# Time Series Econometrics For the 21st Century

by

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# Overview

- Most U.S. undergraduate economic students do not pursue PhDs
- Many work for firms and government
- They see, work with, and analyze time-series data
- Time-series tools are useful for such work

# Core Models

- Autoregressive (AR)

$$y_t = \alpha + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + e_t$$

- Regression

$$y_t = \alpha + \delta_0 x_t + e_t$$

- Distributed Lag (DL)

$$y_t = \alpha + \delta_0 x_t + \delta_1 x_{t-1} + \cdots + \delta_q x_{t-q} + e_t$$

- Autoregressive-Distributed Lag (ADL)

$$y_t = \alpha + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \delta_0 x_t + \delta_1 x_{t-1} + \cdots + \delta_q x_{t-q} + e_t$$

# Core Insights for Core Time-Series Models

1. The coefficients can be estimated by OLS
2. The appropriate standard error (robust or HAC) is important
3. The AR model helps understand serial correlation.
4. The coefficients in the DL and ADL can be interpreted as multipliers.
5. Multipliers are structural under exogeneity
6. Lags may be selected by AIC, not testing.
7. Spurious regression
8. Parameter change.
9. Use ADL model for one-step-ahead point forecasts.
10. Point forecasts should be combined with interval forecasts.
11. Multi-step forecasts can use a multi-step ADL model
12. Multi-step point forecasts should be accompanied by fan charts.

# Standard Errors

- Three methods popular for time-series
  - Classical (homoskedastic)
  - Robust (Heteroskedastic)
  - HAC (Newey-West)
- Classical (old-fashioned) are not used in contemporary economics.
  - Should only be taught as a stepping stone
- Robust
  - Appropriate for AR and ADL
  - Inappropriate for non-dynamic regression or DL
- HAC
  - Important for regression or DL

# Illustration

- Weekly retail gasoline and crude oil prices
- Three standard errors: old-fashioned, robust, and HAC (appropriate)

$$\begin{array}{rccc} gas_t = & 0.029 & + & 0.269 \quad oil_t + \widehat{e}_t \\ & (0.046) & & (0.011) \\ & (0.046) & & (0.015) \\ & (0.073) & & (0.021) \end{array}$$

# t-statistics versus standard errors

- Always report coefficient estimates & standard errors
- Never coefficient estimates & t-statistics
- Standard errors convey degree of uncertainty – always important
- t-statistics concern testing hypothesis of a zero coefficient – rarely of key interest
- De-emphasize significance testing in favor of measurement and analysis

# Autoregressive Models

- Useful for understanding dynamics
- Illustration: U.S. quarterly real GDP growth rates, post-war

$$\begin{aligned} GDP_t &= \underset{(0.32)}{1.93} + \underset{(0.06)}{0.34}\, GDP_{t-1} + \underset{(0.06)}{0.13}\, GDP_{t-2} \\ &\quad - \underset{(0.06)}{0.09}\, GDP_{t-3} + \widehat{e}_t \end{aligned}$$

# Illustration

- Weekly return on S&P 500
- Useful to illustrate test of efficient market hypothesis
- Also illustrates importance of correct (robust) standard errors, otherwise test will falsely reject.

$$\underset{}{return_t} = \underset{(0.04)}{0.16} - \underset{(0.029)}{0.032}\, return_{t-1} + \underset{(0.025)}{0.037}\, return_{t-2} + \widehat{e}_t$$

# Distributed Lag Models

- Useful for understanding multipliers
- Illustration: retail gasoline and crude oil prices

$$
\begin{aligned}
gas_t = & \underset{(0.057)}{-0.009} + \underset{(0.016)}{0.243}\, oil_t + \underset{(0.012)}{0.112}\, oil_{t-1} + \underset{(0.011)}{0.063}\, oil_{t-2} \\
& + \underset{(0.013)}{0.064}\, oil_{t-3} + \underset{(0.010)}{0.030}\, oil_{t-4} + \underset{(0.011)}{0.032}\, oil_{t-5} + \underset{(0.012)}{0.018}\, oil_{t-6}
\end{aligned}
$$

# Auto-Regressive Distributed Lag Models

Phillips Curve: U.S. quarterly inflation and unemployment rate

$$\begin{aligned}
\Delta Inf_t = & \underset{(0.42)}{0.44} - \underset{(0.11)}{0.34}\,\Delta Inf_{t-1} - \underset{(0.09)}{0.39}\,\Delta Inf_{t-2} \\
& - \underset{(0.11)}{0.02}\,\Delta Inf_{t-3} - \underset{(0.07)}{0.17}\,\Delta Inf_{t-4} - \underset{(0.56)}{1.53}\,UR_{t-1} \\
& + \underset{(1.06)}{1.58}\,UR_{t-2} + \underset{(1.03)}{0.11}\,UR_{t-3} - \underset{(0.47)}{0.23}\,UR_{t-4} + \widehat{e}_t
\end{aligned}$$

# Model Selection

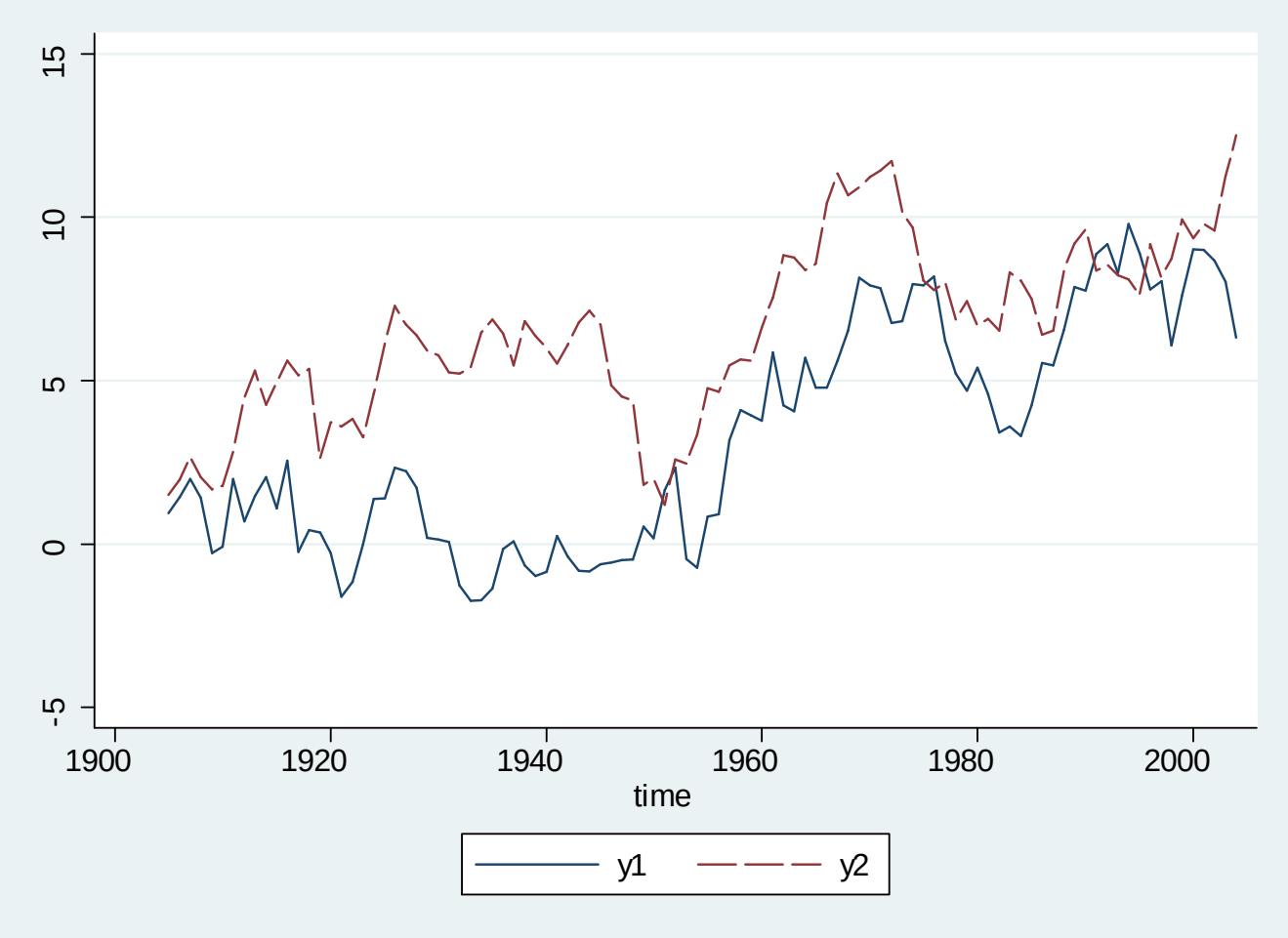
- Economic theory does not inform about lag structure
- In practice, choice implies a bias-variance trade
- Akaike Information Criterion (AIC) is a simple practical tool to compare models
- Testing (t and F) is appropriate for assessing economic hypotheses
- Testing is inappropriate for model selection

# Spurious Regression

- Could be the single most important insight we can teach our students
- Spurious regressions are commonplace
- Teach students to recognize serial correlation and exercise caution when interpreting regressions

# Illustration: Two Annual Series

$$\underset{}{y_{1t}} = \underset{(0.52)}{-2.95} + \underset{(0.07)}{0.95}\ y_{2t} + \widehat{e}_t, \qquad R^2 = 0.54$$



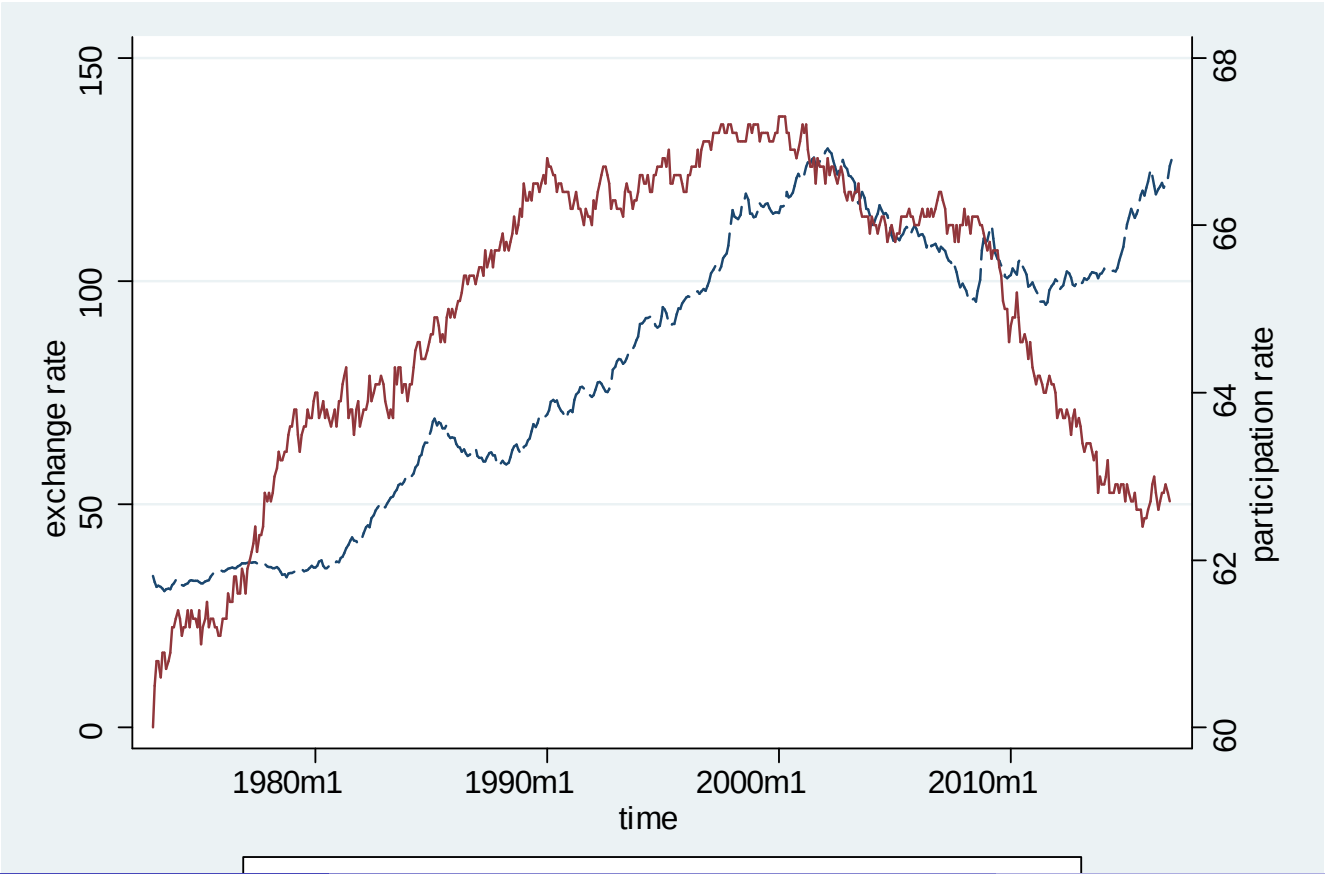
# Spurious!

Both series were generated as independent random walks

# Trump-Era Exchange Rate Theory

- Outsourcing causes displaced workers to be discouraged and leave labor force, and the production shift alters the exchange rate

- $ExchangeRate_t = \underset{(0.56)}{10.2}\ LaborParticipation_t \qquad R^2 = 0.35$



# How to Detect a Spurious Regression

- Dependent variable is highly serially correlated
- Simple regression (no lagged dependent variable)
- Solution: Include at least one lagged dependent variable

$$
\begin{aligned}
Ex_t = \underset{(0.05)}{1.43}\, Ex_{t-1} - \underset{(0.08)}{0.59}\, Ex_{t-2} + \underset{(0.09)}{0.18}\, Ex_{t-3} \\
- \underset{(0.05)}{0.05}\, Ex_{t-4} - \underset{(0.032)}{0.013}\, LaborParticipation_t
\end{aligned}
$$

# Structural Change

Example: U.S. Real GDP Growth Rates

| | Mean | Standard Deviation | AR(1) Coefficient |
|---|---|---|---|
| 1947-1956 | 4.0 | 5.3 | 0.44 |
| 1957-1976 | 3.6 | 4.2 | 0.30 |
| 1977-1996 | 3.2 | 3.5 | 0.31 |
| 1997-2016 | 2.3 | 2.5 | 0.41 |

# Forecasting

- Using $h$-step ADL for multi-step point forecasts
- Example: Inflation given unemployment rates
- Estimates

$$\begin{aligned}\pi_{t+h} - \pi_t &= \widehat{\alpha} + \widehat{\phi}_1 \Delta\pi_t + \cdots + \widehat{\phi}_p \Delta\pi_{t-p+1} \\ &\quad + \widehat{\delta}_1 UR_t + \cdots + \widehat{\delta}_q UR_{t-q+1}.\end{aligned}$$

- Point Forecasts

$$\begin{aligned}\widehat{\pi}_{n+h} &= \pi_n + \widehat{\alpha} + \widehat{\phi}_1 \Delta\pi_n + \cdots + \widehat{\phi}_p \Delta\pi_{n-p+1} \\ &\quad + \widehat{\delta}_1 UR_n + \cdots + \widehat{\delta}_q UR_{n-q+1}\end{aligned}$$

# Forecast Intervals

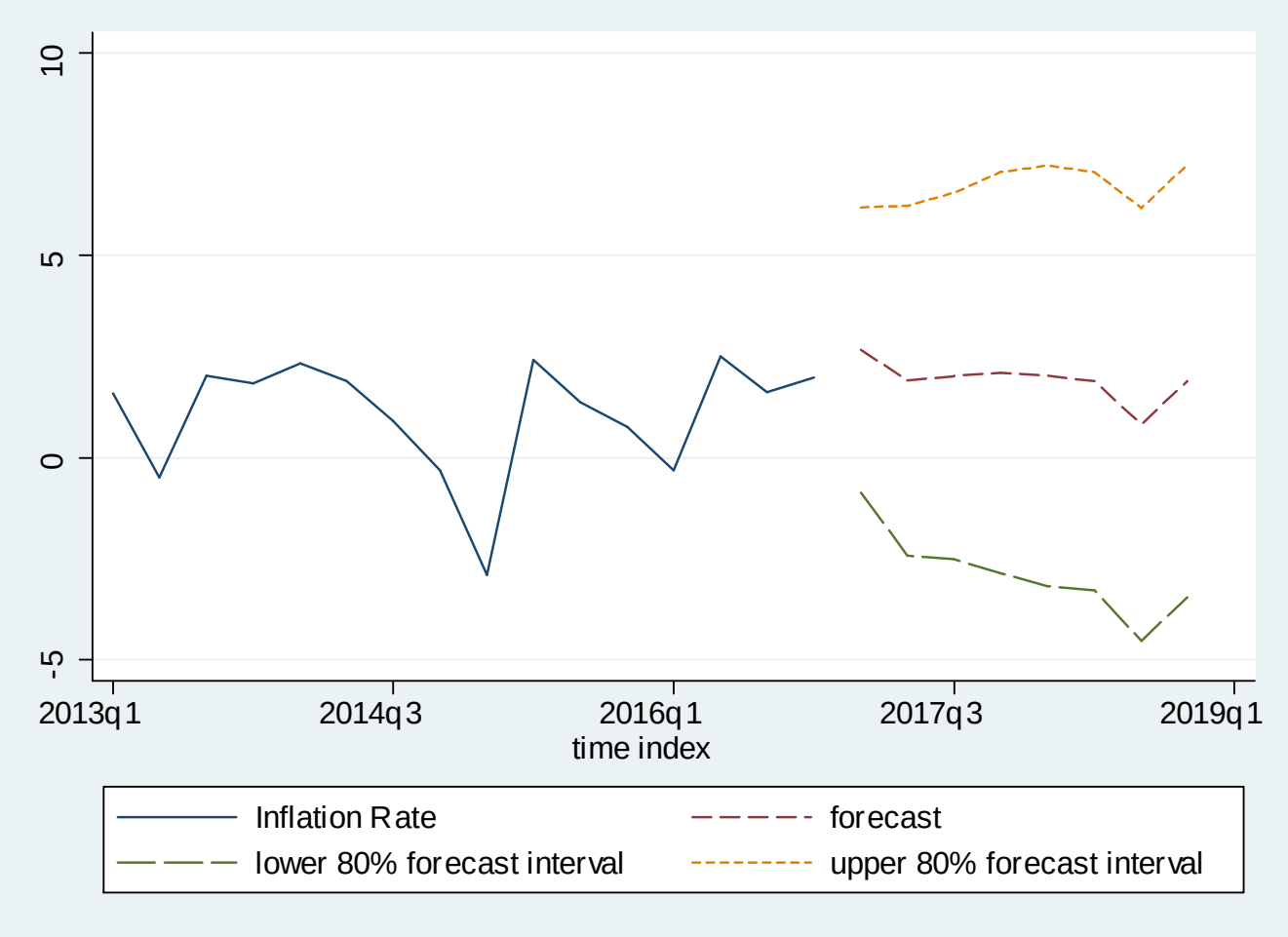
- Uncertainty should be emphasized
- Noise typically dominates signal
- Emphasis on forecasts intervals
- Simple normal approximation interval

$$\widehat{\pi}_{n+h} \pm \widehat{s}_{n+h} z_{1-\alpha/2}$$

- $\widehat{s}_{n+h}$ is the standard error of the forecast
- $\widehat{s}_{n+h} \approx \left(n^{-1} \sum_{t=1}^{n} \widehat{e}_t^2\right)^{1/2}$

# Fan Charts

- Multi-step forecasts are elegantly presented using fan charts
- Illustration: U.S. quarterly inflation using estimated Phillips curve



# Conclusion

- Time-series should be part of econometrics curriculum
- Emphasis on core models used in applications