/4,1) | 0.001 | 0.001 | 0.002 | 0.002 | 0.009 | 0.048 | 0.247 |
| logNormal(0,1) | 0.001 | 0.002 | 0.002 | 0.002 | 0.009 | 0.046 | 0.240 |
| logNormal(0,4) | 0.001 | 0.001 | 0.002 | 0.002 | 0.011 | 0.046 | 0.244 |
| | Four-Parameter Distribution, Number of series terms ($M$) | | | | | | |
| Dummy | 2979 | 13,294 | 56,030 | 249,735 | 1,011,119 | 4,097,671 | $>5,000,00$ |
| Pareto(2) | 62 | 54 | 44 | 55 | 57 | 59 | 61 |
| Gamma(1/4,1) | 51 | 40 | 39 | 41 | 44 | 50 | 57 |
| logNormal(0,1) | 47 | 44 | 45 | 47 | 51 | 55 | 60 |
| logNormal(0,4) | 109 | 81 | 70 | 63 | 60 | 58 | 57 |

Table 8: Actual size using 5% critical values from feasible $G_3/G_4$ distribution

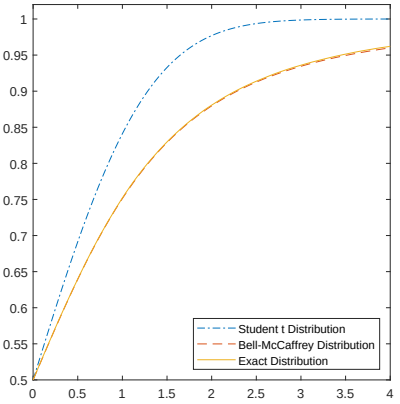
| | $n=30$ | $n=60$ | $n=120$ | $n=250$ | $n=500$ |
|---|---|---|---|---|---|
| | HC1, Homoskedastic Error | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | HC1, Heteroskedastic Error | | | | |
| Dummy | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 |
| Pareto(2) | 0.10 | 0.08 | 0.06 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.09 | 0.07 | 0.06 | 0.05 | 0.05 |
| logNormal(0,1) | 0.09 | 0.06 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.11 | 0.07 | 0.05 | 0.05 | 0.05 |
| | HC2, Homoskedastic Error | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | HC2, Heteroskedastic Error | | | | |
| Dummy | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 |
| Pareto(2) | 0.09 | 0.07 | 0.06 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.09 | 0.07 | 0.06 | 0.05 | 0.05 |
| logNormal(0,1) | 0.08 | 0.06 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.10 | 0.06 | 0.05 | 0.05 | 0.05 |
| | HC3, Homoskedastic Error | | | | |
| Dummy | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,1) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| | HC3, Heteroskedastic Error | | | | |
| Dummy | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 |
| Pareto(2) | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 |
| Gamma(1/4,1) | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 |
| logNormal(0,1) | 0.07 | 0.06 | 0.05 | 0.05 | 0.05 |
| logNormal(0,4) | 0.08 | 0.06 | 0.05 | 0.05 | 0.05 |



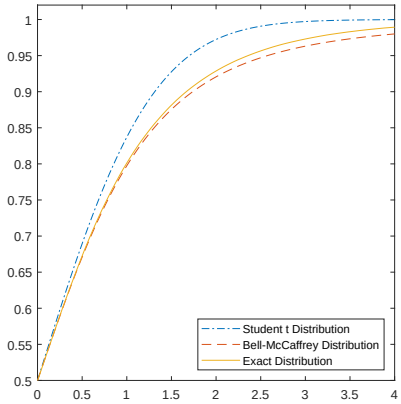

(a) HC1, $n = 30$



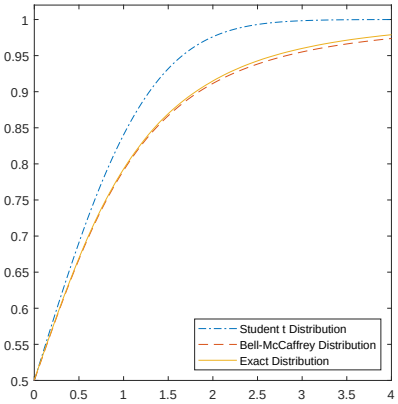

(b) HC1, $n = 120$



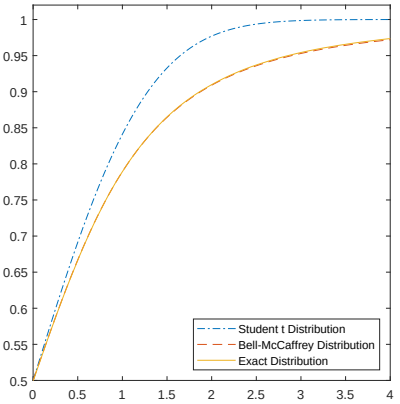

(c) HC1, $n = 500$



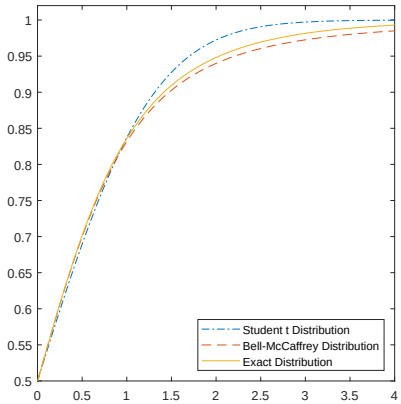

(d) HC2, $n = 30$



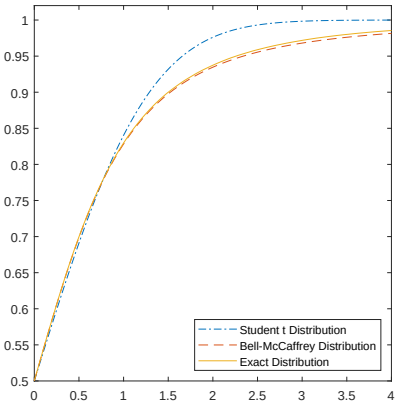

(e) HC2, $n = 120$



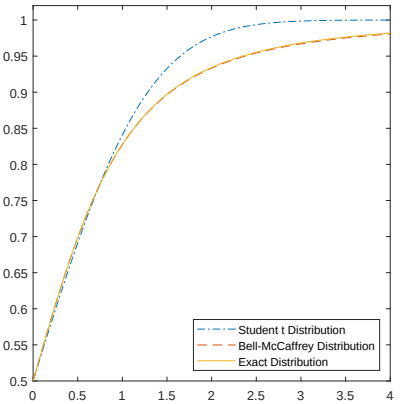

(f) HC2, $n = 500$




(g) HC3, $n = 30$

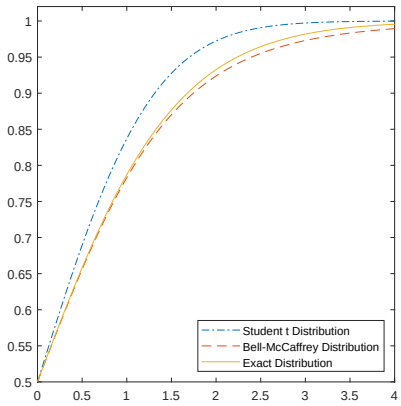



(h) HC3, $n = 120$

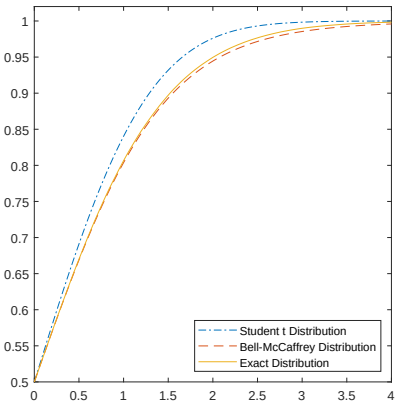



(i) HC3, $n = 500$

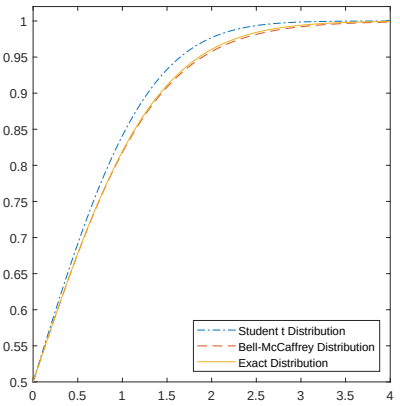
Figure 1: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. Dummy Regressor Design, Homoskedastic Errors



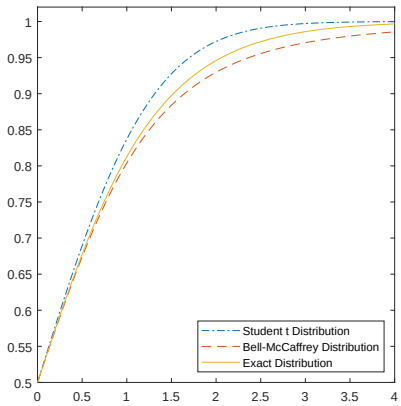

(a) HC1, $n = 30$



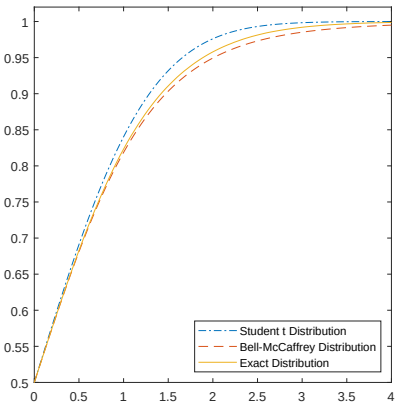

(b) HC1, $n = 120$



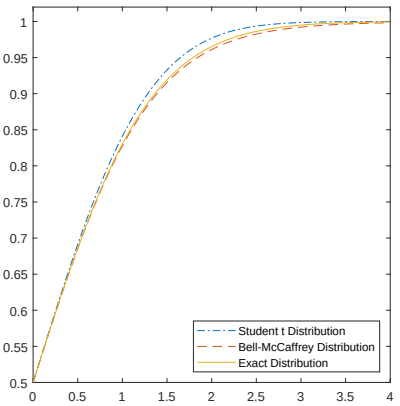

(c) HC1, $n = 500$



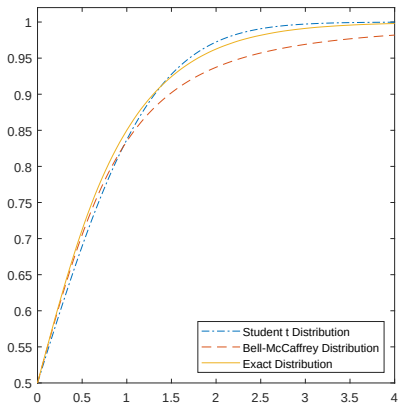

(d) HC2, $n = 30$



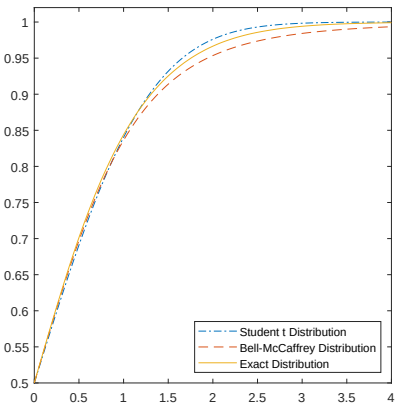

(e) HC2, $n = 120$



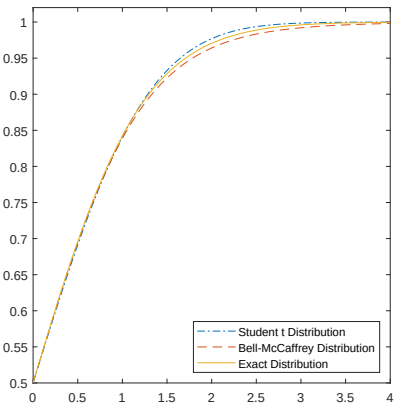

(f) HC2, $n = 500$




(g) HC3, $n = 30$

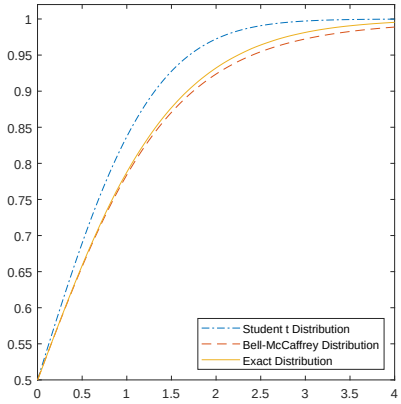



(h) HC3, $n = 120$

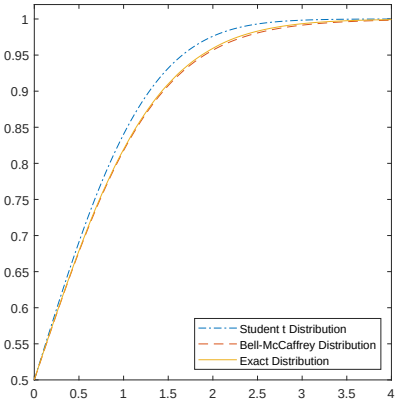



(i) HC3, $n = 500$

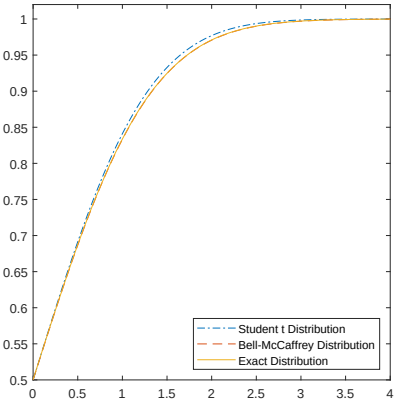
Figure 2: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. Pareto Regressor Design, Homoskedastic Errors



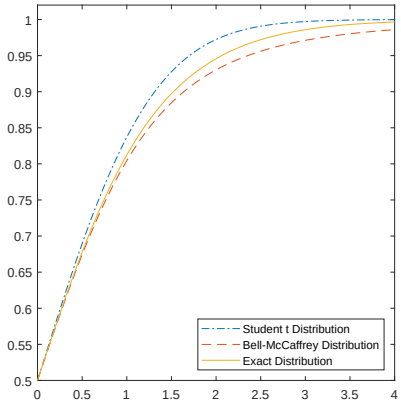

(a) HC1, $n = 30$



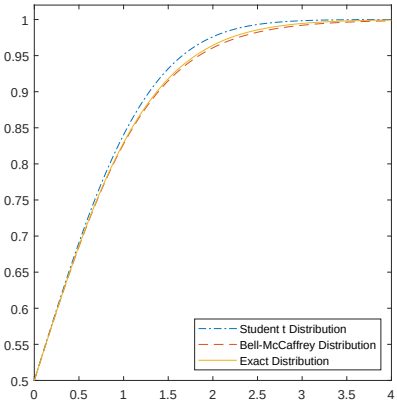

(b) HC1, $n = 120$



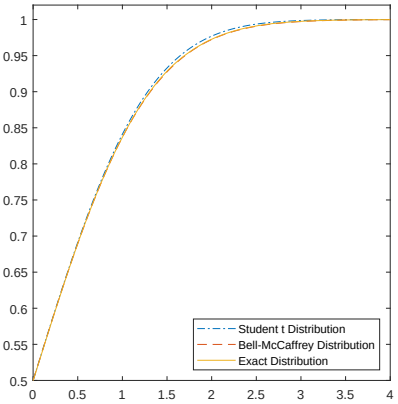

(c) HC1, $n = 500$



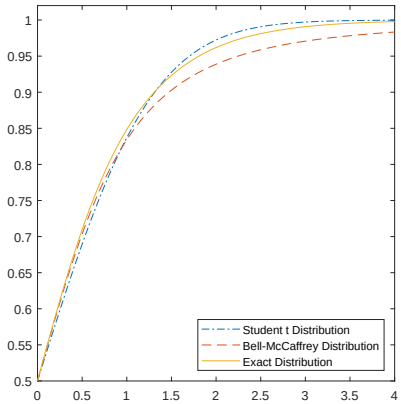

(d) HC2, $n = 30$



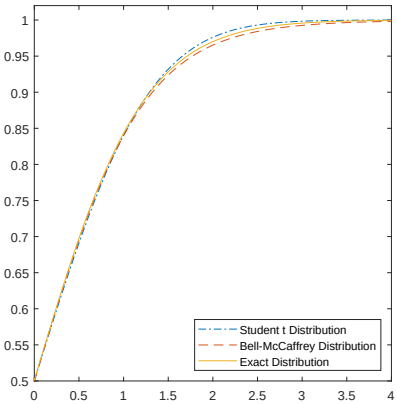

(e) HC2, $n = 120$



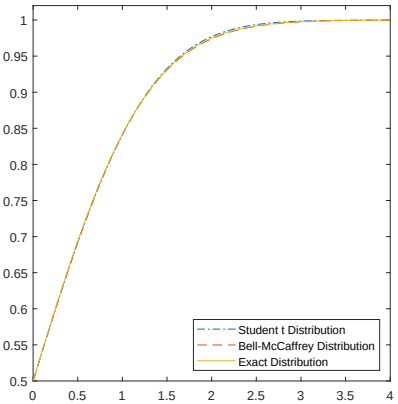

(f) HC2, $n = 500$




(g) HC3, $n = 30$

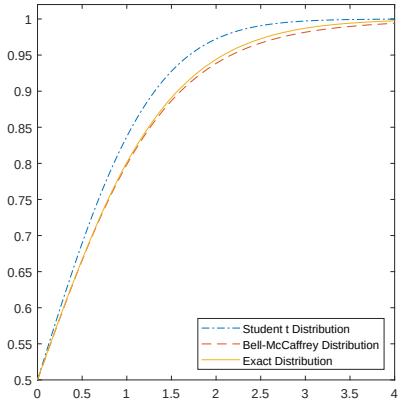



(h) HC3, $n = 120$

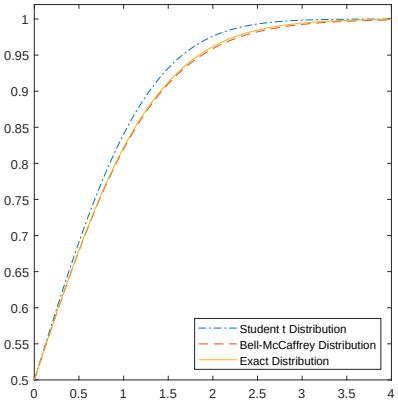



(i) HC3, $n = 500$

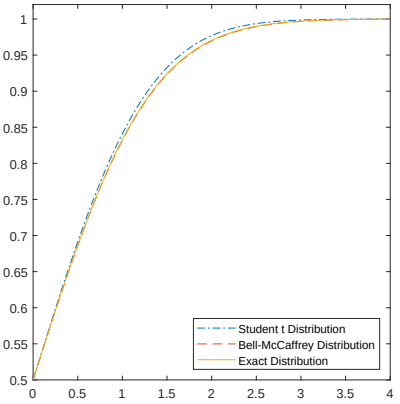
Figure 3: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. Gamma Regressor Design, Homoskedastic Errors



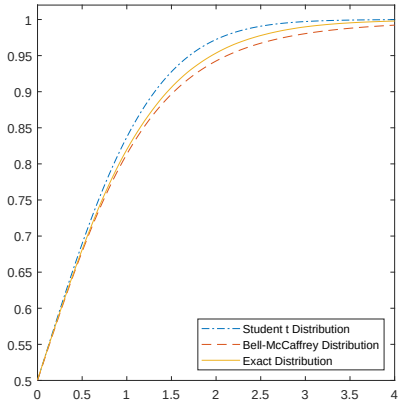

(a) HC1, $n = 30$



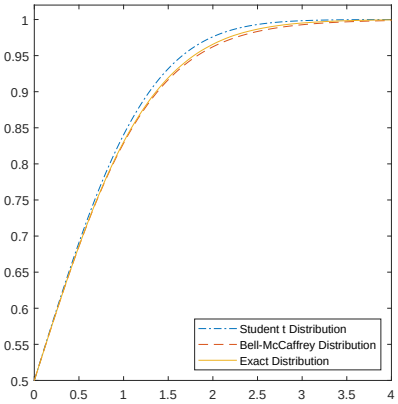

(b) HC1, $n = 120$



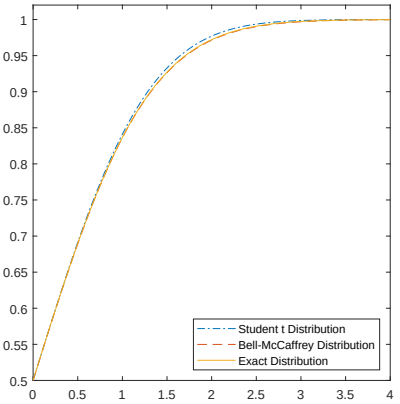

(c) HC1, $n = 500$



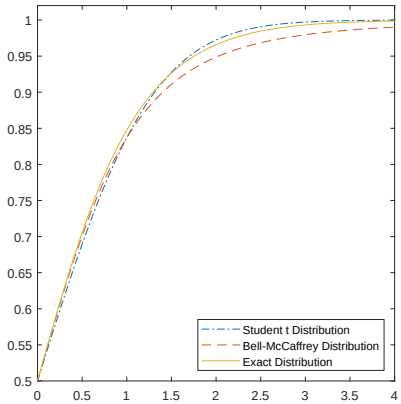

(d) HC2, $n = 30$



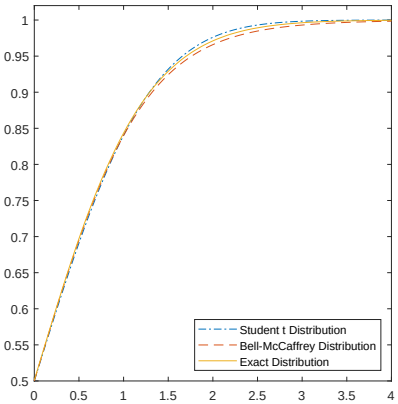

(e) HC2, $n = 120$



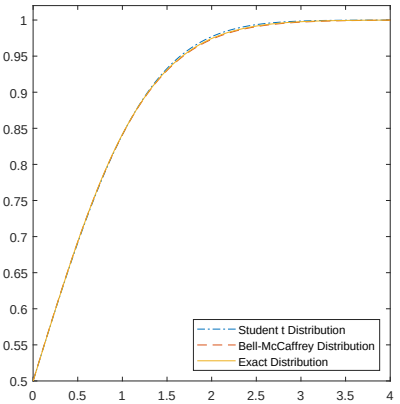

(f) HC2, $n = 500$




(g) HC3, $n = 30$

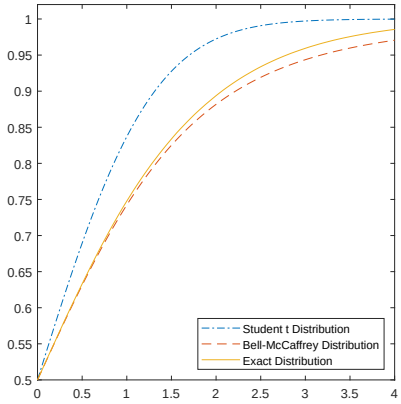



(h) HC3, $n = 120$

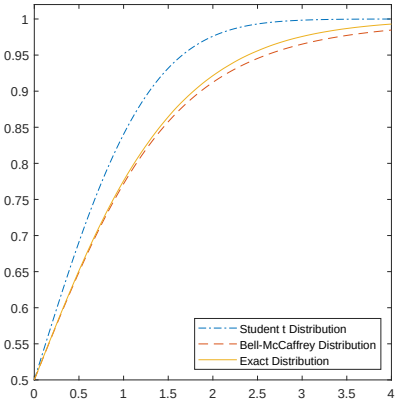



(i) HC3, $n = 500$

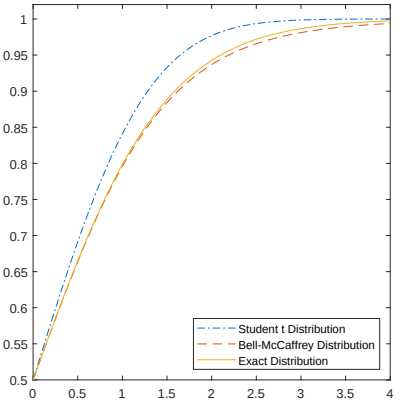
Figure 4: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. logNormal(0,1) Regressor Design, Homoskedastic Errors



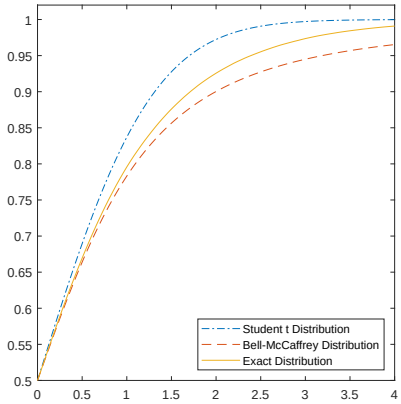

(a) HC1, $n = 30$



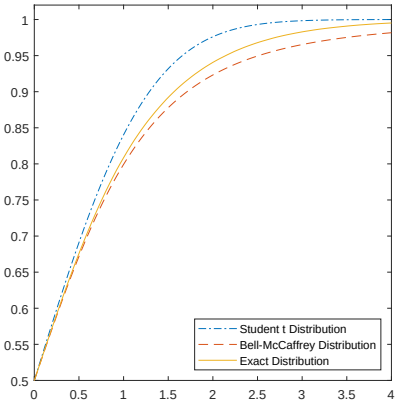

(b) HC1, $n = 120$



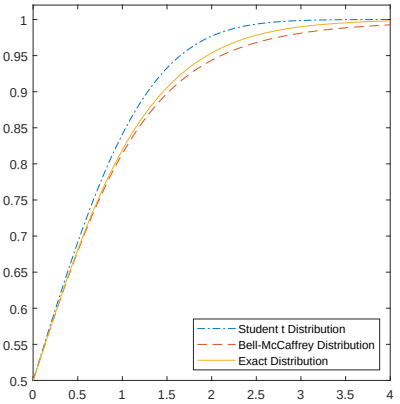

(c) HC1, $n = 500$



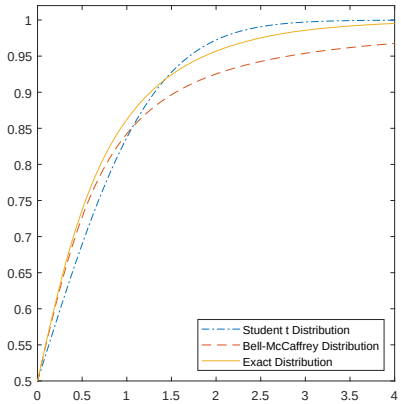

(d) HC2, $n = 30$



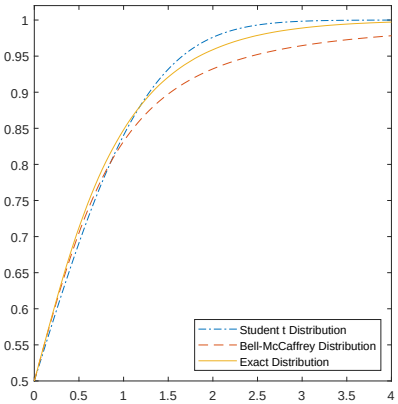

(e) HC2, $n = 120$



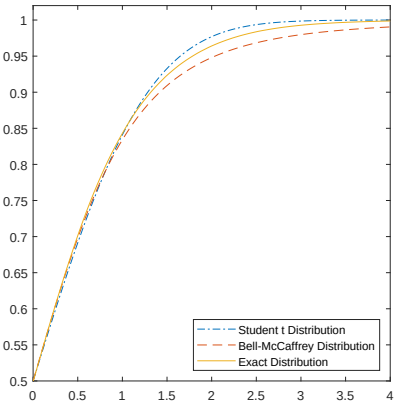

(f) HC2, $n = 500$




(g) HC3, $n = 30$

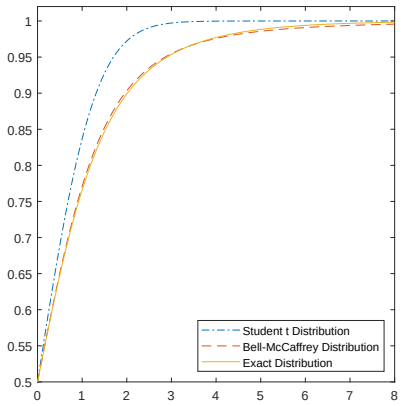



(h) HC3, $n = 120$

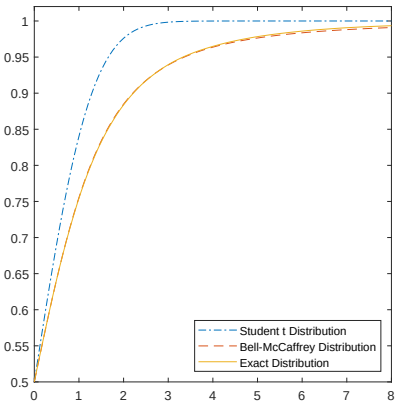



(i) HC3, $n = 500$

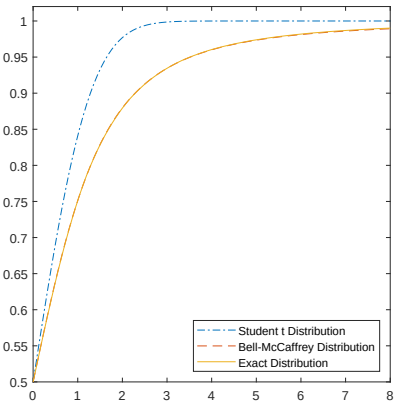
Figure 5: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. logNormal(0,4) Regressor Design, Homoskedastic Errors



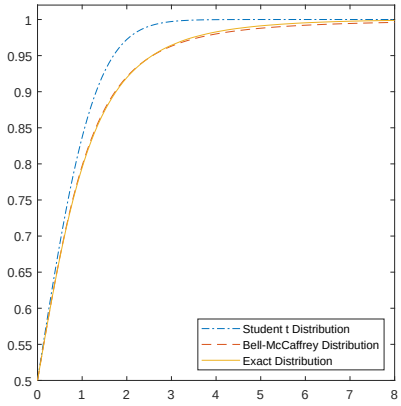

(a) HC1, $n = 30$



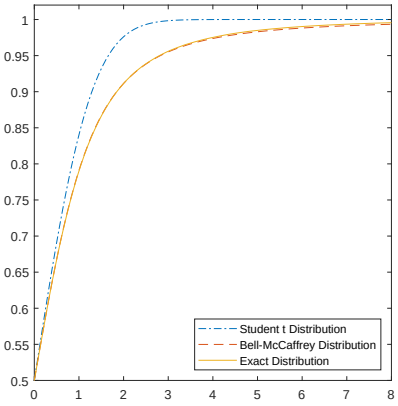

(b) HC1, $n = 120$



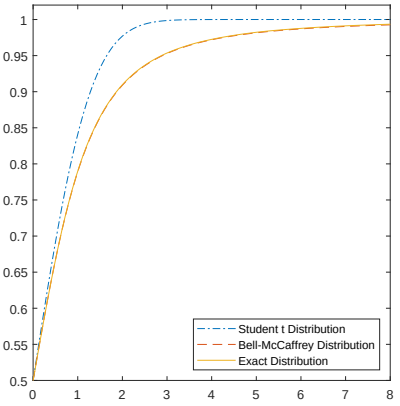

(c) HC1, $n = 500$



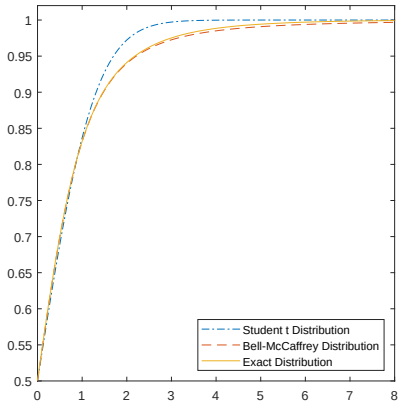

(d) HC2, $n = 30$



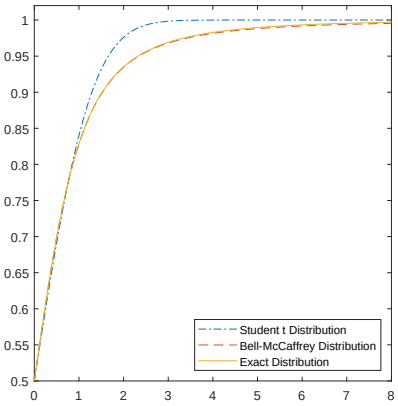

(e) HC2, $n = 120$



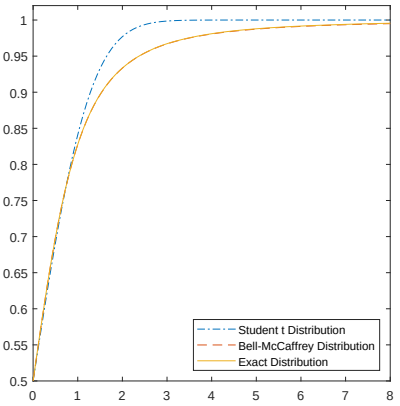

(f) HC2, $n = 500$




(g) HC3, $n = 30$

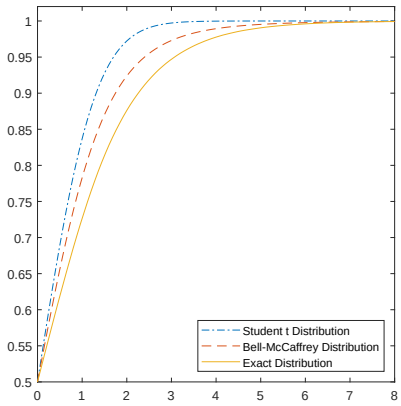



(h) HC3, $n = 120$

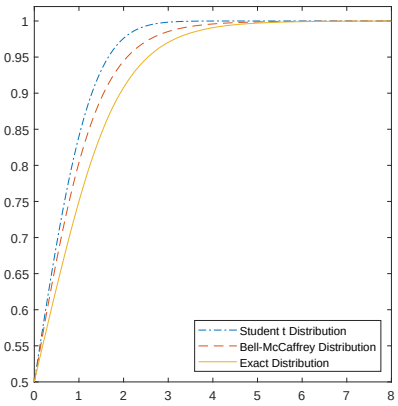



(i) HC3, $n = 500$

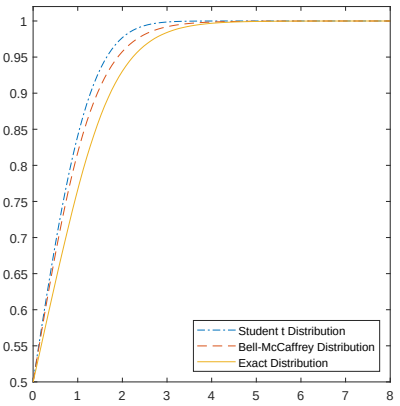
Figure 6: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. Dummy Regressor Design, Heteroskedastic Errors



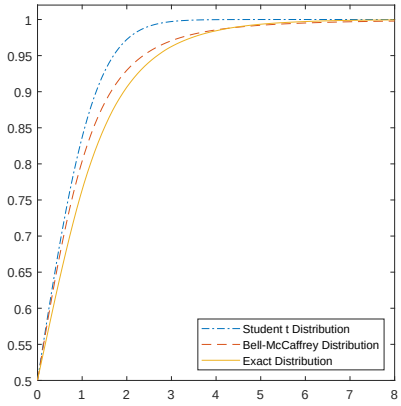

(a) HC1, $n = 30$



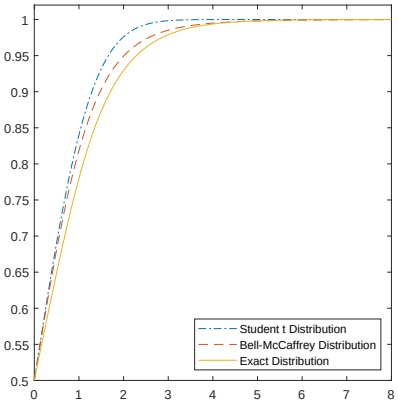

(b) HC1, $n = 120$



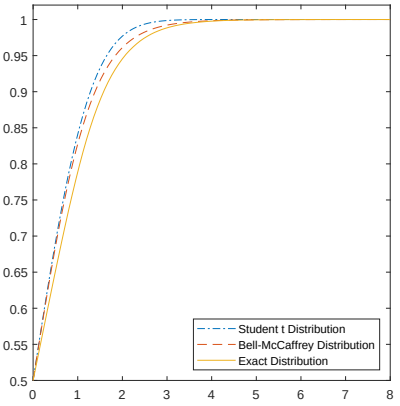

(c) HC1, $n = 500$



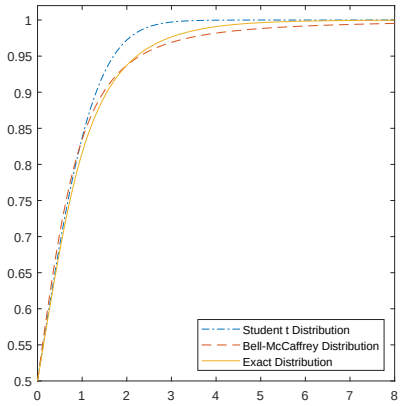

(d) HC2, $n = 30$



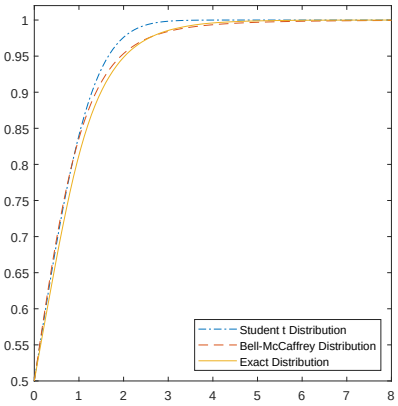

(e) HC2, $n = 120$



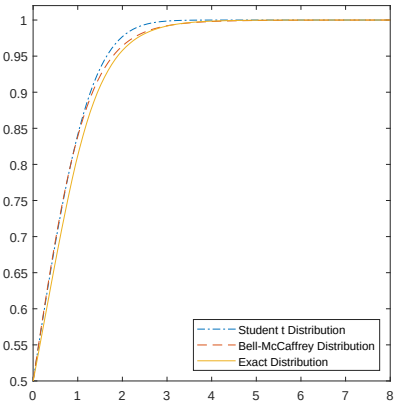

(f) HC2, $n = 500$




(g) HC3, $n = 30$

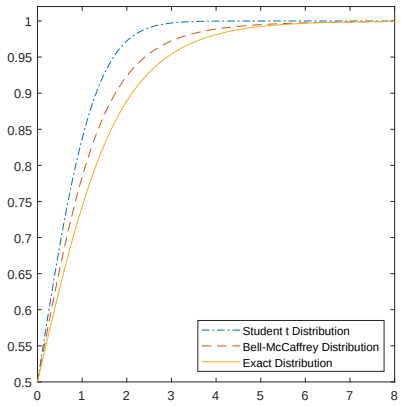



(h) HC3, $n = 120$

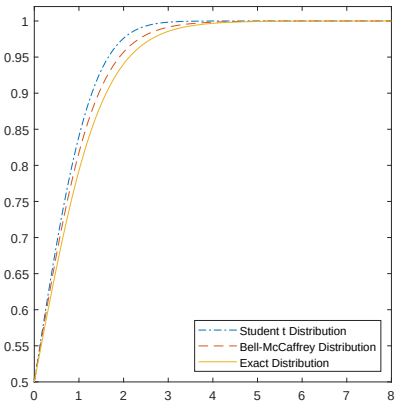



(i) HC3, $n = 500$

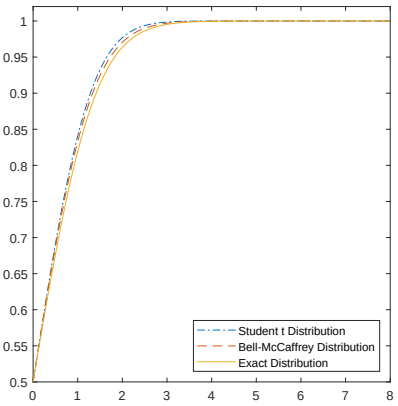
Figure 7: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. Pareto Regressor Design, Heteroskedastic Errors



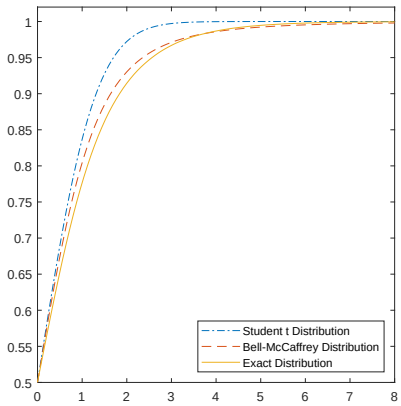

(a) HC1, $n = 30$



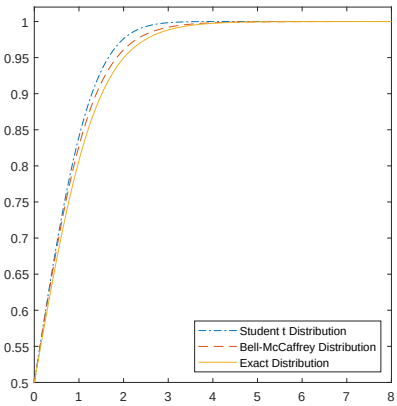

(b) HC1, $n = 120$



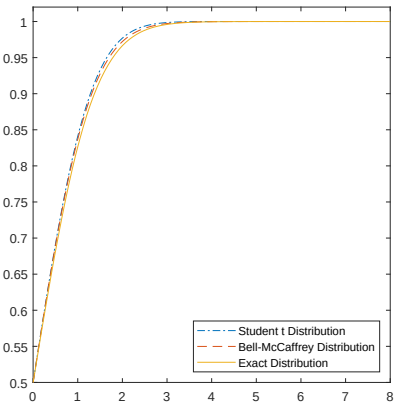

(c) HC1, $n = 500$



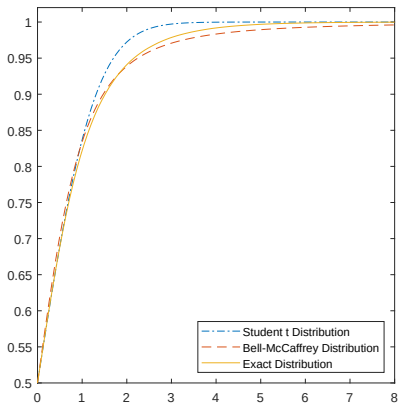

(d) HC2, $n = 30$



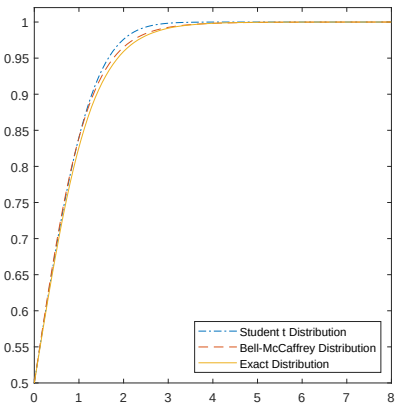

(e) HC2, $n = 120$



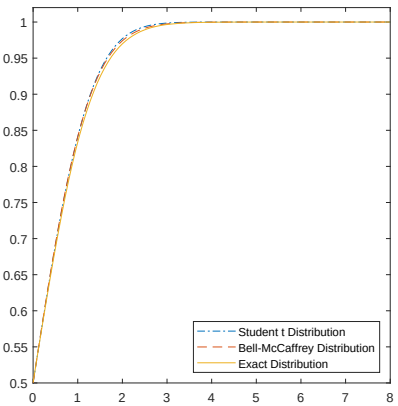

(f) HC2, $n = 500$




(g) HC3, $n = 30$

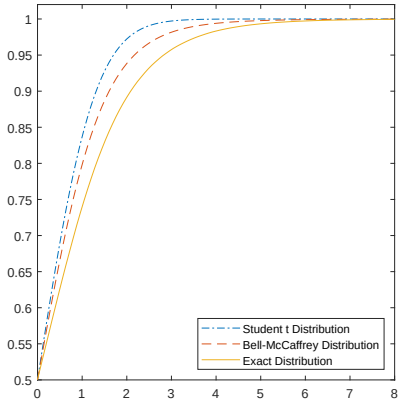



(h) HC3, $n = 120$

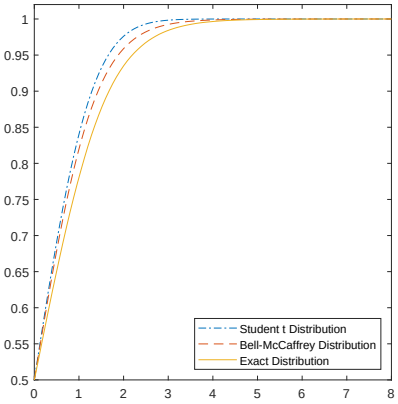



(i) HC3, $n = 500$

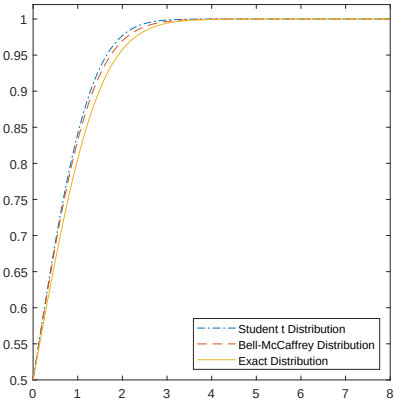
Figure 8: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. Gamma Regressor Design, Heteroskedastic Errors



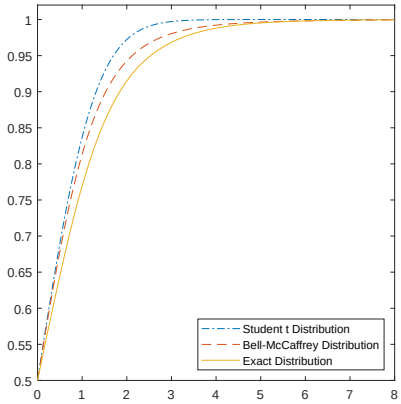

(a) HC1, $n = 30$



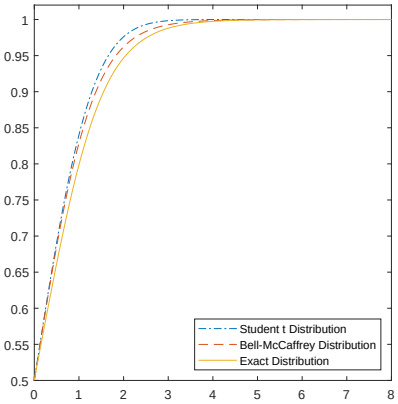

(b) HC1, $n = 120$



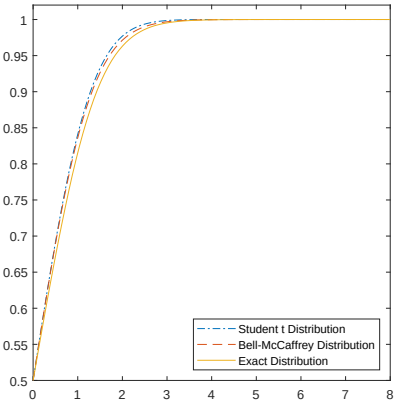

(c) HC1, $n = 500$



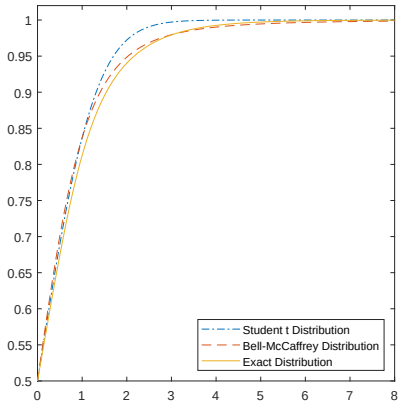

(d) HC2, $n = 30$



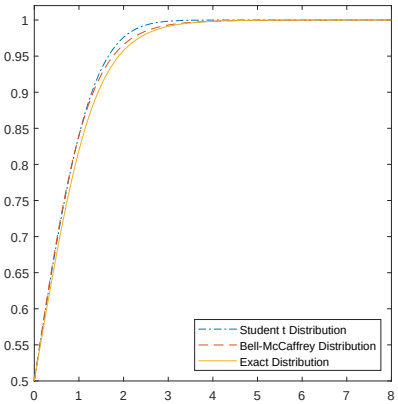

(e) HC2, $n = 120$



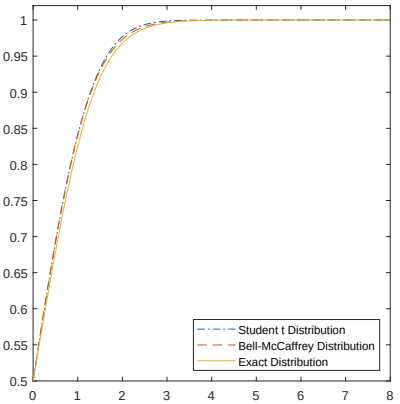

(f) HC2, $n = 500$




(g) HC3, $n = 30$

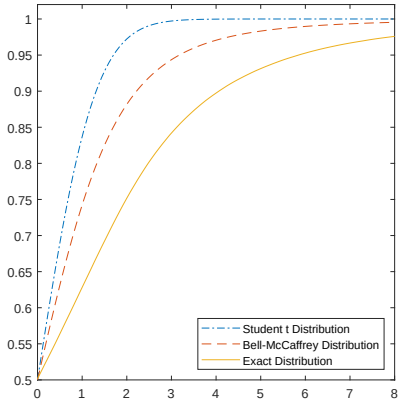



(h) HC3, $n = 120$

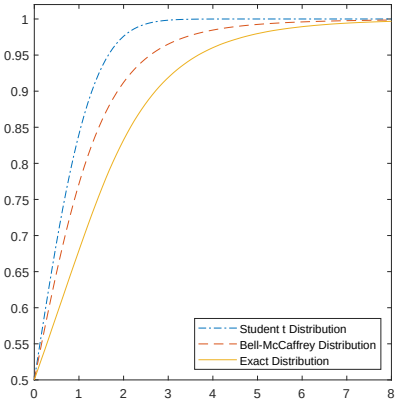



(i) HC3, $n = 500$

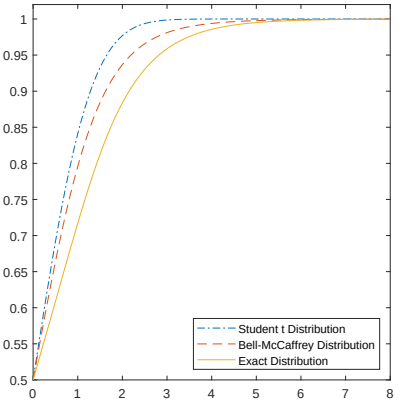
Figure 9: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. logNormal(0,1) Regressor Design, Heteroskedastic Errors



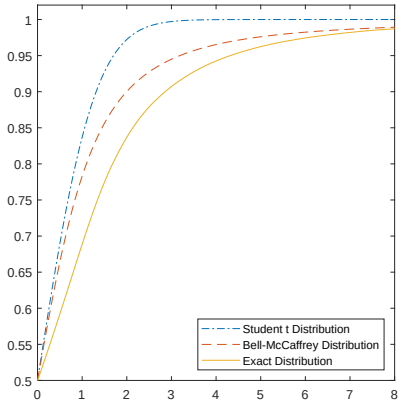

(a) HC1, $n = 30$



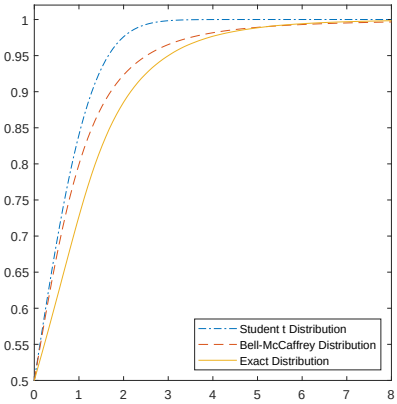

(b) HC1, $n = 120$



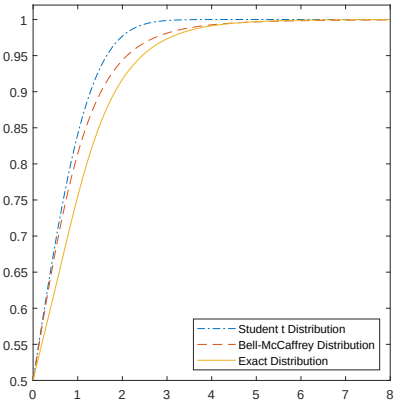

(c) HC1, $n = 500$



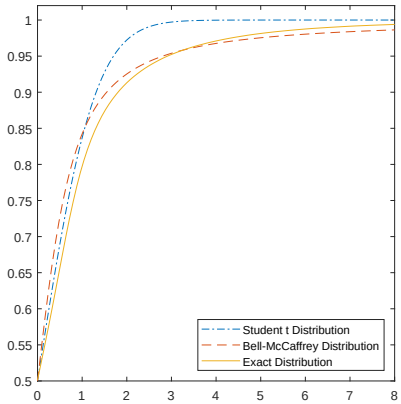

(d) HC2, $n = 30$



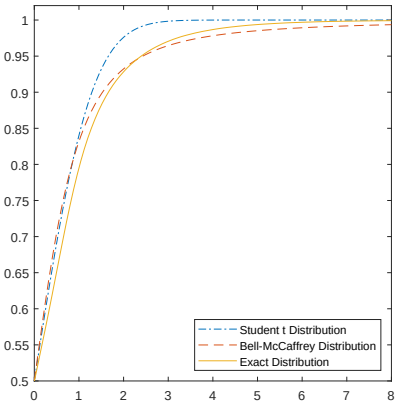

(e) HC2, $n = 120$



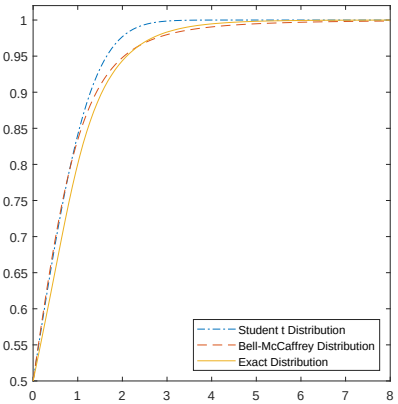

(f) HC2, $n = 500$




(g) HC3, $n = 30$

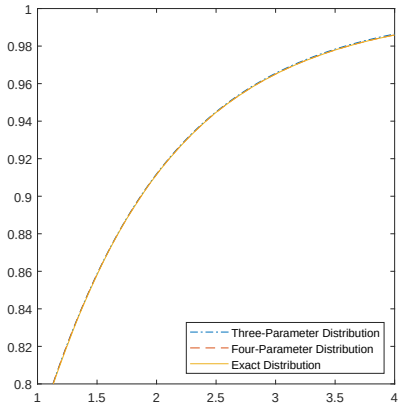



(h) HC3, $n = 120$

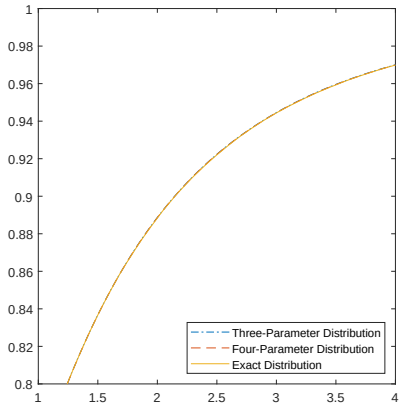



(i) HC3, $n = 500$

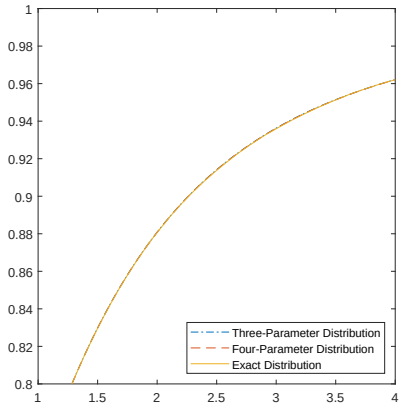
Figure 10: Student t, Bell-McCaffrey, and Exact Distribution of t-ratio. logNormal(0,4) Regressor Design, Heteromoskedastic Errors



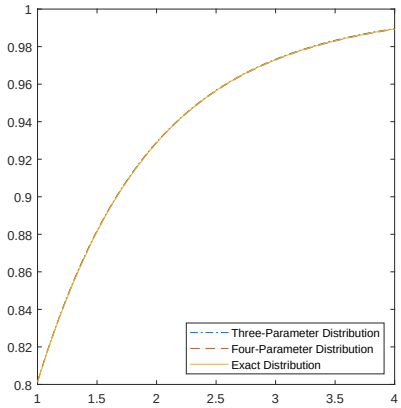

(a) HC1, $n = 30$



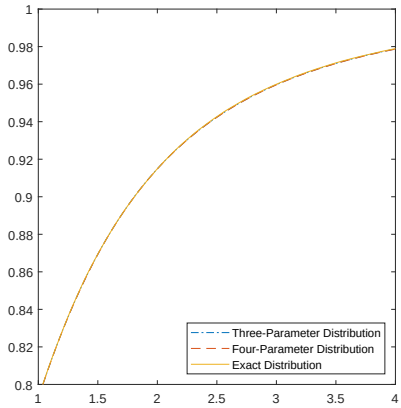

(b) HC1, $n = 120$



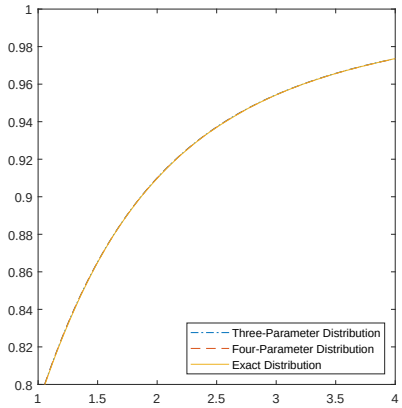

(c) HC1, $n = 500$



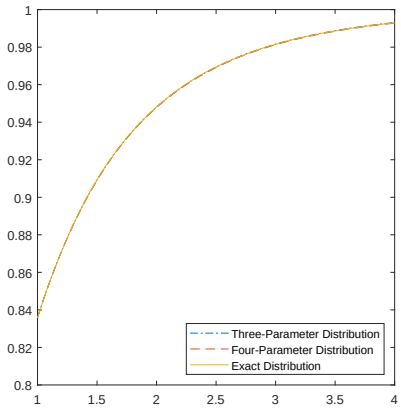

(d) HC2, $n = 30$



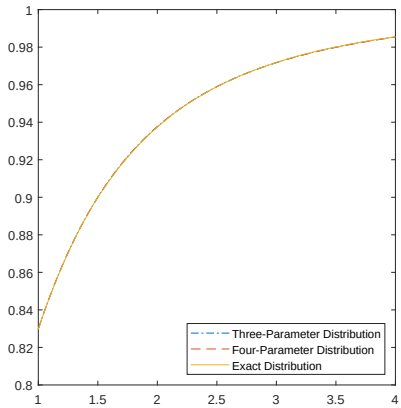

(e) HC2, $n = 120$



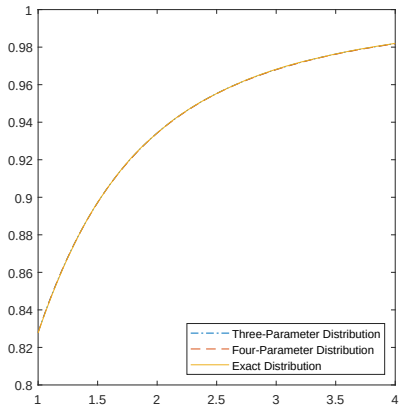

(f) HC2, $n = 500$




(g) HC3, $n = 30$

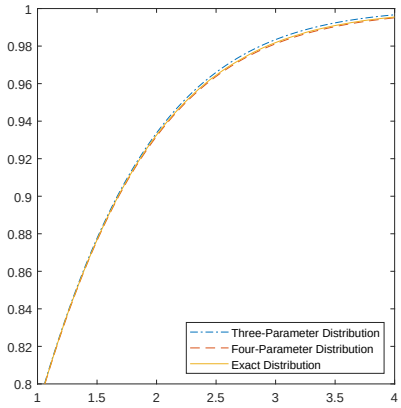



(h) HC3, $n = 120$

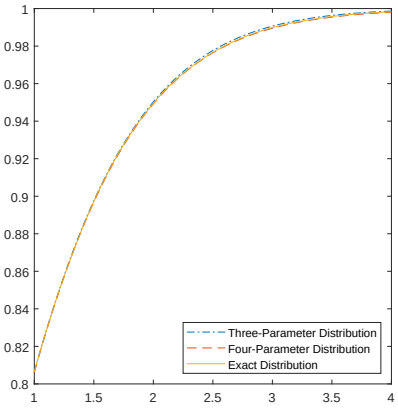



(i) HC3, $n = 500$

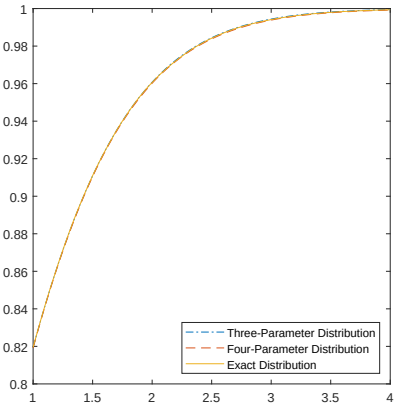
Figure 11: Three-Parameter and Four-Parameter Distributions. Dummy Regressor Design, Homoskedastic Errors



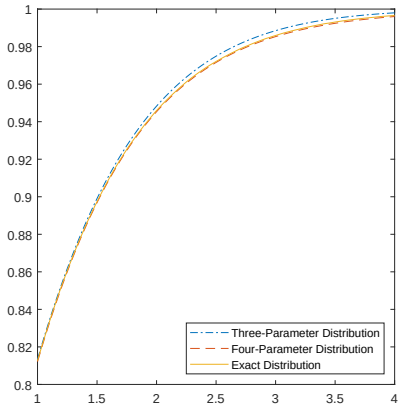

(a) HC1, $n = 30$



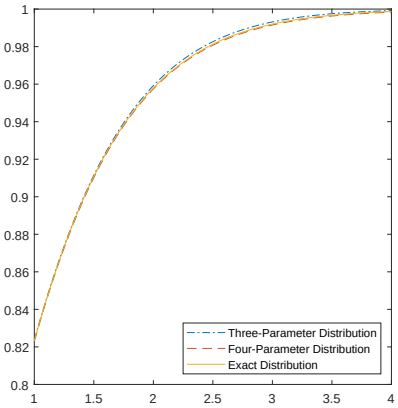

(b) HC1, $n = 120$



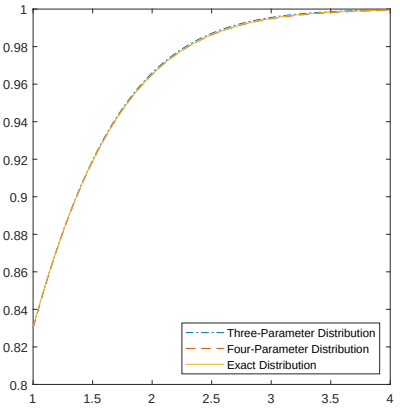

(c) HC1, $n = 500$



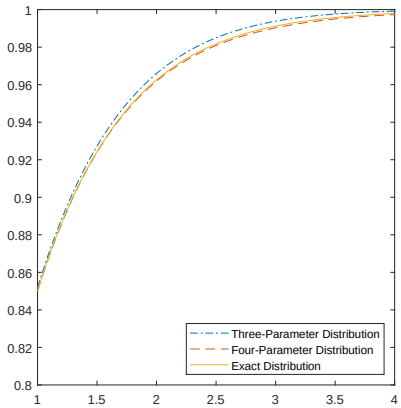

(d) HC2, $n = 30$



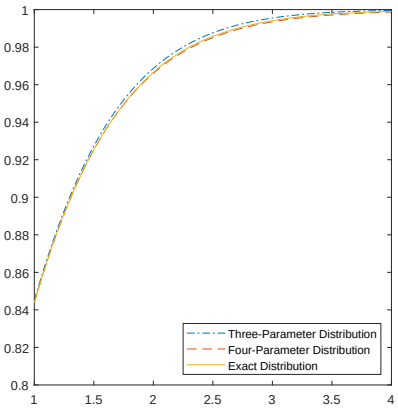

(e) HC2, $n = 120$



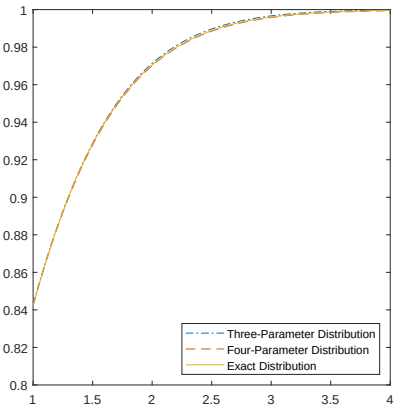

(f) HC2, $n = 500$




(g) HC3, $n = 30$

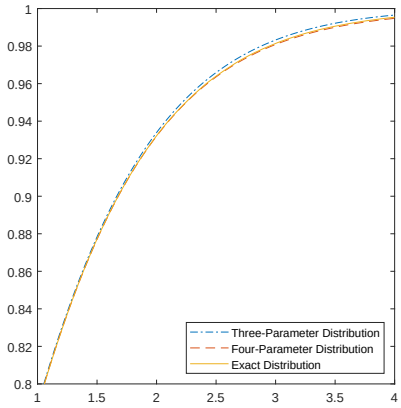



(h) HC3, $n = 120$

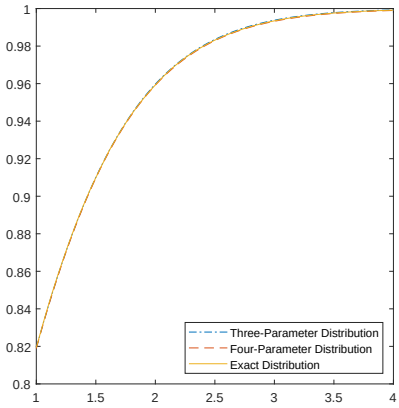



(i) HC3, $n = 500$

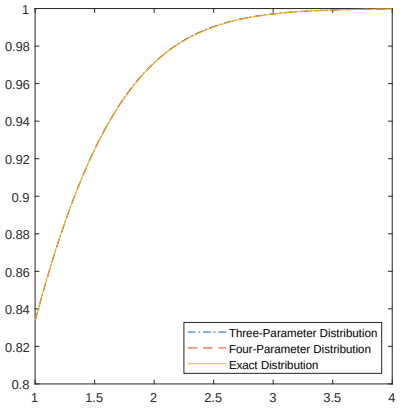
Figure 12: Three-Parameter and Four-Parameter Distributions. Pareto Regressor Design, Homoskedastic Errors



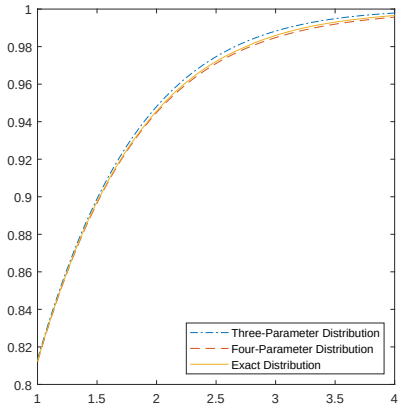

(a) HC1, $n = 30$



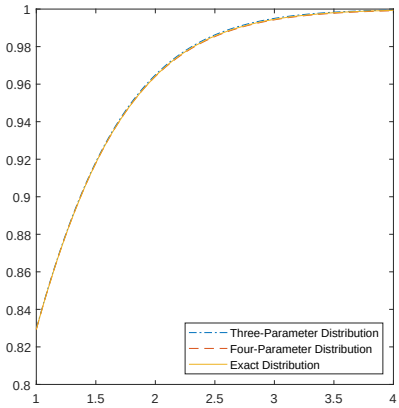

(b) HC1, $n = 120$



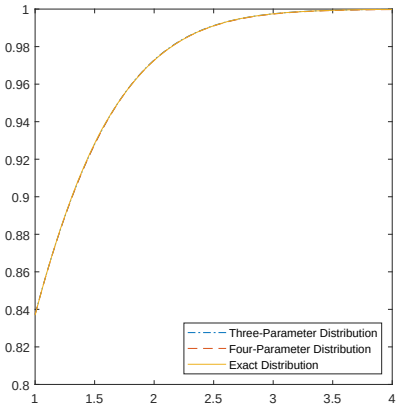

(c) HC1, $n = 500$



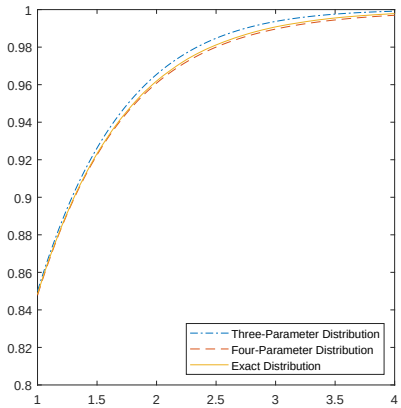

(d) HC2, $n = 30$



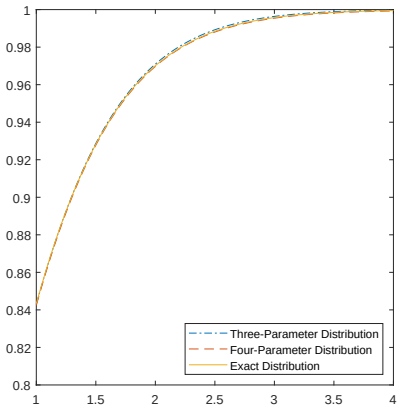

(e) HC2, $n = 120$



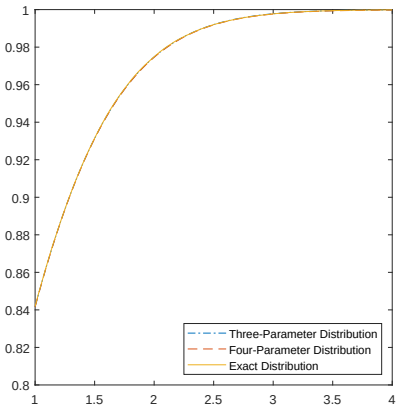

(f) HC2, $n = 500$




(g) HC3, $n = 30$

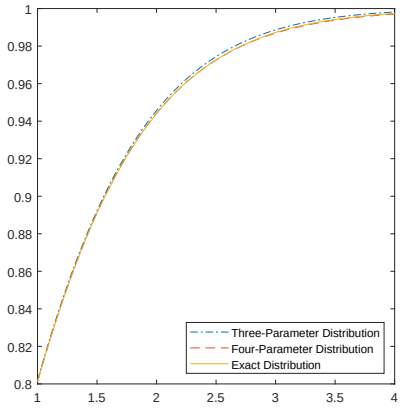



(h) HC3, $n = 120$

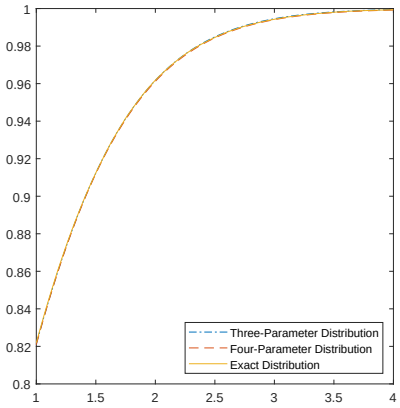



(i) HC3, $n = 500$

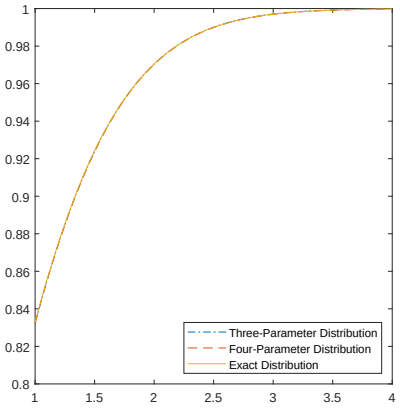
Figure 13: Three-Parameter and Four-Parameter Distributions. Gamma Regressor Design, Homoskedastic Errors



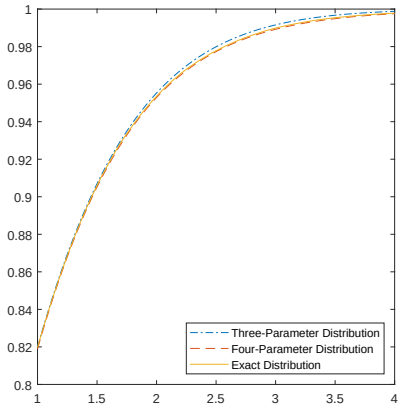

(a) HC1, $n = 30$



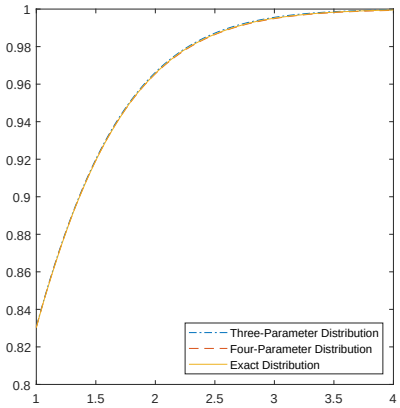

(b) HC1, $n = 120$



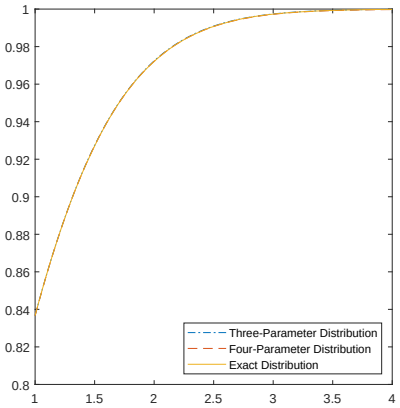

(c) HC1, $n = 500$



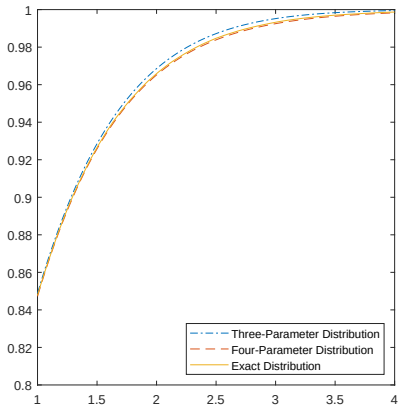

(d) HC2, $n = 30$



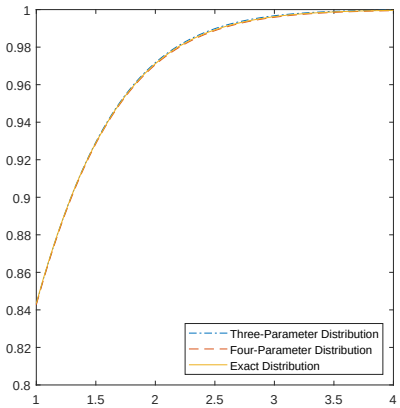

(e) HC2, $n = 120$



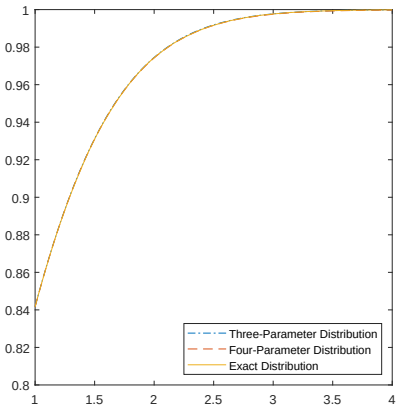

(f) HC2, $n = 500$




(g) HC3, $n = 30$

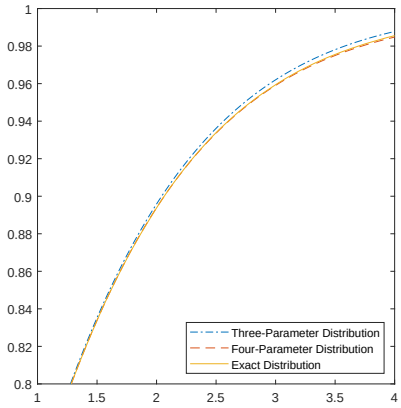



(h) HC3, $n = 120$

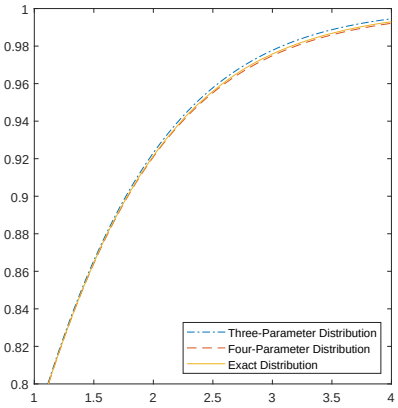



(i) HC3, $n = 500$

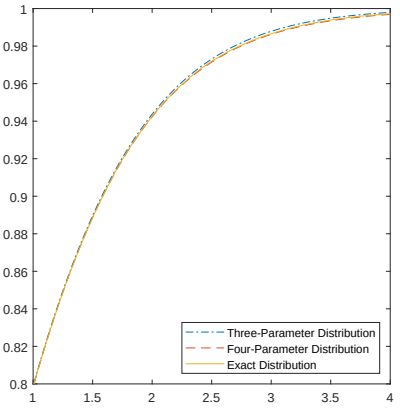
Figure 14: Three-Parameter and Four-Parameter Distributions. logNormal(0,1) Regressor Design, Homoskedastic Errors



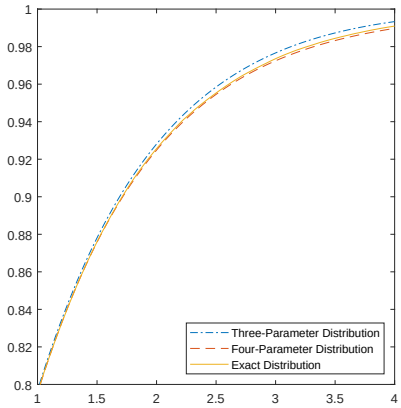

(a) HC1, $n = 30$



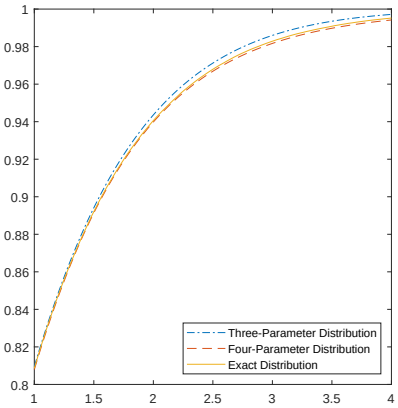

(b) HC1, $n = 120$



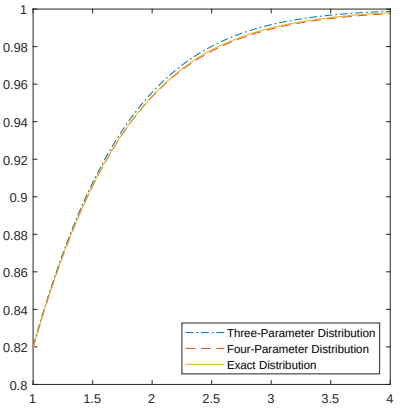

(c) HC1, $n = 500$



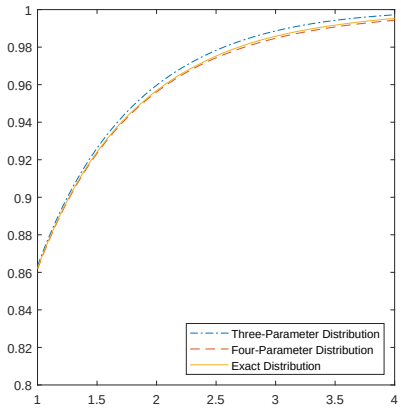

(d) HC2, $n = 30$



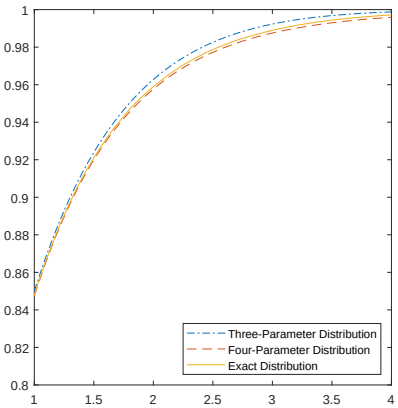

(e) HC2, $n = 120$



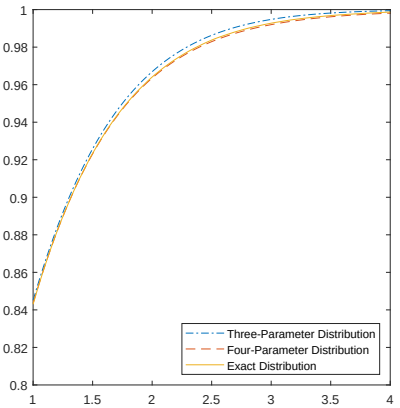

(f) HC2, $n = 500$




(g) HC3, $n = 30$

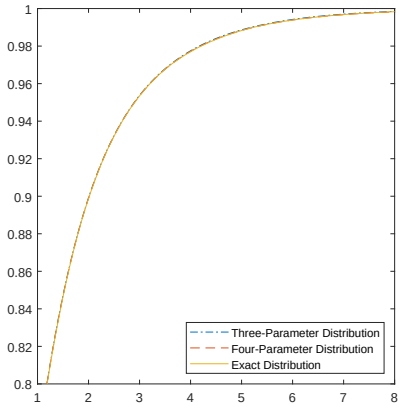



(h) HC3, $n = 120$

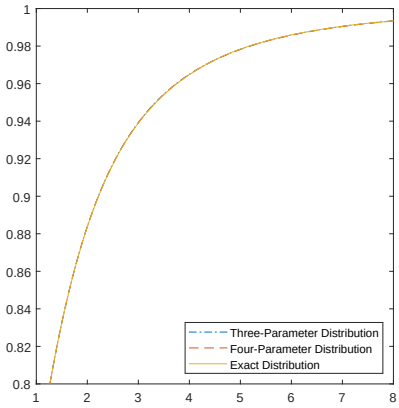



(i) HC3, $n = 500$

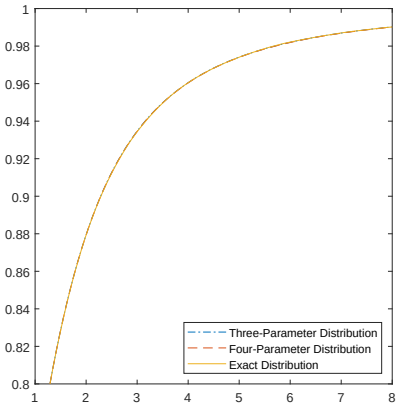
Figure 15: Three-Parameter and Four-Parameter Distributions. logNormal(0,4) Regressor Design, Homoskedastic Errors



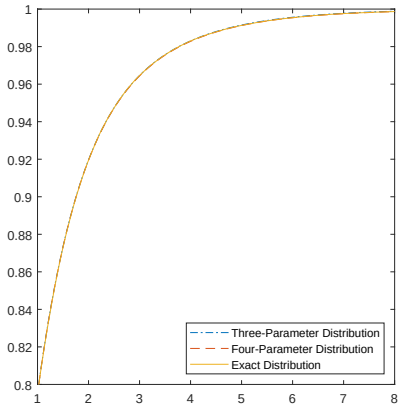

(a) HC1, $n = 30$



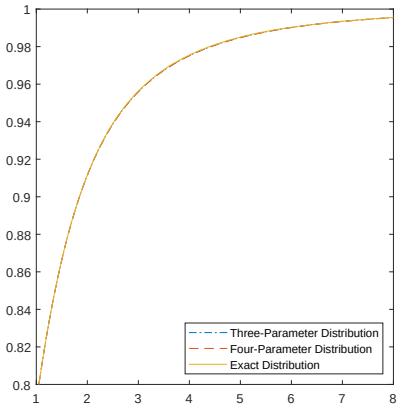

(b) HC1, $n = 120$



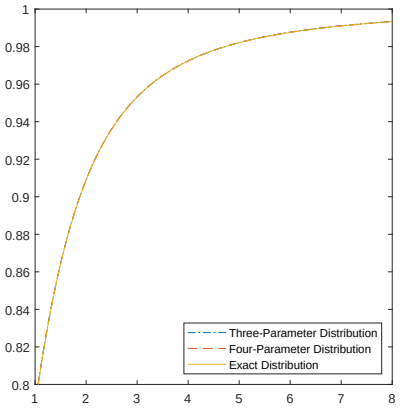

(c) HC1, $n = 500$



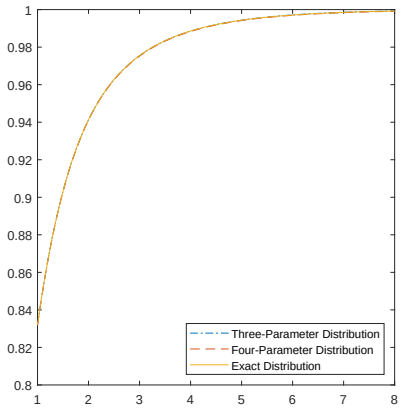

(d) HC2, $n = 30$



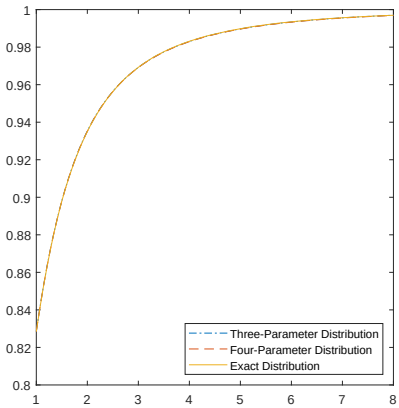

(e) HC2, $n = 120$



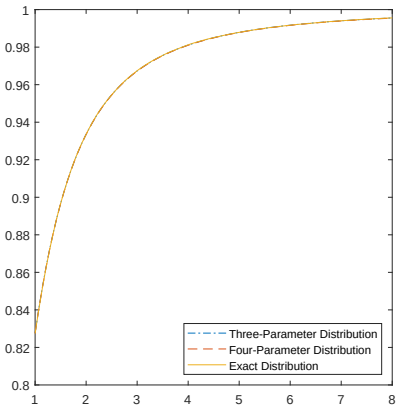

(f) HC2, $n = 500$




(g) HC3, $n = 30$

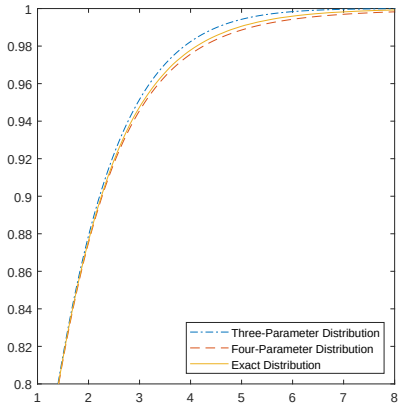



(h) HC3, $n = 120$

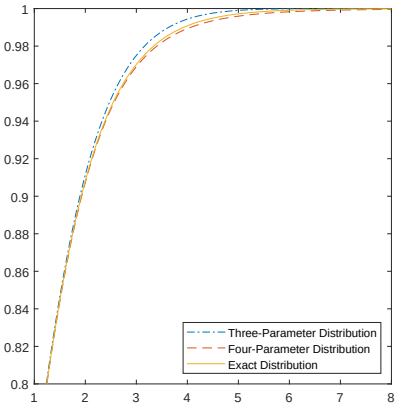



(i) HC3, $n = 500$

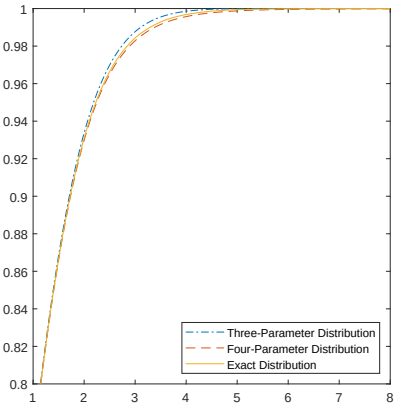
Figure 16: Three-Parameter and Four-Parameter Distributions. Dummy Regressor Design, Heteroskedastic Errors



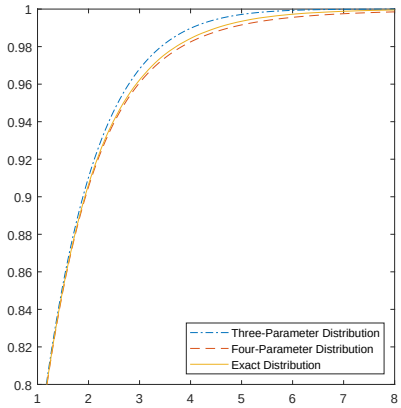

(a) HC1, $n = 30$



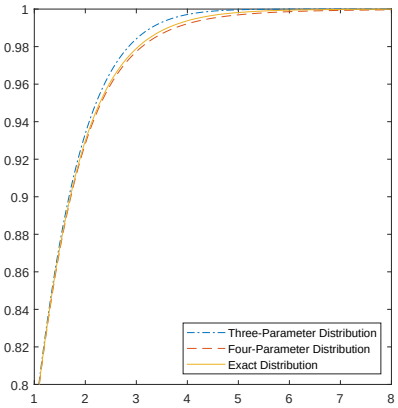

(b) HC1, $n = 120$



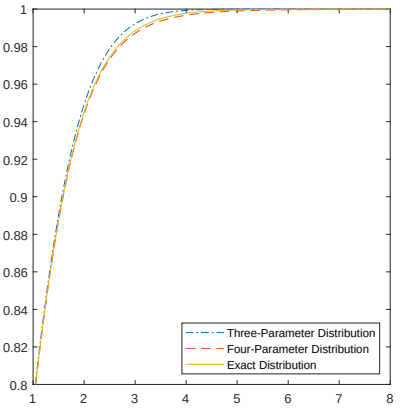

(c) HC1, $n = 500$



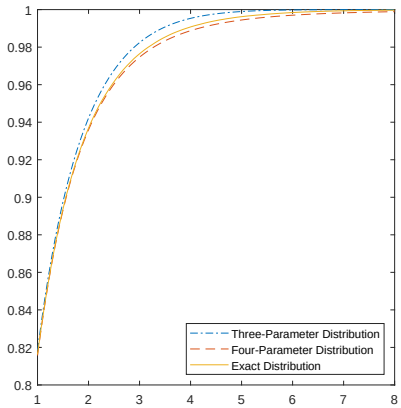

(d) HC2, $n = 30$



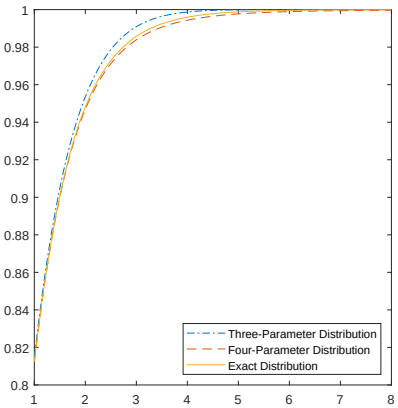

(e) HC2, $n = 120$



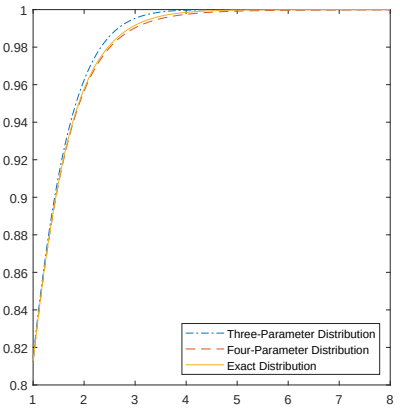

(f) HC2, $n = 500$




(g) HC3, $n = 30$

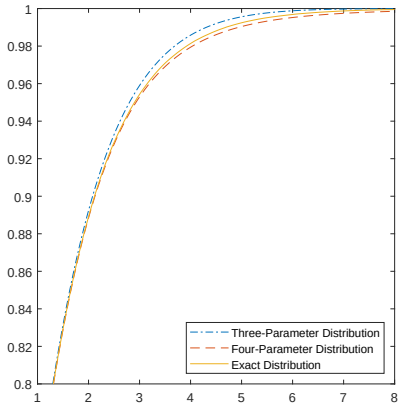



(h) HC3, $n = 120$

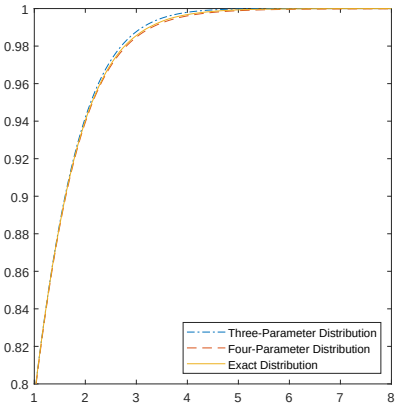



(i) HC3, $n = 500$

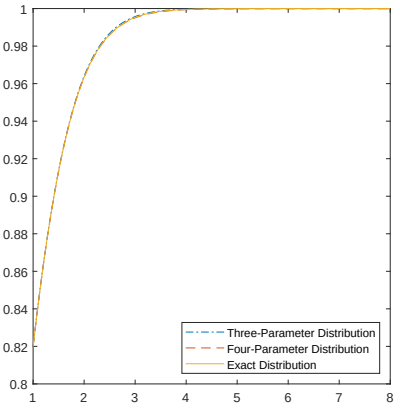
Figure 17: Three-Parameter and Four-Parameter Distributions. Pareto Regressor Design, Heteroskedastic Errors



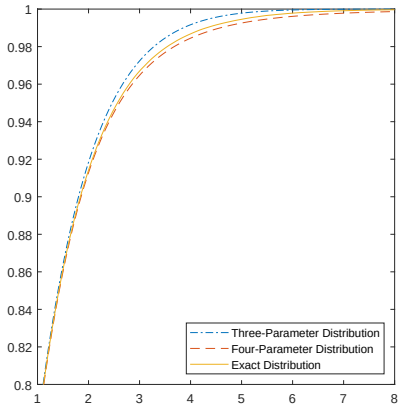

(a) HC1, $n = 30$



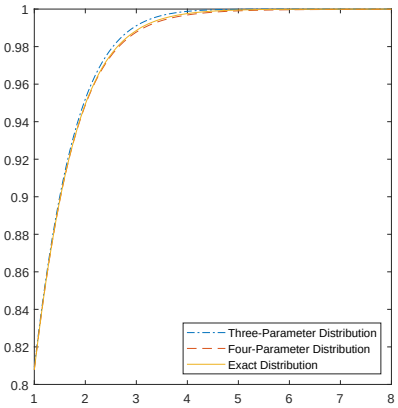

(b) HC1, $n = 120$



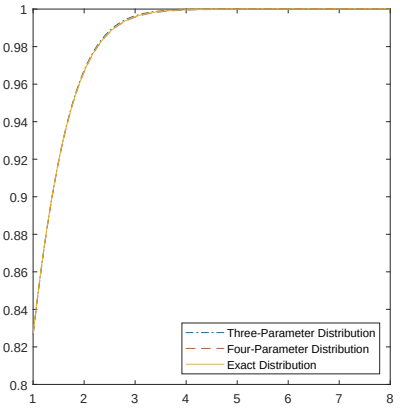

(c) HC1, $n = 500$



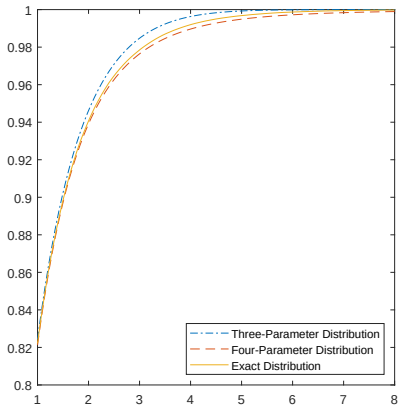

(d) HC2, $n = 30$



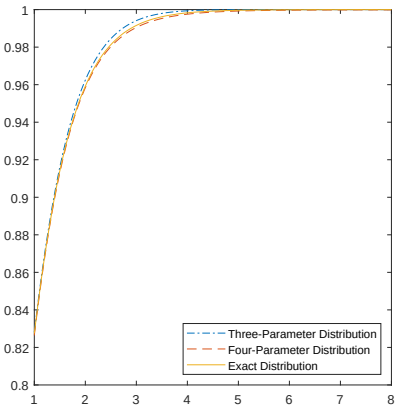

(e) HC2, $n = 120$



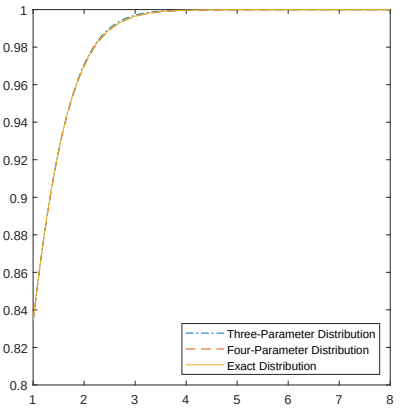

(f) HC2, $n = 500$




(g) HC3, $n = 30$

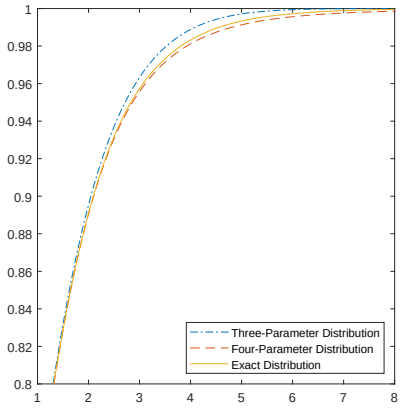



(h) HC3, $n = 120$

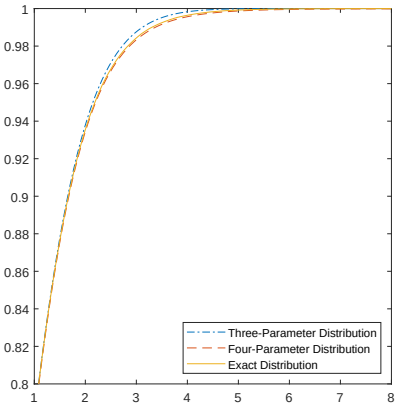



(i) HC3, $n = 500$

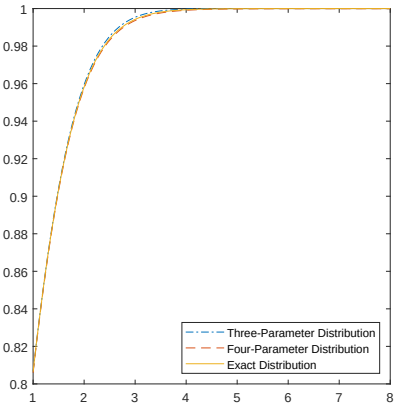
Figure 18: Three-Parameter and Four-Parameter Distributions. Gamma Regressor Design, Heteroskedastic Errors



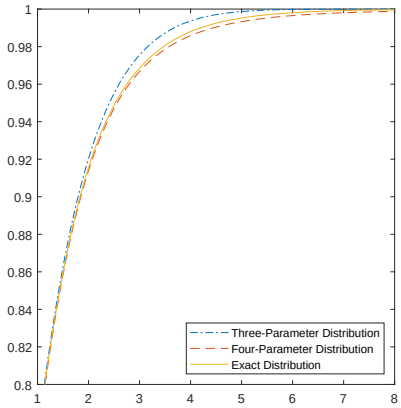

(a) HC1, $n = 30$



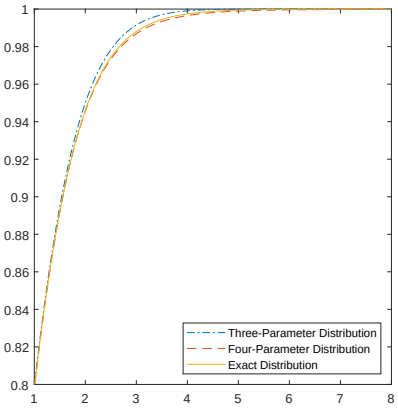

(b) HC1, $n = 120$



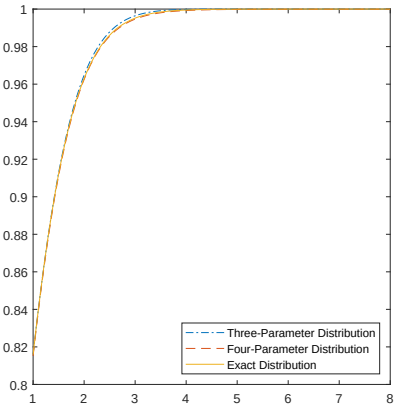

(c) HC1, $n = 500$



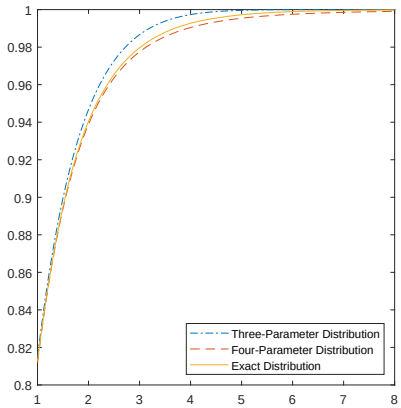

(d) HC2, $n = 30$



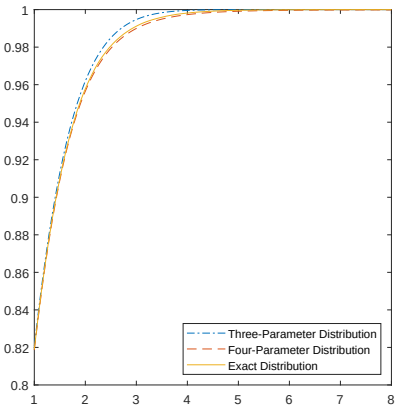

(e) HC2, $n = 120$



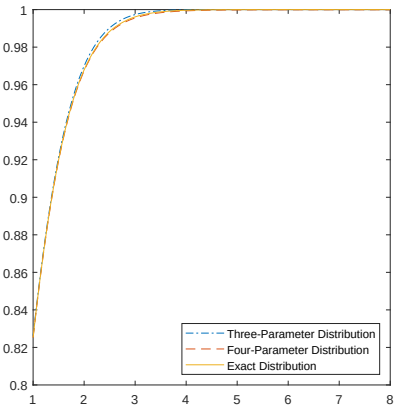

(f) HC2, $n = 500$




(g) HC3, $n = 30$

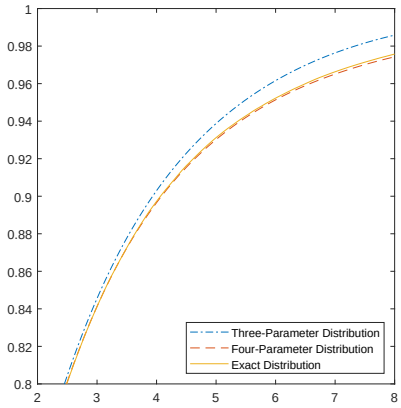



(h) HC3, $n = 120$

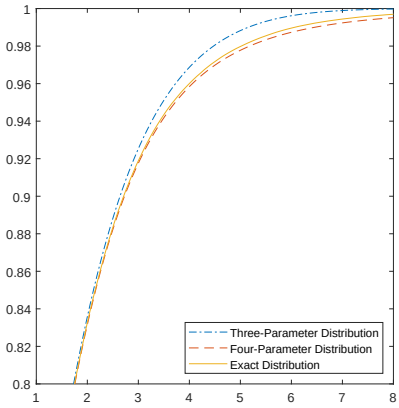



(i) HC3, $n = 500$

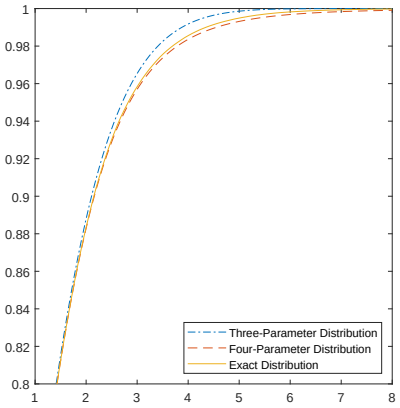
Figure 19: Three-Parameter and Four-Parameter Distributions. logNormal(0,1) Regressor Design, Heteroskedastic Errors



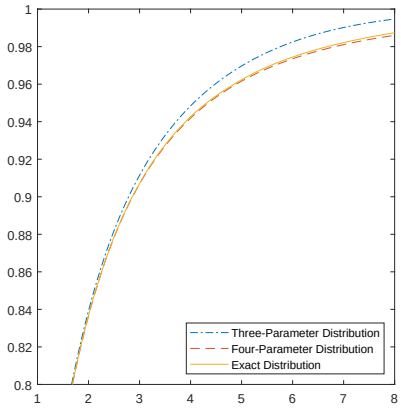

(a) HC1, $n = 30$



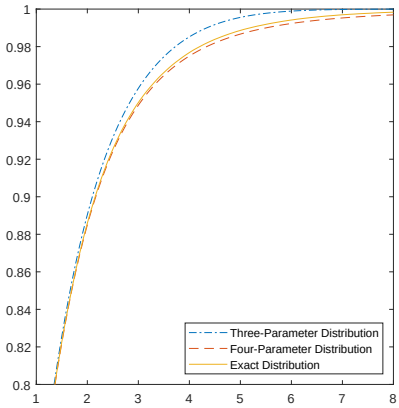

(b) HC1, $n = 120$



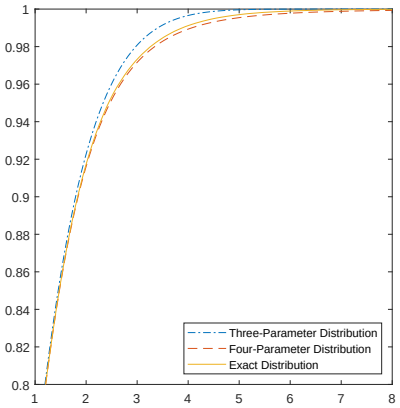

(c) HC1, $n = 500$



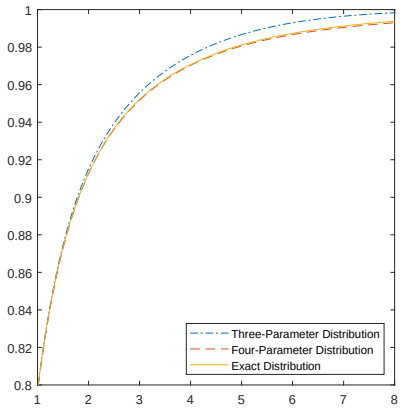

(d) HC2, $n = 30$



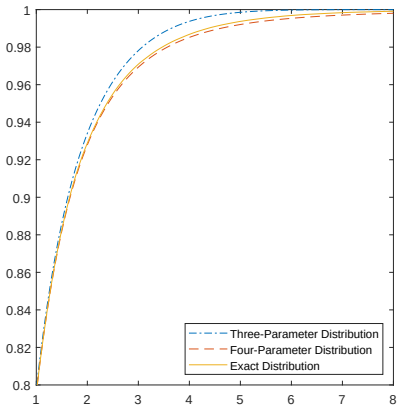

(e) HC2, $n = 120$



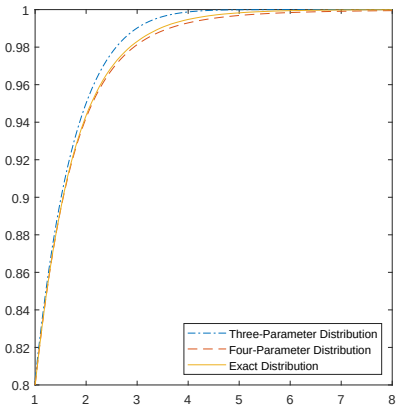

(f) HC2, $n = 500$




(g) HC3, $n = 30$




(h) HC3, $n = 120$




(i) HC3, $n = 500$

Figure 20: Three-Parameter and Four-Parameter Distributions. logNormal(0,4) Regressor Design, Heteromoskedastic Errors