Model Averaging, Asymptotic Risk, and Regressor Groups
Supplemental Appendix

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Simulation Details

The simulation programs were written in R and run under Windows Vista. The programs are available at http://www.ssc.wisc.edu/~bhansen/progs/progs.htm.

The results are presented graphically, with MSE displayed as a function of R^2. The value of R^2 was varied on the 19-point grid $\{0.00, 0.05, 0.10, 0.15, ..., , 0.90\}$. For a fixed α and R^2 the value of c was then determined as

$$c = \sqrt{\frac{R^2}{\sum_{j=1}^{M} j^{-2\alpha} \left(1 - R^2\right)}}.$$

Given c, we then set $\beta_j = cj^{-\alpha}$ and

$$y_i = \beta_0 + \sum_{j=1}^{M} \beta_j x_{ji} + e_i$$

with $\beta_0 = 0$.

We varied $\alpha \in \{0, 1, 2, 3\}$ and $n \in \{50, 150, 400, 1000\}$.

The default model (Model 1) set the errors e_i and regressors x_{ji} as iid $\mathrm{N}(0, 1)$ and set $M = 12$. The remaining models explored the deviations from these default settings.

We explored non-normal errors, heteroskedastic errors, correlated regressors, and $M = 24$.

All models were designed so that the error is conditionally mean zero and has an unconditional variance of one.

The results for model 1 and model 6 are calculated using 10,000 simulation replications. For models 2 through 5, the calculations used 2000 simulation replications.

1. Model 1: Normal Regression

   - $e_i \sim \mathrm{N}(0, 1)$
   - uncorrelated regressors
   - $M = 12$

2. Model 2: Non-Normal Error

   - $e_i \sim \frac{4}{5}\mathrm{N}\left(-\frac{1}{3}, \frac{5}{9}\right) + \frac{1}{5}\mathrm{N}\left(\frac{4}{3}, \frac{5}{9}\right)$

3. Model 3: Heteroskedastic Error

   - $e_i \sim \mathrm{N}\left(0, \frac{1}{2}\left(1 + x_{2i}^2\right)\right)$

4. Model 4: Correlated Regressors

   - $e_i \sim \mathrm{N}(0, 1)$
   - $E\left(x_{ji}^2\right) = 1,\ E\left(x_{ji}x_{ki}\right) = 0.5$ for $j \neq k$

5. Model 5: Increased Number of Regressors
    - $e_i \sim \mathrm{N}(0, 1)$
    - $M = 24$
6. Model 6: Autoregression

In the paper, the figures display the normalized MSE for the estimators MMA_4, MMA, Stein, Lasso, and BMA. Here, we also display the normalized MSE for the estimator SAIC and the the MMA_4 estimator with the regressors ordered in reverse (from smallest to largest coefficients) and is labeled as “Reversed”.

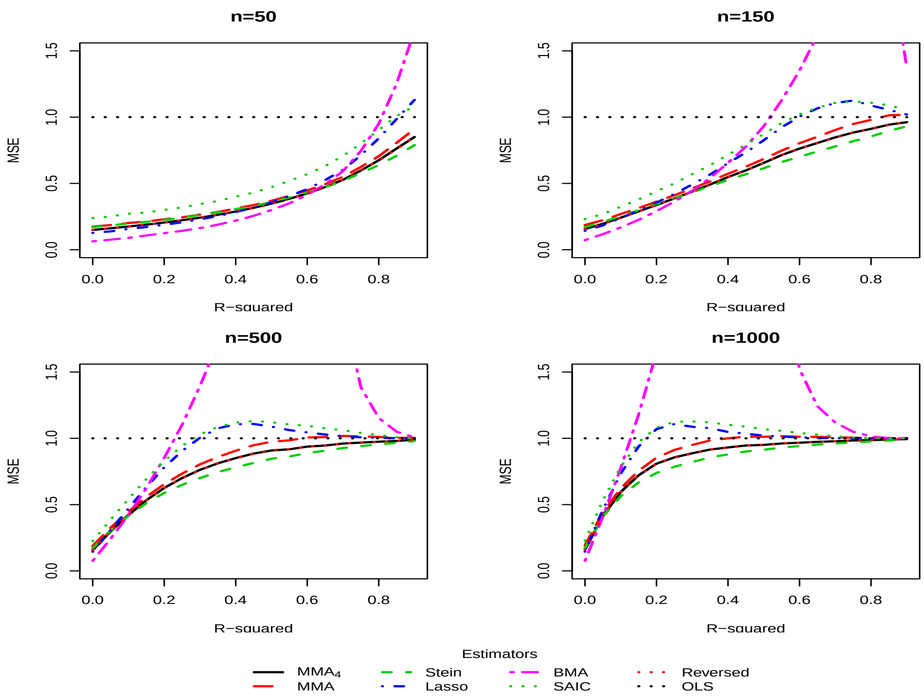

Figure 1: Model 1: $\alpha = 0$

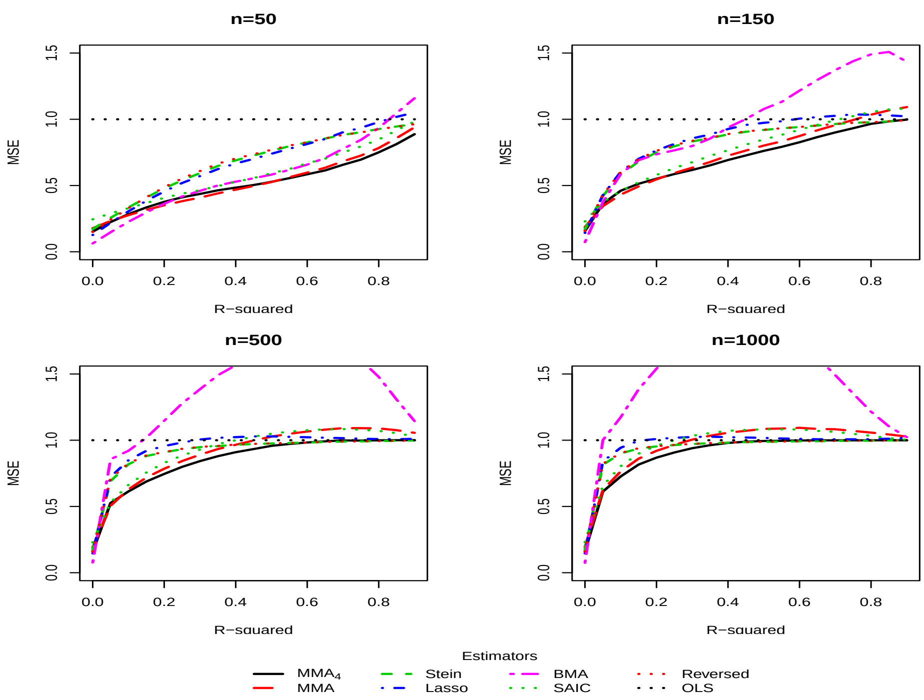

Figure 2: Model 1: $\alpha = 1$

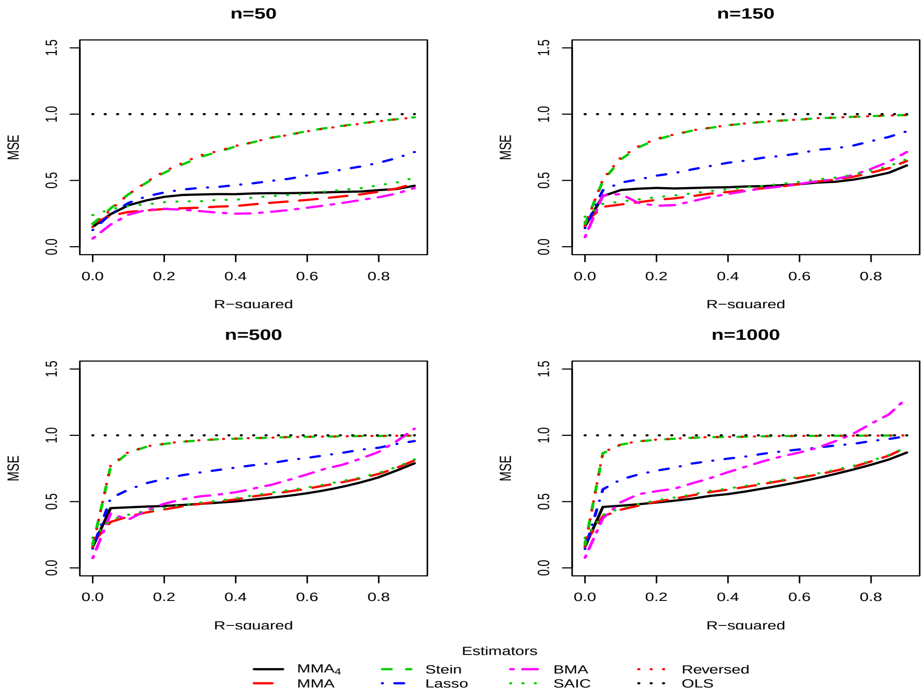

Figure 3: Model 1: $\alpha = 2$

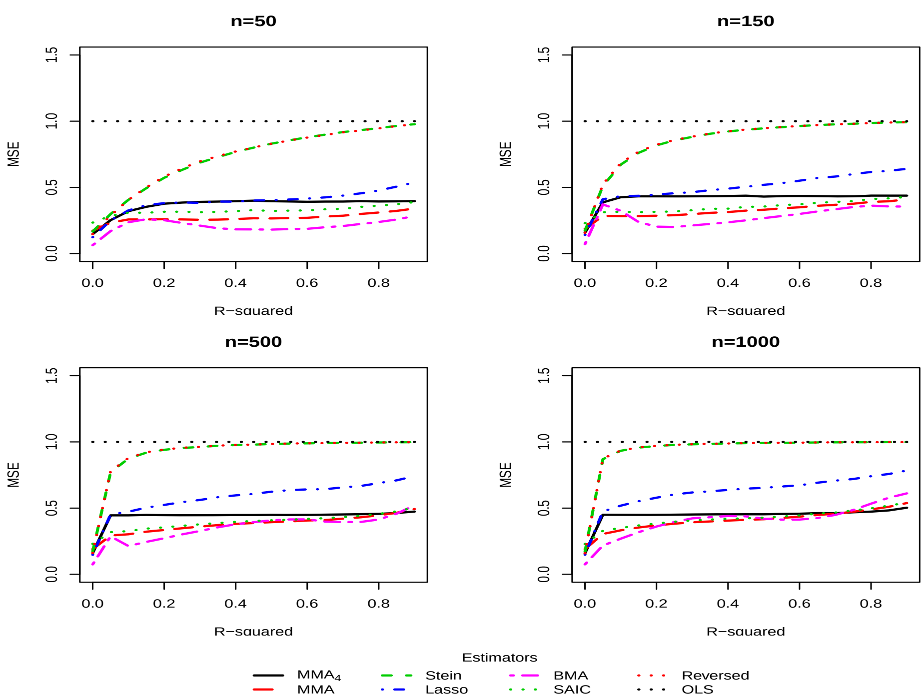

Figure 4: Model 1: $\alpha = 3$

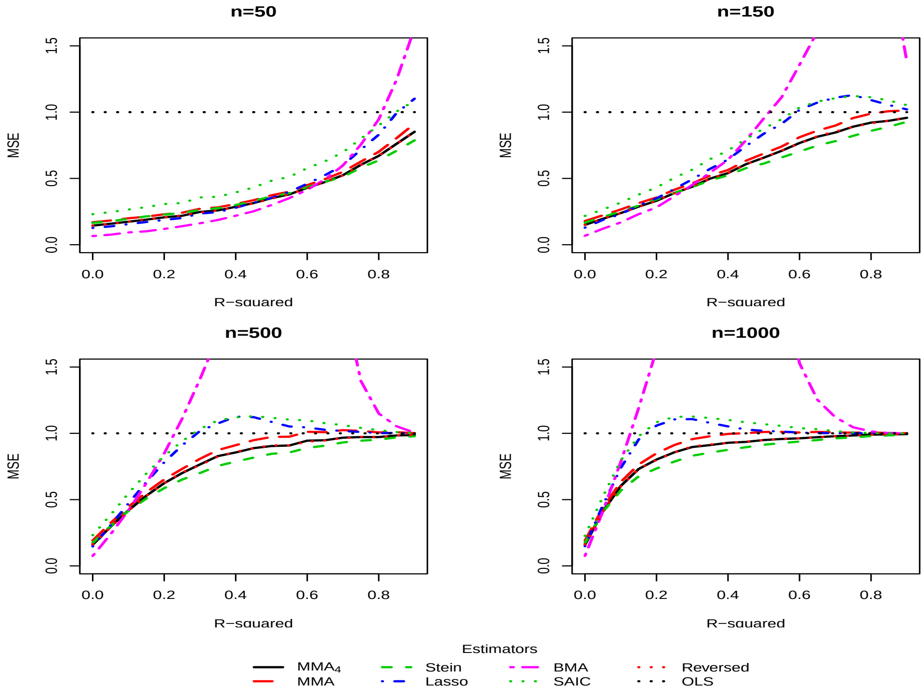

Figure 5: Model 2: $\alpha = 0$

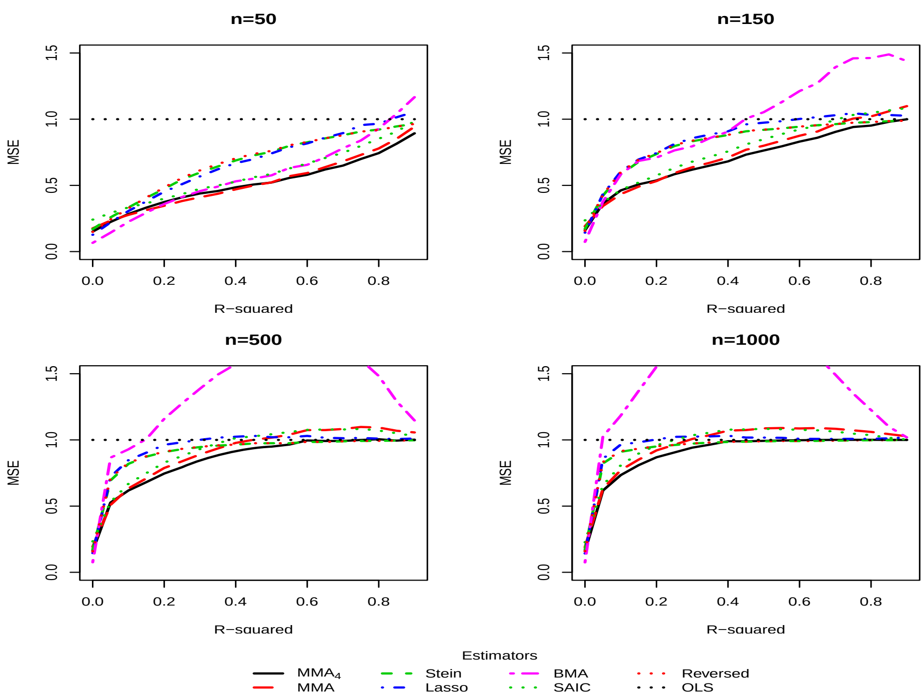

Figure 6: Model 2: $\alpha = 1$

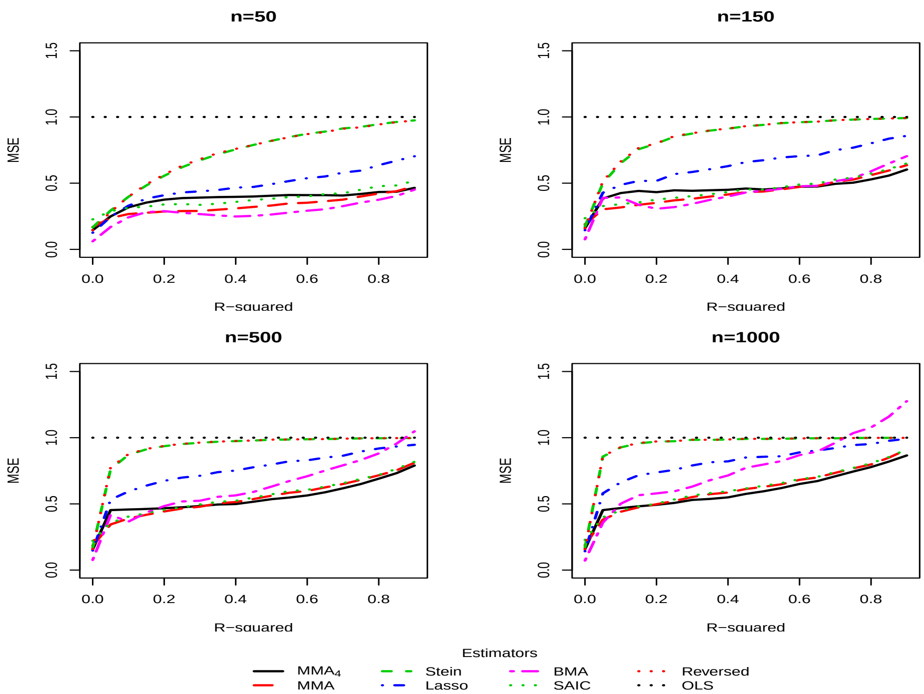

Figure 7: Model 2: $\alpha = 2$

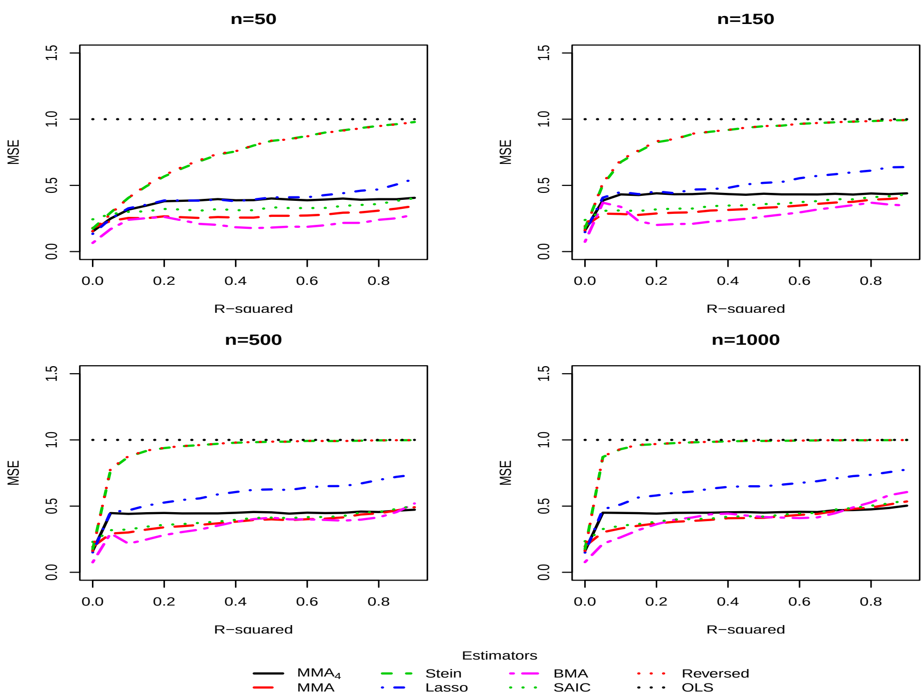

Figure 8: Model 2: $\alpha = 3$

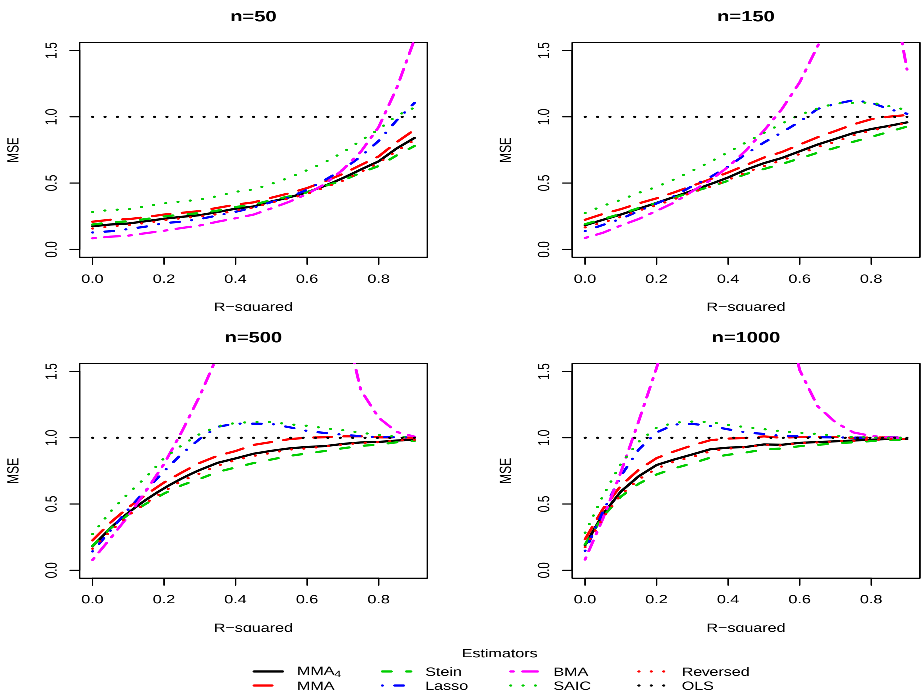

Figure 9: Model 3: $\alpha = 0$

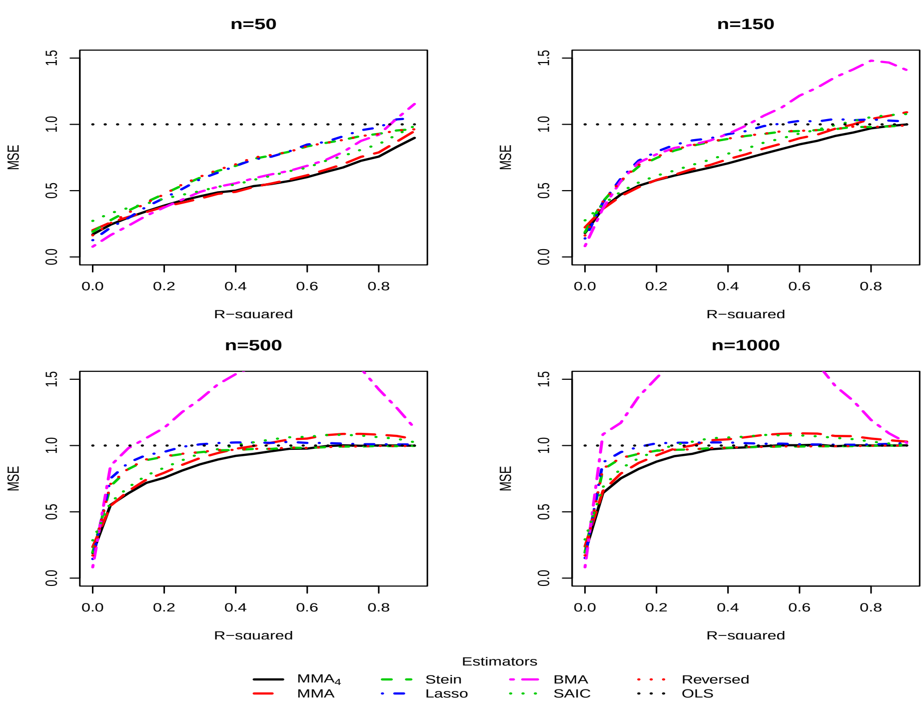

Figure 10: Model 3: $\alpha = 1$

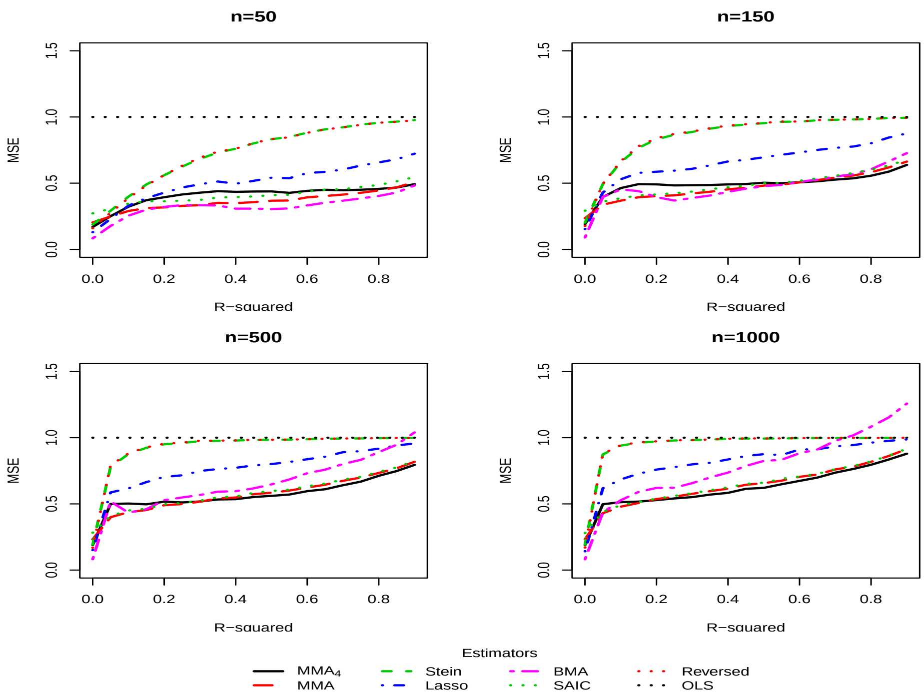

Figure 11: Model 3: $\alpha = 2$

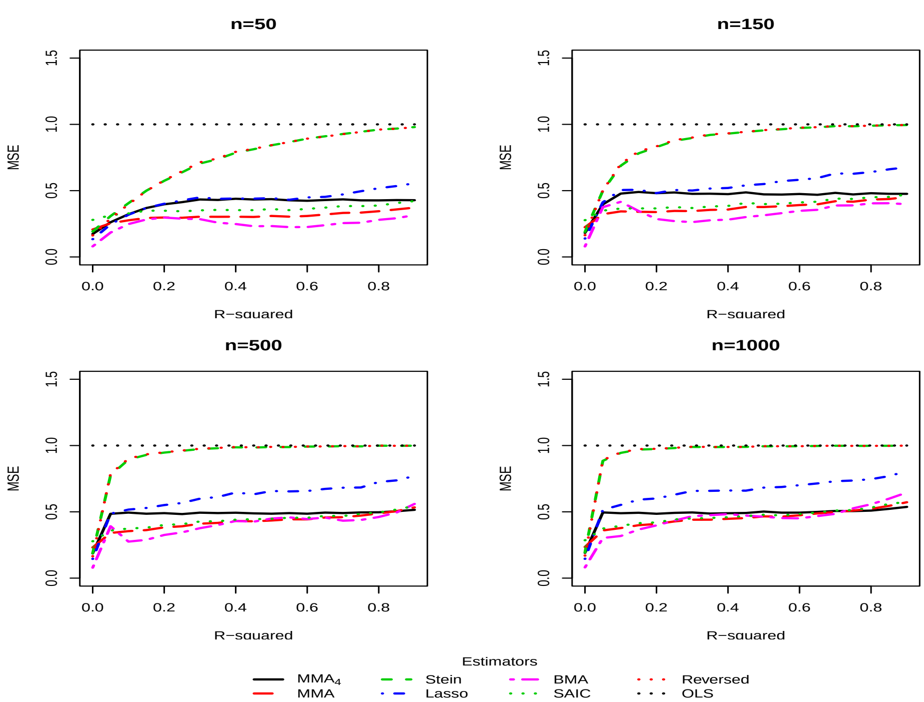

Figure 12: Model 3: $\alpha = 3$

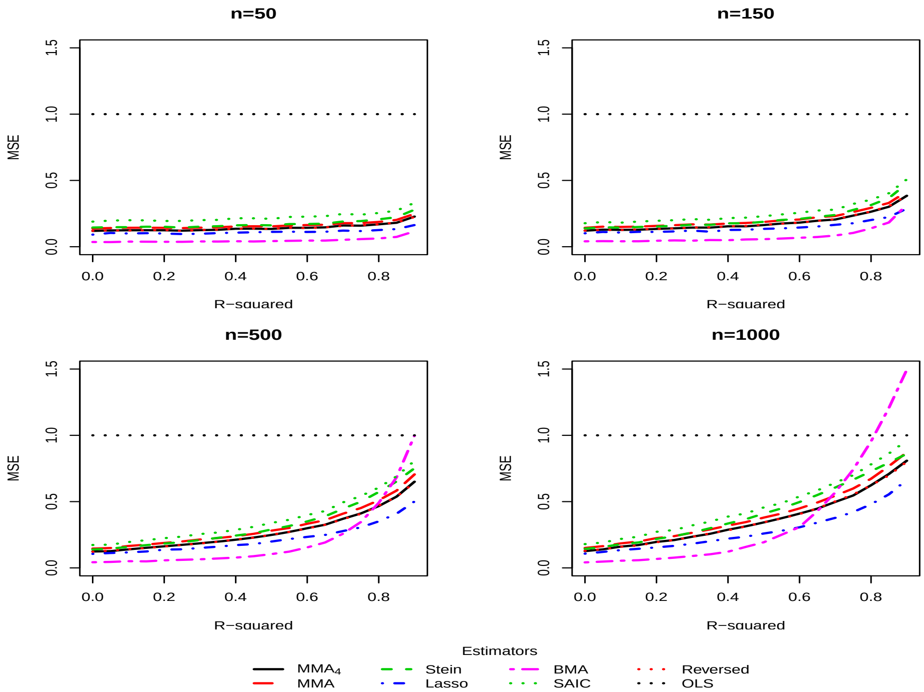

Figure 13: Model 4: $\alpha = 0$

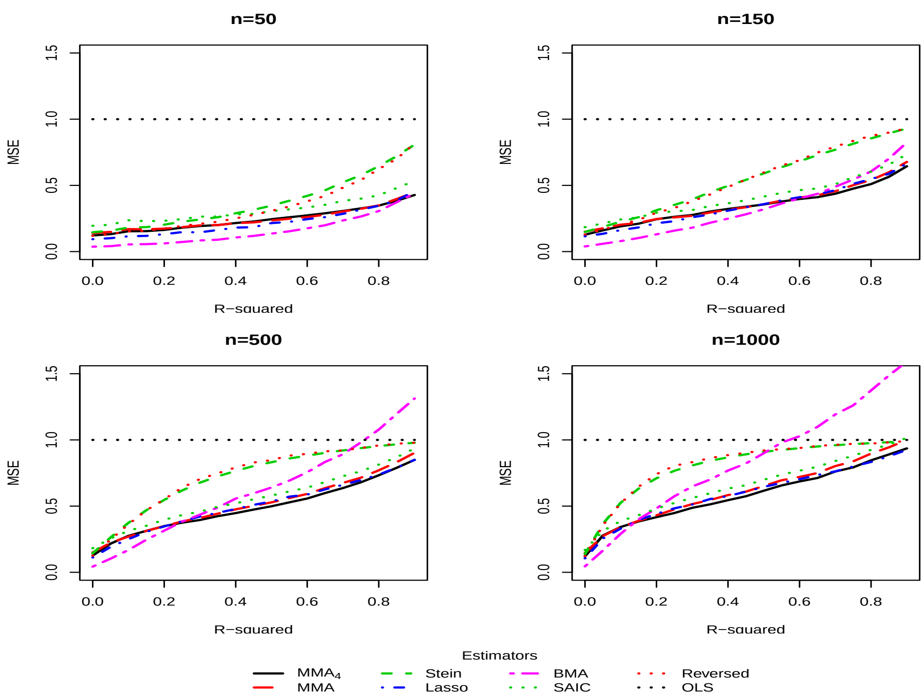

Figure 14: Model 4: $\alpha = 1$

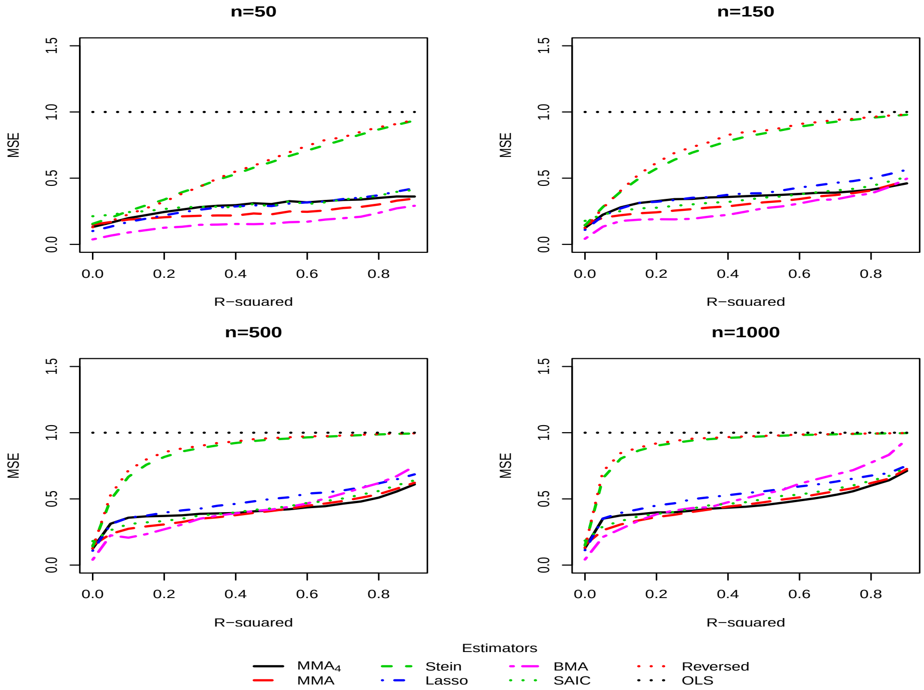

Figure 15: Model 4: $\alpha = 2$

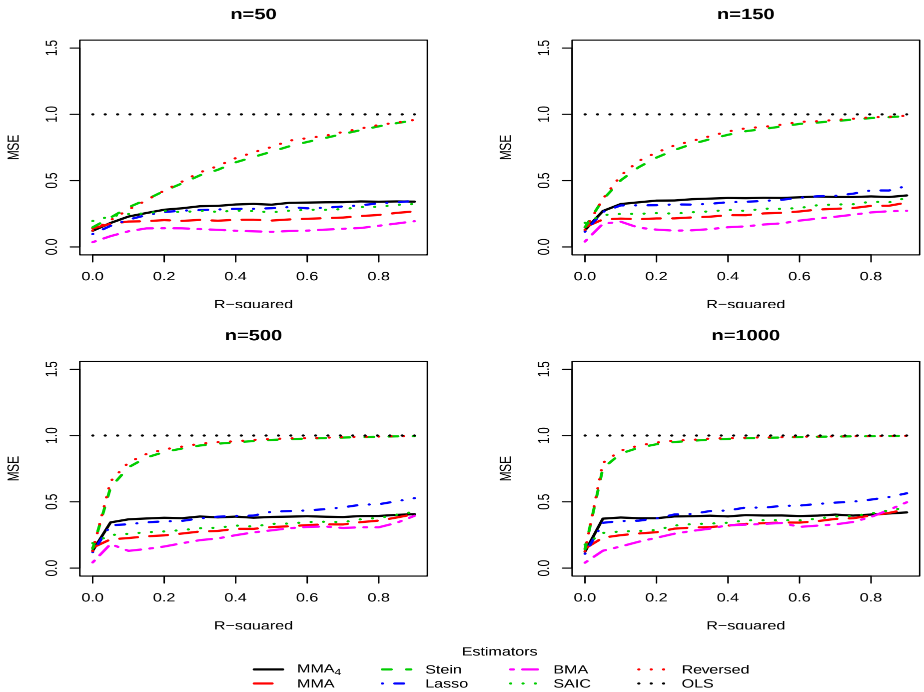

Figure 16: Model 4: $\alpha = 3$

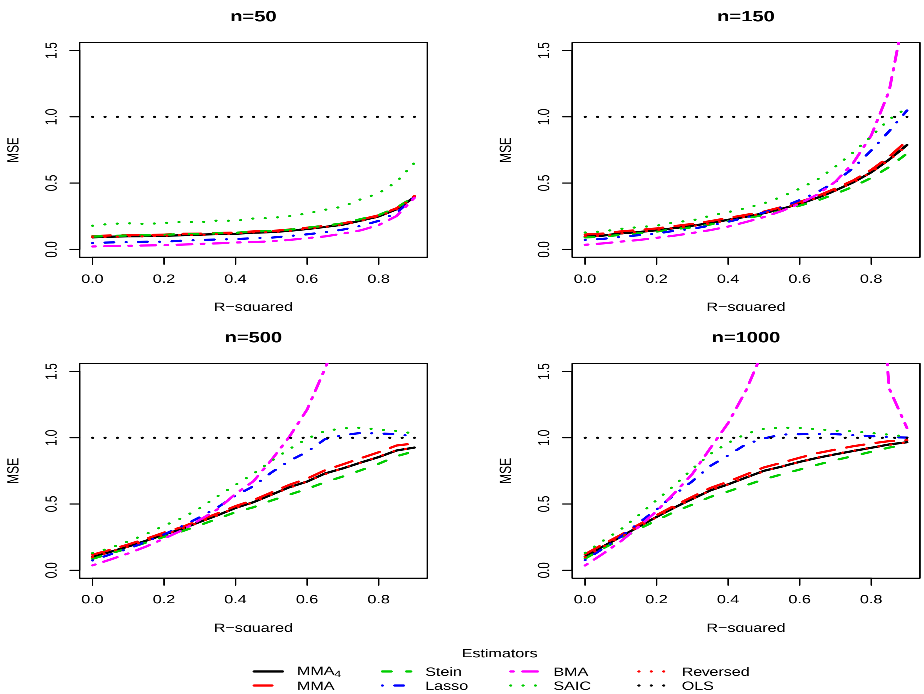

Figure 17: Model 5: $\alpha = 0$

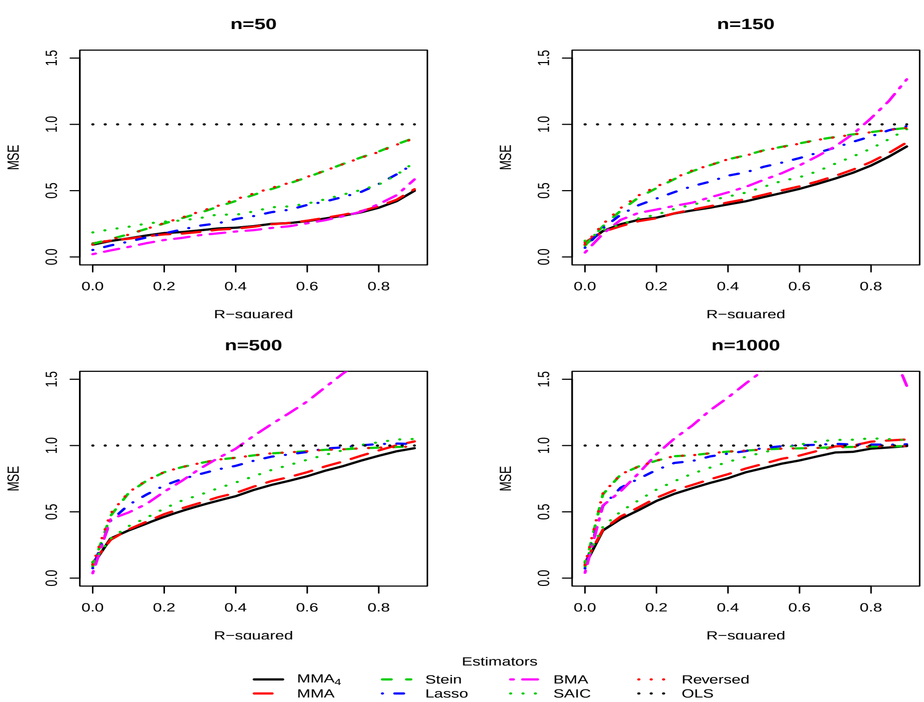

Figure 18: Model 5: $\alpha = 1$

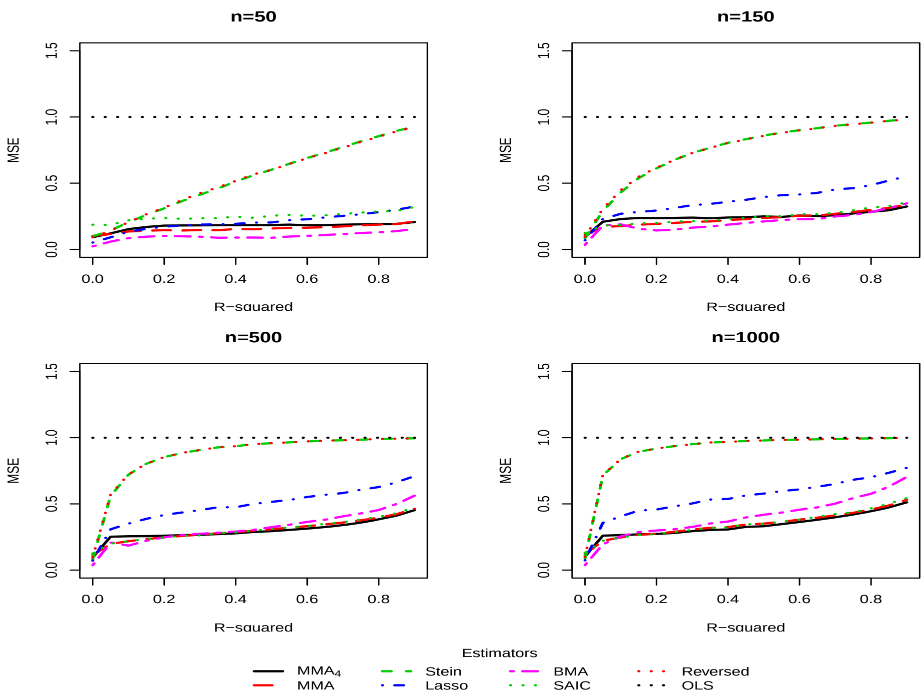

Figure 19: Model 5: $\alpha = 2$

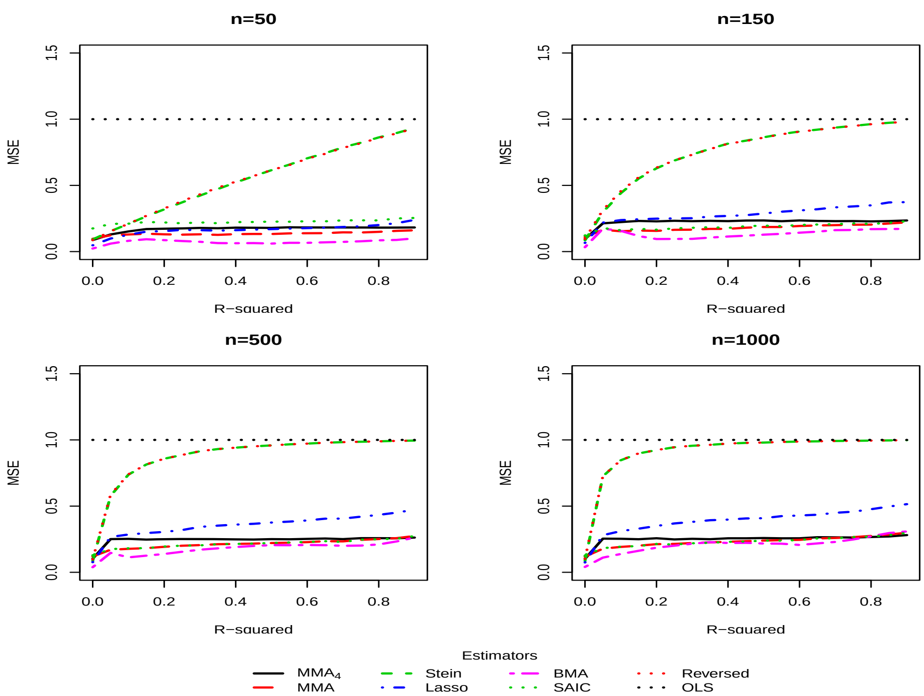

Figure 20: Model 4: $\alpha = 3$

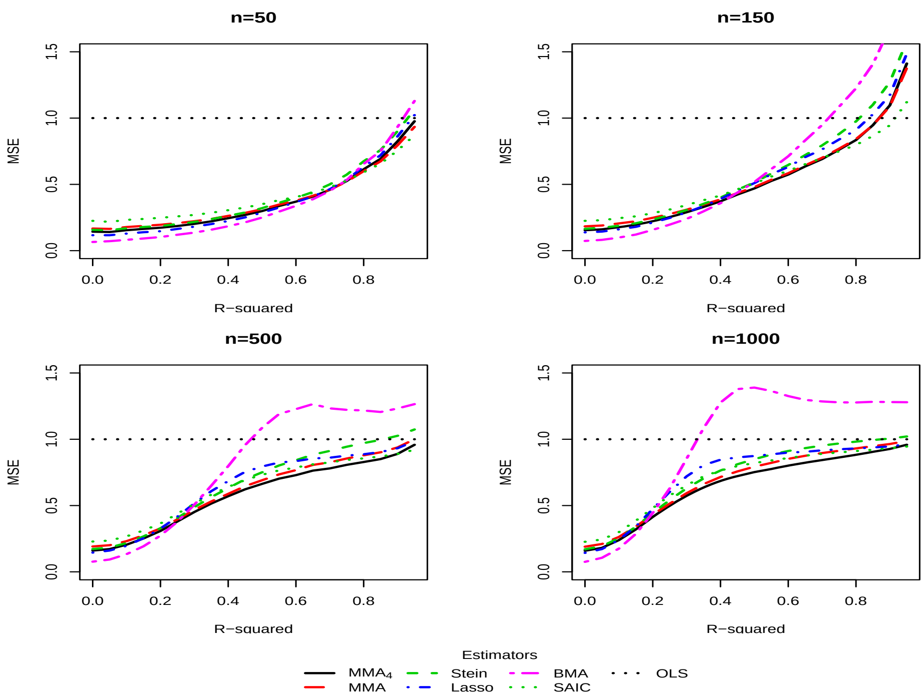

Figure 21: Model 6: Autoregression