Specialization in Education

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Abstract

Private markets promote greater diversity in educational opportunities. Emphasis is placed on the central role that teachers play in producing human capital. Students have different needs and teachers vary in characteristics. An assignment process is developed where students and teachers get matched together. A private system creates better incentives for teachers to acquire the skills that students want. This allows for teachers and students to be matched together better, and hence promotes growth. Though the private economy entails higher inequality within a generation and persistence in human capital across generations, it permits a greater degree of economic mobility from one generation to the next.

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JEL Classification: D63, J13, J41.

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1 Introduction

The American education system is in a state of flux. Discontent with the existing education system is rising. Challenges to the efficiency of the existing system and huge disparities in earnings have led to a serious consideration of the possibility of privatization of education at the primary and secondary levels. A growing body of evidence suggests that interventions beyond the age of fourteen do little to improve an individual's prospects (Heckman 1998). This makes primary and secondary education extremely important tools with which to improve the prospects of the poor. Given the various problems confronting the public education system, it is hardly surprising that the option of private schooling is receiving increased attention.

Even though much has been said about public and private investment in education, there has been little systematic analysis of the supply-side (teachers) of the schooling sector, and of the equity and efficiency aspects of varying supply. This is quite surprising, given that the available evidence is quite strong on one point: teachers exert a considerable influence on the achievement levels of students. To quote Hanushek (1981), "the only reasonably consistent finding seems to be that 'smarter' teachers do better in terms of student achievement." More recently, Rivkin et. al. (1998) provide strong evidence in support of the view that 'teacher effects' are extremely important in driving student achievement. They demonstrate that the effects of schools are mostly driven by variations in teacher quality.[1] This paper fills this void in the analysis of the supply-side, by emphasizing the central role that the human capital of the teacher plays in the *production* of human capital. This is the primary objective of this paper. This immediately raises the following questions: How does one model the supply-side? What would be the effects of a systematic sorting of students to teachers on long-run inequality? If inequality, per se, is harmful for growth and the private economy leads to a higher degree of inequality, will the private

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[1] Further evidence in support of the view that the cognitive skills of teachers influence the learning of students is contained in Ballou and Podgursky (1997) and Hanushek (1971).

economy experience lower long-run growth? Further, if the private economy leads to a higher degree of persistence in human capital, does it exhibit a lower degree of intergenerational mobility? The model presented is well equipped at addressing these issues.

A dynamic general equilibrium assignment model of education is formulated in order to compare public and private systems of *provision* of education. There are two types of agents: workers and teachers. They live for two periods. In the first period, they are attached to their parents and accumulate human capital. An individual's human capital is determined by his own ability, which is idiosyncratic in nature, the human capital of his parent, and the human capital of his teacher. He starts the second period with the human capital acquired, bequests received, if any, and a child with an intrinsic ability that is random. The underlying economic structure for intergenerational transfers follows the tradition set by Becker and Tomes (1981) and Loury (1981). Given that students have different needs and teachers vary in quality, the economy is confronted with a non-trivial assignment problem. An assignment process, along the lines of Tinbergen (1951) and Sattinger (1975), is then explicitly modeled, through which students are assigned to teachers. The assignment rule is then decentralized so as to be consistent with the individual's decision problem. The economy is then cast in an endogenous growth framework along the lines of Lucas (1988) where investments in education spur economic growth. The assignment process is shown to be consistent with a balanced growth path. Two cases are shown to emerge: one in which the assignment process is sustained across time in the face of renewed assignment and another in which the assignment process can, on its own, spur economic growth.

An analysis of the assignment process reveals that the degree of complementarity between student inputs and teacher human capital plays a key role. For instance, a rise in the degree of complementarity would increase wage inequality among teachers. This would, in turn, increase wage inequality among workers in the next generation,

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through the assignment process. The next generation of teachers now have to cater to an even more varied set of students, and will accumulate human capital accordingly. This would have a rippling effect on the economy and, on net, increase wage inequality.

The basic thesis of this paper is that a private market for education promotes the *specialization* of teachers in various skills. Parents would like to have schools tailored to their needs and, thereby, move closer to the desired level of education for their children. Essentially, private markets foster diversity in educational opportunities for students and provide the right incentives for teachers to accumulate human capital. A comparison of the public and private education economies reveals an immense trade-off between growth and inequality. The private system promotes growth by virtue of being better able to match student needs and teacher skills. However, it simultaneously exacerbates inequality by efficiently rewarding merit and talent. This doesn't necessarily imply that the public system is preferable on grounds of equity. While the private system amplifies the persistence in human capital across generations, it also enhances the absolute impact of a child's ability in determining his human capital. This trade-off is captured in an analysis of social mobility.

The analysis of the balanced growth path distinguishes itself from most existing work. The model allows for the possibility that individuals who possess different levels of ability and parental human capital grow at different rates. The growth path is characterized by a non-degenerate cross-sectional distribution of growth rates. This statistic has economic content. It transpires that there is a direct connection between the cross-sectional distribution of growth rates and economic mobility. In a special case, intergenerational mobility turns out to be a measure of *inequality in growth rates*. The analysis further reveals that while the private economy entails higher inequality within a generation, it also promotes a greater degree of social mobility from one generation to the next. While the private economy leads to a greater degree of *persistence* in status across generations, it also increases the importance of ability, the *random* component, in determining a child's earnings. Therefore, equalizing

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opportunity does indeed reduce economic mobility.

The impact of heterogeneity, sorting, and different systems of education has been the subject of several recent papers. Fernandez and Rogerson (1999) and Kremer (1997) examine the impact of sorting on long-run inequality. Caucutt (1997), Durlauf (1996), Epple and Romano (1999), and Fernandez and Rogerson (1996) examine the effects of neighborhood and school sorting. Galor and Zeira (1993) examine the effect of borrowing constraints on aggregate output and investment. Cooper (1992) presents a model where redistribution of human capital expenditures can come about voluntarily. Two other papers, to which this work is closely related examine the effects of public and private systems on long-run growth. Glomm and Ravikumar (1992) suggest that a private economy will grow faster than a public economy since the former provides individuals with the right incentives to accumulate human capital. Several papers have since argued that in the presence of capital market imperfections, inequality is harmful for growth and hence sorting can be detrimental to long-run growth (see, for instance, a very interesting analysis in Benabou, 1996). In all of the above work, the sorting mechanism is exogenous. The key difference between this paper and all of the above lies in endogenizing the sorting mechanism.

Building on these papers, the analysis here shows that an explicit consideration of the *endogenous* nature of the assignment process that leads to the higher inequality, together with the *incentives* provided to teachers to accumulate human capital could likely lead to the conclusion that the private economy grows at a faster rate than the public economy. On the one hand, complementarity between individual ability and teacher quality means that the private system, being more responsive to the needs of parents, will facilitate a better match between students and teachers. This also leads to more heterogeneity. Heterogeneity, however, pulls down growth because each dynasty exhibits diminishing returns to human capital production. This, in turn, argues for some degree of equalization. Equality of opportunity, however, does not come free! Such a cost is captured by the resources needed to be expended

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for all teachers to attain some minimum standard. The analysis reveals that the gains from positive assignment exactly cancel out the losses from the higher degree of heterogeneity. Whether long-run growth is higher in the private economy will then depend entirely on how high the government can set and maintain standards for teachers in the public economy. This suggests that taking into account the incentives provided to teachers to accumulate and supply their services to the education sector can be vital to an examination of the long-run properties of the public and private education economies.

2 Environment

The basic environment is a discrete-time overlapping generations economy. There are four types of players: workers, teachers, government and firms. There is a dynasty of workers and a dynasty of teachers, each of measure one. Workers and teachers live for two periods, the first as a child, the second as an adult. In the first period of life, they receive education and accumulate human capital. The human capital that a worker acquires is a function of his intrinsic ability, the human capital of his parent and the human capital of his teacher. He begins the second period of his life with the human capital acquired in the first period, bequests from parents, and a child with an intrinsic ability that is random in nature. Intrinsic ability is uncorrelated across generations and is observed by parents before they undertake decisions. A parent cannot purchase insurance against the ability of his grandchild. There are, however, no borrowing constraints. All individuals are endowed with one unit of working time. Workers earn a wage proportional to their acquired human capital, and decide how much to consume, spend on the education of their children and to leave for them in bequests.

Tastes: An individual's momentary utility function is given by $U(c_t)$. The function $U(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, satisfies

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$\lim_{c_t \to 0} U_1(c_t) = \infty$. Further, $U(\cdot)$ is bounded below.[2] Preferences are assumed to be time-additive separable. This is done for two reasons: some results on the income-fluctuation problem, that are used in this paper, rely explicitly on time-additive separable preferences (see Aiyagari 1995); and preferences of the general recursive variety are extremely difficult to sustain along an endogenous growth path. Further, assume that parents are altruistic towards their children. Each adult has preferences of the form

$$U(c_t) + \beta E[J_{t+1}],$$

where c_t is his consumption when old and J_{t+1} denotes the lifetime utility that his child will realize upon growing up.

Ability: Children differ according to their ability a_t. A child's ability is drawn from the distribution function $A : [a_{\min}, a_{\max}] \to [0, 1]$.

Human Capital Production: The human capital that a worker acquires is given by

$$h_{t+1} = X\left(s_t, \widetilde{h}_t\right). \tag{1}$$

Here, $s_t = a_t h_t$ denotes his learning ability, where a_t is his intrinsic ability and h_t stands for his parent's human capital. Further, $\widetilde{h}_t$ stands for the human capital of his teacher.[3] Assume that $X(\cdot, \cdot)$ is strictly increasing and strictly concave in each of its arguments. Observe that the human capital of the parent is an input in the production of the human capital for the child. A high human capital parent could spend 'quality time' with his child and this enhances his child's learning.[4] A second interpretation is that a parent with a higher human capital could provide a better

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[2]The assumption that the momentary utility function is bounded below guarantees that the dynamic program is well posed. This rules out, for instance, logarithmic preferences. See Duran (1999), for results on stochastic dynamic programming with non-compact state spaces.

[3]In the entire paper, a variable with a tilda above it refers to a teacher-specific variable.

[4]Evidence in support of a 'quality-time' specification is contained in Smith, Brooks-Gunn, and Klebanov (1997). They demonstrate that a mother's education has a statistically significant impact on her child's cognitive ability even after controlling for family income.

neighborhood and surroundings which would complement a child's learning at school (see Jencks and Mayer, 1990). It is useful to think of s_t and h_t as representing an individual's 'ability to learn' and 'ability to earn' respectively.

Another feature of equation (1) is that teachers serve as the engine of endogenous growth. This paper follows the tradition set by Lucas (1988) in modeling endogenous growth through human capital accumulation. Since investments in teachers spur growth, it is important at the very outset to defend this assumption on empirical grounds. In the late nineteenth century, women in the US were discriminated against from entering into professions such as medicine and law. In face of such severe labor market conditions, many talented women took up teaching at public schools (Carter, 1986). Both the quantity as well as quality of schooling rose, and this contributed, in part, to the high growth rates experienced by the US economy in the early part of the twentieth century (Engerman, 1971).

A key assumption on the function $X\left(\cdot,\cdot\right)$ is that $X_{12} > 0$. Individual learning ability and teacher human capital are complementary in 'producing' human capital. Complementarity captures the idea that higher learning ability children lose more from a marginal decline in teacher quality than lower learning ability children might gain from a corresponding marginal increase. As a consequence, any assignment that does not exhibit strictly positively matching is inefficient as a reassignment would increase aggregate human capital. Note that it is the human capital of the teacher, and not educational expenditures per se that influences the human capital of the child.[5]

Assume that the underlying teaching technology is such that one student needs to be educated by one teacher. This restricts the set of feasible assignment rules to the one-to-one variety. This assumption is not essential to the main results of the paper but simplifies the analysis. Further, the evidence about improvements in student

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[5]The implications of aggregate spillovers in the human capital production function are explored in section 5. Aggregates will nevertheless affect individual decisions through relative prices.

achievement that can be attributed to smaller classes is meager and unconvincing (see Hanushek, 1999).

The human capital that a teacher acquires is a function of his learning ability and the expenditures undertaken. Underlying this specification is the assumption that the human capital accumulation process for teachers takes the form of a consumption cost. Denote these goods inputs into a teacher's child by $\widetilde{e}_{t+1}$.[6] Then the human capital that a teacher acquires is given by

$$\widetilde{h}_{t+1} = \widetilde{X}\left(\widetilde{a}_t \widetilde{h}_t, \widetilde{e}_{t+1}\right). \tag{2}$$

Final Goods Production: Final goods are produced according to a constant-returns-to-scale production function $F\left(\mathbf{k_t} + \widetilde{\mathbf{k}}_t, \mathbf{h}_t\right)$. Here, $\mathbf{k}_t$ and $\widetilde{\mathbf{k}}_t$ denote aggregate physical capital hired from workers and teachers respectively, while $\mathbf{h}_t$ stands for aggregate human capital hired from workers. The function $F\left(\cdot, \cdot\right)$ is assumed to be strictly increasing and strictly concave.

2.1 The Private Education Economy

In the private economy, parents have complete freedom to choose the teacher they want their child to be educated by. Further, teachers are free to 'set up' schools of their own in order to sell their services to the diverse population of students. The provision of education is taken care of directly by teachers. The parent then shops around for a teacher. An exact description of the assignment process is postponed to Section 3.

Workers: There are a continuum of workers distributed uniformly on the unit interval. Workers care about the welfare of their progeny in the sense of 'pure' altruism. The

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[6]This form for a production function essentially ensures that teacher's children are 'kept out' of the assignment problem. Such a specification is not essential to the main results in the paper.

dynamic-programming problem faced by a worker is given by

$$J\left(b_t, h_t, a_t; Z_t, \widetilde{Z}_t\right) = \max_{c_t, b_{t+1}, \widetilde{h}_t} \left\{ U\left(c_t\right) + \beta \int J\left(b_{t+1}, h_{t+1}, a_{t+1}; Z_{t+1}, \widetilde{Z}_{t+1}\right) dA\left(a_{t+1}\right) \right\},$$

$$(3)$$

subject to the constraints

$$c_t + b_{t+1} + p_t\left(\widetilde{h}_t\right) = w_t h_t + b_t\left(1 + r_t\right),$$

and equations (1), (**??**) and (**??**). Here, $p_t\left(\widetilde{h}_t\right)$ is the tuition cost that a parent incurs in order to have his child educated by a teacher of human capital $\widetilde{h}_t$. Further, Z_t stands for the joint distribution of worker human capital and assets, while $\widetilde{Z}_t$ is the corresponding statistic for teachers.

The Euler equation governing teacher quality is given by

$$\frac{p_{1t}\left(\widetilde{h}_t\right)}{\beta X_2\left(s_t, \widetilde{h}_t\right)} = \underbrace{\left(1 - \tau_h\right) w \int \frac{U_1\left(c_{t+1}\right)}{U_1\left(c_t\right)} dA\left(a_{t+1}\right)}_{Wage\ Effect} + \underbrace{\int p_{1t}\left(\widetilde{h}_{t+1}\right) \frac{X_1\left(s_{t+1}, \widetilde{h}_{t+1}\right)}{X_2\left(s_{t+1}, \widetilde{h}_{t+1}\right)} a_{t+1} \frac{U_1\left(c_{t+1}\right)}{U_1\left(c_t\right)} dA\left(a_{t+1}\right)}_{Intergenerational\ Effect}.$$

$$(4)$$

Observe that the accumulation of human capital serves two roles: first, an extra unit of human capital acquired by an individual today yields him an additional income of w_t tomorrow and second, it aids in the intergenerational transmission of human capital.

Teachers: Teachers have an incentive to accumulate and supply their services to the education sector since they get paid on the basis of their human capital. The dynamic-programming problem facing a teacher is given by

$$\widetilde{J}\left(\widetilde{b}_t, \widetilde{h}_t, \widetilde{a}_t; Z_t, \widetilde{Z}_t\right) = \max_{\widetilde{c}_t, \widetilde{b}_{t+1}, \widetilde{e}_{t+1}} \left\{ U\left(\widetilde{c}_t\right) + \beta \int \widetilde{J}\left(\widetilde{b}_{t+1}, \widetilde{h}_{t+1}, \widetilde{a}_{t+1}; Z_t, \widetilde{Z}_{t+1}\right) dA\left(\widetilde{a}_{t+1}\right) \right\},$$

$$(5)$$

subject to the constraints

$$\widetilde{c}_t + \widetilde{b}_{t+1} + \widetilde{e}_{t+1} = p_t\left(\widetilde{h}_t\right) + \widetilde{b}_t\left(1 + r_t\right),$$

and equations (2), (**??**), and (**??**). The Euler equation governing educational expenditures is given by

$$\frac{U_1\left(\widetilde{c}\right)}{\beta \widetilde{X}_2\left(\widetilde{s},\widetilde{e}'\right)} = \int U_1\left(\widetilde{c}'\right) p_1\left(\widetilde{h}'\right) d\widetilde{A}\left(\widetilde{a}'\right) + \int \frac{\widetilde{X}_1\left(\widetilde{s}',\widetilde{e}''\right)}{\widetilde{X}_2\left(\widetilde{s}',\widetilde{e}''\right)} \widetilde{a}' U_1\left(\widetilde{c}'\right) d\widetilde{A}\left(\widetilde{a}'\right).$$

Firm: The firm's optimization problem is given by

$$\max_{\mathbf{k}_t, \widetilde{\mathbf{k}}_t, \mathbf{h}_t} \left\{ F\left(\mathbf{k_t}+\widetilde{\mathbf{k}}_t, \mathbf{h}_t\right) - r_t\left(\mathbf{k_t}+\widetilde{\mathbf{k}}_t\right) - w_t \mathbf{h}_t \right\}.$$

The first-order conditions for the firms problem are $r_t = F_1\left(\mathbf{k_t}+\widetilde{\mathbf{k}}_t, \mathbf{h}_t\right)$ and $w_t = F_2\left(\mathbf{k_t}+\widetilde{\mathbf{k}}_t, \mathbf{h}_t\right).$

Competitive Equilibrium: The competitive equilibrium for the private education economy is reasonably standard and hence a precise definition is omitted. Essentially, households and firms solve their optimization problems, markets clear and the assignment of students to teachers must be feasible.

3   The Assignment Problem

This section analyzes the assignment problem in detail and demonstrates how the assignment rule may be decentralized and the price function for teachers computed. It helps to bridge the gap between the first two sections and the analysis of the balanced growth path by demonstrating how the assignment rule can be decentralized. The static problem of assigning students to teachers, to be detailed below, closely follows the assignment problem analyzed in Sattinger (1975) and Jovanovic (1998). Such a problem was first considered by Tinbergen (1951) where the assignment is in 'fixed proportions'. The assignment rule must be feasible, consistent with the efficiency conditions governing the worker's dynamic programming problem, and be sustained across time along a balanced growth path in face of renewed assignment. The price function for teachers must be such that, given the price function and interest and wage

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rates, the consumer solves his optimization problem and the resulting demand for teachers exactly coincides with that implied by the assignment in 'fixed proportions'.

Since teachers get paid on the basis of their human capital, the best teacher attracts the best student and earns the highest possible income. The next best teacher gets the next best student and so on. The teacher with the lowest human capital gets to teach the lowest learning ability student at the lowest possible wage. Denote the distribution functions for worker and teacher human capital by $\Psi\left(h_t\right)$ and $\widetilde{\Psi}\left(\widetilde{h}_t\right)$ respectively. The market clearing condition for the private school market is given by

$$\int\int I\left(\phi\left(a_t h_t\right)\leq\widetilde{h}_t\right)A_1\left(a_t\right)\Psi_1\left(h_t\right)da_t dh_t = \widetilde{\Psi}\left(\widetilde{h}_t\right) \ \text{ for all } \widetilde{h}_t\in\left[\widetilde{h}_{\min},\widetilde{h}_{\max}\right],$$

where $\phi\left(a_t h_t\right)$ is the assignment rule and $I\left(\cdot\right)$ is an indicator function that assumes a value 1 if the statement is true and 0 otherwise. The derivatives $A_1\left(\cdot\right)$ and $\Psi_1\left(\cdot\right)$ represent the pdf's of a_t and h_t respectively. The above equation states that the mass of students assigned to teachers of quality less than $\widetilde{h}_t$ exactly equals the mass of teachers of that quality range. It defines a feasible assignment rule and essentially states that the assignment process is one-to-one. Recall that it was assumed that the teaching technology required that one teacher educate one student. Now, given that individual learning ability and teacher human capital are complementary in determining a child's human capital, the assignment rule will be strictly increasing.[7]

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[7]This fact dates back at least to Becker (1973), who reproduces a proof due to William Brock. An alternative way (following Fernandez and Gali, 1998, Theorem 2.2, who use results on multivariate stochastic dominance due to Levy and Parousch, 1974 and Atkinson and Bourguignon, 1981) is to demonstrate that complementarity is a sufficient condition to guarantee that the joint distribution of student learning ability and teacher human capital generated by a strictly positive assignment rule, stochastically dominates, in the first-order sense, the joint distribution generated by any other feasible assignment rule. As an immediate consequence, aggregate human capital is higher with a strictly positive assignment than any other assignment.

Then, the assignment rule $\widetilde{h}_t = \phi(s_t)$, will be given by

$$\int\limits_{a_{\min}}^{a_t} \int\limits_{h_{\min}}^{h_t} A_1(a_t)\, \Psi_1(h_t)\, da_t dh_t = \int\limits_{\widetilde{h}_{\min}}^{\phi(s_t)} \widetilde{\Psi}_1\left(\widetilde{h}_t\right) d\widetilde{h}_t \text{ for all } s_t \in [s_{\min}, s_{\max}]$$

The condition above states that, for any given level of student intrinsic ability a_t and parental human capital h_t, the mass of students who have learning abilities greater than $a_t h_t$ must exactly equal the mass of teachers who possess a human capital level greater than $\phi(s_t)$ to teach them. Thus, the fact that the assignment is in fixed proportions, together with the monotonicity of the assignment, completely characterizes the assignment rule. It is important to note that complementarity is sufficient to guarantee a strictly positive assignment so long as different levels of human capital are perfect substitutes in determining aggregate human capital.[8]

Before proceeding, it is important to make clear what exactly is a school in this environment. A school is characterized by a teacher of human capital $\widetilde{h}_t$. Further, the set of students in a school is given by $\left\{(a_t, h_t) : a_t h_t = \phi^{-1}\left(\widetilde{h}_t\right)\right\}$. Even though all students attending a given school necessarily have the same learning ability, they possess different values of a_t and h_t. The equilibrium entails (borrowing the terminology of Epple and Romano, 1998) an infinite number of schools, each serving an infinitely refined peer group.[9]

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[8] What if they are imperfect substitutes in determining aggregate human capital? For example, $G(h_t) = \left[\int h_t^{\frac{\nu-1}{\nu}} d\Psi(h_t)\right]^{\frac{\nu}{\nu-1}}$ ($\nu = \infty$ corresponds to linear aggregation). In that case, complementarity in the determination of *aggregate* human capital is sufficient to guarantee that strictly positive assignment maximizes $G(h_t)$. The relevant condition is given by

$$G_1(\cdot) \underbrace{X_{12}(\cdot)}_{Complementarity} \geq - \underbrace{G_{11}(\cdot)}_{Concavity} X_1(\cdot) X_2(\cdot), \tag{6}$$

where the function $G(\cdot)$ stands for the manner in which different levels of human capital are aggregated. For the example considered above, equation (6) reduces to $\nu > 1$. In this case, however, the wage rates will be a function of the human capital of the individual's human capital.

[9] The case in which a continuum of schools exists in an equilibrium is not analyzed in Epple and Romano (1998). In the model economy they present, a competitive equilibrium fails to exist.

Recall that equation (4) represents the Euler equation for teacher human capital $\widetilde{h}_t$ from the worker's dynamic-programming problem. To derive closed-form solutions, some assumptions regarding functional forms will be made. Let $X(\cdot)$ be linearly homogeneous and the assignment rule be given by $\widetilde{h}_t = \frac{1}{x} s_t = \phi(s_t)$, where x is a constant across schools.[10] Conjecture that $p\left(\widetilde{h}_t\right)$ is linear. Then, the equilibrium voucher function is given by $v(s_t) = p(\phi(s_t)) = p(s_t/x)$. Equation (4) then implies that

$$p_1\left(\widetilde{h}_t\right) = \left[\frac{\frac{w}{(1+r)}}{1 - \beta\left\{X_1(x,1) + X_2(x,1)\frac{1}{x}\right\}\int a_{t+1}\frac{U_1(c_{t+1})}{U_1(c_t)}dA(a_{t+1})}\right]X_2\left(s_t,\widetilde{h}_t\right).$$

The term in the brackets above, say q, will be constant along a balanced growth path. The above equation then becomes

$$p_1\left(\widetilde{h}_t\right) = qX_2\left(s_t,\widetilde{h}_t\right), \tag{7}$$

where q reflects the 'right' value of acquiring an additional unit of human capital. Observe that the derivative of the price function $p(\cdot)$ is constant thereby verifying the conjecture that the price function is indeed linear.

Remark 1 *Note that the assignment is between learning abilities s on the one hand and human capital levels of the teachers $\widetilde{h}$ on the other. This helps make another point: the equilibrium allocation demands that the best teacher be matched not to the student with the highest intrinsic ability but to the student with the best combination of intrinsic ability and parental human capital. This result holds in general, so long as parental human capital is an input in producing human capital for a child.*

Caucutt (1997) demonstrates that the usage of lotteries leads to the existence of a competitive equilibrium. An equilibrium in which a continuum of schools exist would arise if schools were thought to be clubs with peer-group effects and faced constant-returns-to-scale cost functions.

[10] Given two continuous random variables x and y, the assignment rule (which assigns to each element in the support of x, one and only one element in the support of y) may be obtained as follows: let $F(x)$ and $G(y)$ be their respective cdf's. Then the random variables $F(x)$ and $G(y)$ have the $UNIF[0,1]$ distribution. The assignment rule is given by $F(x) \equiv G(y)$ or $x \equiv G^{-1}(F(y))$.

The analysis now turns to establishing some properties regarding the price function. The price function for teachers can be obtained by integrating equation (7) to yield

$$p\left(\widetilde{h}_t\right) = p\left(\widetilde{h}_{\min}\right) + q \int_{\widetilde{h}_{\min}}^{\widetilde{h}_t} X_2\left(\phi^{-1}\left(h_1\right), h_1\right) dh_1 \qquad \forall\, \widetilde{h}. \tag{8}$$

Recall that the equilibrium assignment was conjectured to be $s_t = x\widetilde{h}_t$.[11] Since X is assumed to be linearly homogeneous, it is easy to see that the above equation may be written as

$$p\left(\widetilde{h}_t\right) = p\left(\widetilde{h}_{\min}\right) + q\left[f\left(x\right) - xf'\left(x\right)\right]\left(\widetilde{h}_t - \widetilde{h}_{\min}\right), \tag{9}$$

where $f\left(x\right) = X\left(x, 1\right)$. Note that the wages paid to the teachers are increasing in their human capital. In order to completely characterize the price function, $P\left(\widetilde{h}_{\min}\right)$ needs to be computed. Since the voucher system eliminates the role played by borrowing constraints, all parents will invest up to the point where the rate of return equals the real interest rate. Therefore

$$qX\left(s_t, \widetilde{h}_t\right) = qf\left(x\right)\widetilde{h}_t = \left(1 + r\right)\left\{p\left(\widetilde{h}_{\min}\right) + q\left[f\left(x\right) - xf'\left(x\right)\right]\left(\widetilde{h}_t - \widetilde{h}_{\min}\right)\right\}.$$

Evaluating the above equation at $\widetilde{h}_{\min}$ implies that

$$p\left(\widetilde{h}_{\min}\right) = \frac{qf\left(x\right)}{\left(1 + r\right)}\widetilde{h}_{\min}. \tag{10}$$

Therefore, the price function is given by

$$p\left(\widetilde{h}_t\right) = \frac{qf\left(x\right)}{\left(1 + r\right)}\widetilde{h}_{\min} + q\left[f\left(x\right) - xf'\left(x\right)\right]\left(\widetilde{h}_t - \widetilde{h}_{\min}\right).$$

Observe that the price function is increasing in the teacher human capital. The assignment process is thus able to draw distinctions among teachers and suggests that private markets would make teacher's salaries responsive to market forces. One can

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[11] In fact, it turns out that along a balanced growth path, x will be constant across matches at a point in time as well as across time.

stop at this stage and question the very capability of a private system to implement a merit pay system for teachers. In other words, from a historical perspective, where private systems did exist, were they capable of drawing such distinctions among teachers? The evidence suggests that private markets were indeed capable of differentiating teachers by their quality.[12] For instance, in the late seventeenth century Scotland, salaries for teachers depended on direct payments from their customers. Discriminatory pricing developed to such an extent that Robert Lowe observed "In Scotland they sell education like a grocer sells figs".[13]

If the private system is indeed capable of drawing such distinctions among teachers, the next question is what happens to inequality? More important, what would be the main determinant of inequality? The next proposition helps to provide an answer.

Proposition 1 *Effect of Complementarity: In the private regime, wage inequality among teachers at a point in time will rise when individual ability and teacher human capital are stronger complements in the sense of a larger X_{12}.*

Proof. Using equation (9), the ratio of the wages received by the teacher with a human capital level $\widetilde{h}$ to that received by the teacher with the lowest human capital is given by

$$\frac{p\left(\widetilde{h}_t\right)}{p\left(\widetilde{h}_{\min}\right)} = 1 + \frac{1}{p\left(\widetilde{h}_{\min}\right)} \int_{\widetilde{h}_{\min}}^{\widetilde{h}_t} X_2\left(\phi^{-1}\left(h_1\right), h_1\right) dh_1.$$

Using equation (10) to eliminate $p\left(\widetilde{h}_{\min}\right)$, the above equation may be re-written as

$$\frac{p\left(\widetilde{h}_t\right)}{p\left(\widetilde{h}_{\min}\right)} = 1 + \frac{1+r}{q f'(x)} \left(\frac{\widetilde{h}_t - \widetilde{h}_{\min}}{\widetilde{h}_{\min}}\right) \underbrace{\left[\left(1 - \frac{x f'(x)}{f(x)}\right) f'(x)\right]}_{=X_{12}}.$$

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[12] The interested reader is referred to West (1964) and Coulson (1999).

[13] As reported by West (1964)

The above proposition suggests that private markets would increase wage inequality among teachers and that the extent of this inequality would depend upon the degree of complementarity between individual ability and teacher human capital. The diversity in educational opportunities would further induce greater inequality among workers through the assignment process. Teachers would then accordingly increase investments in the human capital of their children since they will have to educate an even more diverse set of students next period. This will induce greater inequality among teachers in the next period and so on. The extent to which this inequality will exceed that of the public economy will depend on the importance of own ability vis-à-vis teacher human capital in the production of human capital, and more generally, the curvature exhibited by the assignment process. The dynamics of the assignment problem are analyzed in greater detail in the next section where the balanced growth paths of the public and private economies are cast out.

3.1   The Public Education Economy

The economy considered is an idealized world where the government essentially provides education on an 'equal basis' to all. Workers pay taxes on labor and capital income and in return, get their children educated at a public school. The government runs the public schools. It sets a minimum human capital requirement for teachers, and all teachers satisfying this criterion get to work at the public school. The worker's problem is identical to the one in the private economy except that education is now provided by the government.

Teachers: Teachers work at a public school so long as they satisfy a minimum human capital requirement. They are paid a flat wage independent of their human capital which is given by

$$\widetilde{y}_t = \begin{cases} \underline{p}_t, & \widetilde{h}_t \geq \underline{h}_t \\ 0, & otherwise \end{cases}, \tag{11}$$

where $\underline{p}_t$ is the income that a teacher obtains from working at a public school and $\underline{h}_t$ is the minimum human capital required to work at a public school. The cut-off level

$\underline{h}_t$, may be thought of as the minimum human capital required to obtain a teaching certificate. This institutional restriction on the wage structure may be thought to be a result of powerful teachers unions.[14] This wage structure is the easiest way to capture the fact that public schools, being pressured by teachers unions, do not provide teachers with the right incentives to accumulate human capital.

The inability on the part of the government to implement a merit pay system, as reflected in equation (11), is taken as a given. Merit pay is a rather old idea, dating back to the turn of the century. Empirical evidence suggests that no other reform has been tried more often and failed time and again (see Murnane et. al. 1991, p. 117-119). This is, in large part, due to opposition by teachers unions. More recently, Ballou and Podgursky (1997, p. 109) note that 'The history of efforts to introduce merit pay and other performance incentives into public schools does not leave grounds for much optimism'.

An interesting justification for such a rigid pay structure is equality of opportunity. As noted earlier, a uniform pay structure gives rise to equal opportunity in the sense that all public schools would be able to provide students with a teacher of the same human capital. The costs of doing so are clear-cut: the deadweight loss associated with the pay structure and the resources needed to be invested to ensure that all teachers attain that cut-off level. As will be seen, it provides teachers with the wrong incentives to accumulate and supply human capital.

Assume that the government runs a teacher's training program and invests in all teachers such that they are able to attain the cut-off level $\underline{h}$. Such expenditures are financed by payments from taxes on teacher's income. Further, assume that the government gets to observe the learning ability of every teacher and can thereby invest different amounts in different teachers. Further, given the wage structure specified by equation (11), teachers will not have an incentive to supplement the goods inputs

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[14]For an interesting analysis of the impact of Teachers' unions on the production of education, see Hoxby (1996).

from the government.

While one may debate the realism of this rule, the key features of the public education system (as represented here) lie elsewhere: all students will get the same teacher quality $\underline{h}_t$, and all teachers will receive the same wage.

While salary structures represented by equation (11) are the norm in the US education system, a great deal of variation in teacher quality is nevertheless seen. This would be expected if different schools provide different working environments and other intangible benefits. Ballou and Podgursky (1997) suggest that this may very well be as important as the salary and benefits provided to teachers.

Government: The government collects taxes on labor and capital income from workers and teachers and distributes the proceeds partly to pay teacher's salaries and partly to invest in teacher's children who would be teachers next period.[15] It's budget constraint reads

$$\underline{p}_t + \widetilde{\mathbf{e}}_{t+1} = \tau_h w_t \mathbf{h_t} + \tau_k r_t \mathbf{k_t} + \tau_{\tilde{h}} \underline{p}_t + \tau_{\tilde{k}} r_t \widetilde{\mathbf{k}}_t, \tag{12}$$

where $\mathbf{h}_t$ denotes aggregate worker human and $\mathbf{k}_t$ and $\widetilde{\mathbf{k}}_t$ are aggregate worker and teacher physical capital respectively. Public schools hire teachers who satisfy a minimum human capital requirement and pay them each the same wage $\underline{p}_t$. Public schools make no profits, they merely receive the tax revenues from the government and use the same to pay the teachers. The tax rates τ_h and τ_k and the minimum human capital level $\underline{h}_t$ are exogenous. As generations go by, the human capital that workers acquire will grow, and $\underline{h}_t$ will have to be raised to sustain economic growth.[16]

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[15]Note that taxes are distortional. Given incomplete markets, the optimal tax (in the Ramsey sense) on capital income would be strictly positive in the steady state in order to bring the interest rate to equality with the rate of time preference (see Aiyagari 1995). Moreover, if taxes were lump-sum, the private education economy would have the poor low ability agents subsidizing the rich high ability agents, which is clearly unrealistic.

[16]An exact description of the balanced growth path is left to section 4.

4   The Balanced Growth Path

This section characterizes the balanced growth paths of the public and private education regimes and establishes conditions under which long-run growth can be sustained. The heterogeneity in intrinsic abilities necessitates a characterization with different growth rates for dynasties with different intrinsic abilities and human capital levels. Essentially, it amounts to specifying the cross-sectional distribution of growth rates of individual-specific variables and the growth rate of aggregate variables. Along a balanced growth path, these would be time-invariant and the distribution of growth rates of all individual-specific variables will be identical. Such a complication helps to yield joint predictions on long-run growth, inequality and intergenerational mobility.

The private economy can experience balanced growth even if the human capital production function exhibits decreasing returns to scale (!!!). This results if the assignment process exhibits increasing returns. Two cases will be distinguished: one in which the assignment process can be sustained across time as a process of renewed assignment along a balanced growth path, and another in which the assignment process, can on its own, *spur* economic growth. As will be seen, the latter turns out to be the more interesting and plausible case. One could also draw parallels with the convergence results in Tamura (1991). In Tamura (1991), a heterogeneous society converges to a homogeneous outcome. Here, a heterogeneous society converges to a more homogeneous but non-degenerate outcome, if a dynasty effectively exhibits diminishing returns to human capital accumulation.

In order to derive some analytical properties of the public and private education economies, assume that the human capital production functions take the form

$$X\left(s,\widetilde{h}\right) = s^{\gamma_1}\left(\widetilde{h}\right)^{\gamma_2}; \widetilde{X}\left(\widetilde{s},\widetilde{e}\right) = \left(\widetilde{s}\right)^{\gamma_1}\left(\widetilde{e}\right)^{\gamma_2},$$

and the distributions of intrinsic abilities are given by

$$a \sim \log N\left(\mu_a; \sigma_a^2\right); \widetilde{a} \sim \log N\left(\mu_{\widetilde{a}}; \sigma_{\widetilde{a}}^2\right).$$

Further, assume that the momentary utility function is given by

$$U\left(c\right) = \frac{c^{\zeta}}{\zeta}, \; 0 < \zeta < 1.$$

A few comments are in order. First, the distribution of abilities is assumed to be lognormal. The functional forms for the human capital production functions guarantee that if one were to start with a lognormal distribution of human capital, future distributions will remain lognormal. This permits an easy characterization of the properties of the distribution across generations.[17] Second, the coefficients on the human capital production functions for workers and teachers (γ_1 and γ_2) are assumed to be identical. Relaxing this is easy, but does not affect any of the results. Third, the sum $\gamma_1 + \gamma_2$, together with the curvature of the assignment rule in the private economy, will determine whether or not the economy is capable of exhibiting balanced growth. The analysis considers cases in which the human capital production function exhibits decreasing and constant returns to scale. Fourth, the distribution of intrinsic abilities of workers and teachers are allowed to be different. Finally, the assumption on the utility function is quite standard in the endogenous growth literature.

4.1   The Public Education Economy

The balanced growth path will be uncovered through a 'guess and verify' procedure. Conjecture that the cross-sectional distributions of growth rates of c_t, b_{t+1} and h_{t+1} are identical and that aggregate variables $\mathbf{k}_t$, $\mathbf{h}_t$ and $\mathbf{c}_t$ grow at the constant rate $\mathbf{g}$. In other words, once the shocks a_{t+1} have been realized, the cross-sectional distributions of $\frac{C(b_{t+1},h_{t+1},a_{t+1};\mathbf{z}_{t+1})}{C(b_t,h_t,a_t;\mathbf{z}_t)}$, $\frac{B(b_{t+1},h_{t+1},a_{t+1};\mathbf{z}_{t+1})}{B(b_t,h_t,a_t;\mathbf{z}_t)}$, and $\frac{H(h_{t+1},a_{t+1};\mathbf{z}_{t+1})}{H(h_t,a_t;\mathbf{z}_t)}$ are identical. In fact, the growth path is 'balanced' precisely because the cross-sectional distribution of growth rates of all variables is exactly the same and time-invariant. Conjecture that the distribution of worker human capital is given by $h_t \sim \log N\left(\mu_{h_t}; \sigma_{h_t}^2\right)$.[18] A dynasty

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[17]These functional forms are quite standard in the literature. See Glomm and Ravikumar (1992) and Benabou (1996).

[18]If the initial distribution of human capital is log-normal, so will the equilibrium distribution.

which currently comprises a parent with a human capital h_t and a child with ability a_t will experience a growth rate in human capital given by

$$g(a_t, h_t) = h_{t+1}/h_t = X(s_t, \underline{h}_t)/h_t = (a_t)^{\gamma_1} (\underline{h}_t)^{\gamma_2} /h_t^{1-\gamma_1}.$$

If the distribution of $g(a_t, h_t)$, which is the cross-sectional distribution of growth rates, is to remain time invariant (a requirement for balanced growth), so must the statistic $(\underline{h}_t)^{\gamma_2} /h_t^{1-\gamma_1}$. Further, note that all else equal, the higher the parental human capital, the lower the growth rate. The public education regime serves to provide the highest growth rates of human capital to the poor high ability children and the lowest growth rates to the rich low ability children. The distribution of human capital next period will be given by

$$h_{t+1} \sim \log N \left(\gamma_2 \log (\underline{h}_t) + \gamma_1 \left(\mu_{h_t} + \mu_a \right); \gamma_1^2 \left(\sigma_{h_t}^2 + \sigma_a^2 \right) \right). \tag{13}$$

The cross-sectional distribution of growth rates is then given by

$$g_t \sim \log N \left(\gamma_2 \log (\underline{h}_t) + \gamma_1 \mu_a - (1 - \gamma_1) \mu_{h_t}; (1 - \gamma_1)^2 \sigma_{h_t}^2 + \gamma_1^2 \sigma_a^2 \right). \tag{14}$$

In order to ensure that the above distribution will be time-invariant, some restrictions need to be in place. Specifically, the distribution of growth rates next period needs to be computed, and conditions imposed to ensure that it coincides with the current period's distribution of growth rates. Let g_{t+1} denote next period's cross-sectional distribution of growth rates. Further, let $\underline{h}_t$ grow at the rate $\mathbf{g}_{\underline{h}_t}$. Then

$$g_{t+1} \sim \log N \left(\mu_{g_{t+1}}; (1 - \gamma_1)^2 \gamma_1^2 \left(\sigma_{h_t}^2 + \sigma_a^2 \right) + \gamma_1^2 \sigma_a^2 \right), \tag{15}$$

where

$$\mu_{g_{t+1}} = \gamma_2 \log \left(\mathbf{g}_{\underline{h}_t} \underline{h}_t \right) + \gamma_1 \mu_a - (1 - \gamma_1) \left[\gamma_2 \log (\underline{h}_t) + \gamma_1 \left(\mu_{h_t} + \mu_a \right) \right].$$

Equating means across the two cross-sectional distributions in equations (14) and (15) yields

$$\begin{aligned} \gamma_2 \log \mathbf{g}_{\underline{h}_t} &= (1 - \gamma_1) \left[\gamma_2 \log (\underline{h}_t) + \gamma_1 \mu_a - (1 - \gamma_1) \mu_{h_t} \right] \tag{16} \\ &= (1 - \gamma_1) \left[\mu_{h_{t+1}} - \mu_{h_t} \right]. \end{aligned}$$

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Next, equating variances yields

$$\sigma_h^2 = \gamma_1^2 \left(\sigma_h^2 + \sigma_a^2 \right). \tag{17}$$

Using equation (13), the aggregate growth rate of the economy $\mathbf{g}$ can be expressed as

$$\mathbf{g} = \frac{\mathbf{h}'}{\mathbf{h}} = \frac{E\left(h'\right)}{E\left(h\right)} = \frac{\exp\left(\gamma_2 \log\left(\underline{h}\right) + \gamma_1\left(\mu_h + \mu_a\right) + \frac{1}{2}\sigma_{h'}^2\right)}{\exp\left(\mu_h + \frac{1}{2}\sigma_h^2\right)}. \tag{18}$$

Note from equations (17) and (13) that $\sigma_{h'}^2 = \sigma_h^2$. Using this fact, and taking logarithms on both sides of equation (18) yields

$$\log \mathbf{g} = \gamma_2 \log\left(\underline{h}\right) + \gamma_1 \mu_a - \left(1 - \gamma_1\right)\mu_h. \tag{19}$$

Substituting the above equation in equation (16), it transpires that

$$\gamma_2 \log \mathbf{g}_{\underline{h}} = \left(1 - \gamma_1\right)\log \mathbf{g}. \tag{20}$$

This is rather intuitive. As generations go by, standards for teachers must be raised in order to sustain long-run growth. The rate at which these standards must be raised depends on the returns-to-scale exhibited by the human capital production function. This raises the question: is it possible to sustain long-run growth if $\mathbf{g}_{\underline{h}} \neq \mathbf{g}$? Equivalently, is it possible to sustain long-run growth if $\gamma_1 + \gamma_2 \neq 1$? The next Lemma demonstrates that the answer is indeed in the negative.

Lemma 2 *The public education economy can exhibit balanced growth if and only if* $\gamma_1 + \gamma_2 + 1$ *and* $\underline{h}$ *grows at the same rate as the aggregate growth rate of the economy.*

Proof. The 'if' part is quite standard. As regards the 'only if' part, recall that

$$\widetilde{e} = \left[\frac{\mathbf{g}\underline{h}}{\left(\widetilde{a}\underline{h}\right)^{\gamma_1}}\right]^{1/\gamma_2} = \left[\frac{\mathbf{g}\underline{h}^{1-\gamma_1}}{\left(\widetilde{a}\right)^{\gamma_1}}\right]^{1/\gamma_2}.$$

It follows that aggregate goods inputs into teachers is given by

$$\log \widetilde{\mathbf{e}} = \left(\frac{1-\gamma_1}{\gamma_2}\right)\log \underline{h} + \left(\frac{1}{\gamma_2}\right)\log \mathbf{g} - \left(\frac{1}{\gamma_2}\right)\mu_{\widetilde{a}} + \left(\frac{1}{\gamma_2^2}\right)\sigma_{\widetilde{a}}^2/2. \tag{21}$$

From the above equation, it is easy to discern that

$$\log \mathbf{g_{\tilde{e}}} = \left(\frac{1 - \gamma_1}{\gamma_2} \right) \log \mathbf{g_{\underline{h}}}.$$

However, all aggregate variables must grow at the same rate. Hence, $\mathbf{g_{\tilde{e}}}$ equals $\mathbf{g}$ and the above equation reduces to

$$\gamma_2 \log \mathbf{g} = (1 - \gamma_1) \log \mathbf{g_{\underline{h}}}. \tag{22}$$

Now, comparing equations (20) and (22), it is easy to see that the condition $\gamma_1 + \gamma_2 = 1$ must hold along a balanced growth path so that $\mathbf{g_{\underline{h}}} = \mathbf{g}$. ∎

It is easy to see that the expression on the right hand side of equation (16) exactly equals the mean of $\log g$, the cross-sectional distribution of growth rates. That condition ensures that the logarithm of the aggregate growth rate of the economy exactly equals the expected value of the logarithm of the growth rate of human capital in the economy. Further, notice from equation (17) that the variance of worker human capital is completely pinned down by the importance of individual learning ability in producing human capital and the variance in intrinsic abilities. It is easy to see from equations (13) and (17) that the variance of log-human capital will be time-invariant along a balanced growth path.

Proposition 3 *The variance of log-human capital (wage income) in the public education regime is time-invariant and equals $\frac{\gamma_1^2}{1-\gamma_1^2}\sigma_a^2$.*

Recall that γ_1 represents the importance of individual characteristics relative to school characteristics in determining human capital. Thus, a higher γ_1 leads to higher income inequality since $\frac{\gamma_1^2}{1-\gamma_1^2}$ increases in γ_1.[19] As the importance of a school diminishes, income inequality rises. This result stems from the fact that public education

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[19] For a log-normally distributed variable $h \sim \log N\left(\mu; \sigma^2\right)$, the Gini coefficient depends only on σ. Hence, a higher variance may be interpreted as higher inequality. Moreover, ranking log-normal distributions by σ is equivalent to ranking them by the Lorenz criterion (see Aitchison and Brown p. 112-113).

system serves to equalize opportunities, and that a higher γ_1 implies that this equalizing factor diminishes in importance relative to individual characteristics. Thus, a system of provision in which the government makes a conscious attempt to provide everyone in society with the exact same resources (equality of opportunity) results in wage inequality being independent of the importance of teacher human capital in the human capital production function. The variance of human capital converges to a constant primarily because each dynasty effectively exhibits diminishing returns to human capital production.

The preceding analysis also permits an easy characterization of the cross-sectional distribution of growth rates.

Lemma 4 *The time-invariant cross-sectional distribution of growth rates in the public education economy is*

$$g \sim \log N \left(\log \mathbf{g}; \frac{2\gamma_1^2}{1 + \gamma_1} \sigma_a^2 \right) \tag{23}$$

where $\mathbf{g}$ *is the aggregate growth rate of the economy and given by equation (16).*

Proof. The mean of the cross-sectional distribution of growth rates is characterized by equation (16). The variance is given by

$$\sigma_{\mathbf{g}}^2 = (1 - \gamma_1)^2 \sigma_h^2 + \gamma_1^2 \sigma_a^2 = \frac{(1 - \gamma_1)^2}{1 - \gamma_1^2} \gamma_1^2 \sigma_a^2 + \gamma_1^2 \sigma_a^2 = \frac{2\gamma_1^2}{1 + \gamma_1} \sigma_a^2.$$

∎

In order to maintain budget balance, aggregate physical and human capital must grow at the rate $\mathbf{g}$. Hence, prices w and r will be constant across generations. As regards teachers, the value $\underline{h}$ cannot be set arbitrarily high. While there seems to be no easy way to obtain an upper bound for $\underline{h}$, a very high value implies that a substantial amount of resources need to be spent in order to get all teachers up to that level. Since the initial distribution of teacher human capital is assumed to be log-normal, there is a cost involved in getting all these teachers up to a level $\underline{h}$. The

expenditures required to educate all teachers upto a level $\underline{h}$ is given by

$$\log \widetilde{\mathbf{e}}_0 = (1/\gamma_2) \log \underline{h} - \left(\frac{\gamma_1}{\gamma_2}\right)\left(\mu_{\widetilde{a}} + \mu_{\widetilde{h}_0}\right) + \left(\frac{\gamma_1}{\gamma_2}\right)^2 \left(\sigma_{\widetilde{a}}^2 + \sigma_{\widetilde{h}_0}^2\right). \tag{24}$$

Further, a cost needs to be incurred in each subsequent period. As will be seen in the next section, the level at which standards can be maintained will be critical in determining which economy grows at a faster rate.

4.2   The Private Education Economy

Conjecture that the distribution of worker and teacher human capital are given by $h \sim \log N\left(\mu_h; \sigma_h^2\right)$ and $\widetilde{h} \sim \log N\left(\mu_{\widetilde{h}}; \sigma_{\widetilde{h}}^2\right)$. The assignment rule is then given by $\widetilde{h} \equiv x_1 s^{x_2}$ according to the conjecture.[20] The growth rate of a dynasty is then given by $g(a, h) = X\left(s, \widetilde{h}\right)/h$. The curvature of the assignment process x_2 will play a key role in the rest of the analysis. It follows that

$$g(a, h) = (ah)^{\gamma_1}(x_1 s^{x_2})^{\gamma_2}/h = x_1^{\gamma_2} a^{\gamma_1 + x_2 \gamma_2}(h)^{\gamma_1 + x_2 \gamma_2 - 1}. \tag{25}$$

Along a balanced growth path, the distribution of $g(a, h)$ must be time-invariant. The following Lemma establishes conditions under which the distribution of $g(a, h)$ will remain time-invariant.

Lemma 5 *The cross-sectional distribution of growth rates in the private economy will remain time-invariant if any one of the following conditions are satisfied.*
(i) $\gamma_1 + \gamma_2 < 1$ and $x_2 = (1 - \gamma_1)/\gamma_2 > 1$, and x_1 constant,
(ii) $\gamma_1 + \gamma_2 = 1$ and $x_2 = 1$, and x_1 constant,
(iii) $\gamma_1 + \gamma_2 > 1$ and $x_2 = (1 - \gamma_1)/\gamma_2 < 1$, and x_1 constant,
(iv) $\gamma_1 + \gamma_2 \leq 1$, x_2 constant and x_1 grows at a constant rate $\mathbf{g}_{x_1}$ (to be determined).

Proof. In cases (i) through (iii), $\gamma_1 + x_2 \gamma_2 = 1$, so that $g(a, h) = x_1^{\gamma_2} a$. Since x_1 is a constant, the distribution of $g(a, h)$ will be time-invariant. The proof of case (iv) is relegated to the end of this section when it is analyzed in great detail. ∎

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[20] In fact, given the two log-normal distributions, $\widetilde{h} = x_1 s^{x_2}$ transforms one into the other.

Observe that even if the human capital production function were to exhibit increasing (or decreasing) returns to scale, balanced growth could result provided there is some degree of concavity (or convexity) in the assignment process x_2. Notice that in cases (i), (ii), and (iii), the growth rate of a particular dynasty equals a constant times the idiosyncratic shock (intrinsic abilities). A balanced growth path would then be characterized by a distribution of growth rates which is in fact a constant times the distribution of intrinsic abilities. In particular $g\left(a, h\right) = x_1^{\gamma_2} a$. Recall, however, that the public economy can exhibit balanced growth if and only if $\gamma_1 + \gamma_2 = 1$. Since the end objective is to compare the public and private economies, cases (i), (iii), and a part of case (iv) (where $\gamma_1 + \gamma_2 < 1$) are not considered. From here on, case (iv) must read

(iv)$'$ $\gamma_1 + \gamma_2 = 1$, x_2 constant (to be determined) and x_1 grows at a constant rate $\mathbf{g}_{x_1}$ (again to be determined).

In order to map out the balanced growth path of the private economy, consider the assignment rule $\widetilde{h} \equiv x_1 s^{x_2}$. The values x_1 and x_2 will now be determined. The assignment rule implies that

$$\log N\left(\mu_{\widetilde{h}}; \sigma_{\widetilde{h}}^2\right) \overset{\mathcal{D}}{\equiv} \log N\left(\log x_1 + x_2\left(\mu_h + \mu_\alpha\right); x_2^2\left(\sigma_h^2 + \sigma_a^2\right)\right).$$

In other words, the distribution of $x_1 s^{x_2}$ must coincide with the distribution of $\widetilde{h}$. This, in turn, is possible only if the means and variances are identical, implying that

$$\log x_1 + x_2\left(\mu_h + \mu_a\right) = \mu_{\widetilde{h}}, \tag{26}$$

and

$$x_2^2\left(\sigma_h^2 + \sigma_a^2\right) = \sigma_{\widetilde{h}}^2. \tag{27}$$

Observe that the value of x_2 will be higher, the greater the inequality in teacher human capital. This curvature parameter reflects the ability of the education system to draw distinctions among teachers. Having described the assignment process, it is now possible to analyze the remaining cases (ii) and (iv).

Case (ii): This is the most standard case considered in much of the endogenous growth literature. When $x_2 = 1$, the model reduces to the standard A-K variety, and balanced growth obtains. From equation (27), it follows that the variance in teacher human capital equals the sum of the variances in worker human capital and the variance in intrinsic abilities. Further, human capital next period is given by

$$h' = x_1^{\gamma_2} a^{\gamma_1 + x_2 \gamma_2} (h)^{\gamma_1 + x_2 \gamma_2} = x_1^{\gamma_2} ah, \tag{28}$$

since $\gamma_1 + x_2 \gamma_2 = 1$. It is easy to see from the above equation that wage inequality will blow up to infinity.[21]

Case (iv)′: This is the more interesting case and represents one of the singular contributions of this paper. The rest of this section will be devoted to a detailed analysis of this case. Here, each dynasty effectively exhibits diminishing returns to human capital production since $\gamma_1 + x_2 \gamma_2 < 1$. Hence, just as in the public economy, wage inequality will be time-invariant. It follows that the condition

$$(\gamma_1 + x_2 \gamma_2)^2 \left(\sigma_h^2 + \sigma_a^2 \right) = \sigma_h^2. \tag{29}$$

must hold. The following Lemma characterizes the impact of the assignment process on long-run inequality.

Lemma 6 *Inequality in the private education regime tends to ∞ in the long run in case (ii), and converges to $\sigma_h^2 = (\gamma_1 + x_2 \gamma_2)^2 \sigma_a^2 / \left[1 - (\gamma_1 + x_2 \gamma_2)^2 \right]$ in case (iv).*

Proof. Equation (28) implies that $\sigma_{h_{t+1}}^2 = \sigma_{h_t}^2 + \sigma_a^2$. Hence, in case (ii), the variance of human capital tends to ∞ as $t \to \infty$. As regards case (iv), the result follows directly from equation (29). ∎

Now, since $x_2 < 1$, each dynasty effectively exhibits diminishing returns to human capital production. How can growth be sustained? It is easy to see that if x_1 were to

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[21] The addition of an aggregate spillover will change this prediction and lead to convergence to a finite variance.

grow, then balanced growth can be sustained. Conjecture that x_1 grows at the rate $\mathbf{g}_{x_1}$. Then, writing equation (26) one period ahead implies that

$$\log x_1 + \log \mathbf{g}_{x_1} = \mu_{\widetilde{h}} + \log \mathbf{g} - x_2 \left(\mu_h + \log \mathbf{g} + \mu_\alpha \right),$$

which together with equation (26) yields

$$\log \mathbf{g}_{x_1} = (1 - x_2) \log \mathbf{g}, \tag{30}$$

where x_2 is given by equation (27). The distribution of human capital next period is given by

$$h' \sim \log N \left(\gamma_1 \left(\mu_h + \mu_a \right) + \gamma_2 \mu_{\widetilde{h}}; \left(\gamma_1 + x_2 \gamma_2 \right)^2 \left(\sigma_a^2 + \sigma_h^2 \right) \right)$$

Lemma 7 *The cross-sectional distribution of growth rates in the private economy is given by*

$$g \sim \log N \left(\gamma_2 \left(\mu_{\widetilde{h}} - \mu_h \right) + \gamma_1 \mu_a; 2 \left(\gamma_1 + x_2 \gamma_2 \right)^2 \sigma_a^2 / \left(1 + \gamma_1 + x_2 \gamma_2 \right) \right).$$

Proof. Recall that equation (25) characterizes the evolution of human capital in the private economy. The distribution of growth rates can then be written as

$$g \sim \log N \left(\begin{array}{c} \gamma_2 \log x_1 + \left(\gamma_1 + x_2 \gamma_2 \right) \mu_a + \left(\gamma_1 + x_2 \gamma_2 - 1 \right) \mu_h; \\ \left(\gamma_1 + x_2 \gamma_2 \right)^2 \sigma_a^2 + \left(\gamma_1 + x_2 \gamma_2 - 1 \right)^2 \sigma_h^2 \end{array} \right).$$

Substituting for $\log x_1$ from equation (26) and for σ_h^2 from the previous Lemma, the above equation reduces to

$$g \sim \log N \left(\begin{array}{c} \gamma_2 \left[\mu_{\widetilde{h}} - x_2 \left(\mu_h + \mu_\alpha \right) \right] + \left(\gamma_1 + x_2 \gamma_2 \right) \mu_a + \left(\gamma_1 + x_2 \gamma_2 - 1 \right) \mu_h; \\ \left(\gamma_1 + x_2 \gamma_2 \right)^2 \sigma_a^2 \left[1 + \frac{\left(\gamma_1 + x_2 \gamma_2 - 1 \right)^2}{1 - \left(\gamma_1 + x_2 \gamma_2 \right)^2} \right] \end{array} \right),$$

which after a little algebra yields the result. $\blacksquare$

Now consider the teachers. Conjecture that $\widetilde{e}' = x_3 \left(\widetilde{a}\widetilde{h} \right)^{x_4}$. Human capital will evolve according to $\widetilde{h}' = x_3^{\gamma_2} \left(\widetilde{a}\widetilde{h} \right)^{\gamma_1 + x_4 \gamma_2}$. The long-run variance of teacher human capital is then given by

$$\sigma_{\widetilde{h}}^2 = \left(\gamma_1 + x_4 \gamma_2 \right)^2 \sigma_{\widetilde{a}}^2 / \left[1 - \left(\gamma_1 + x_4 \gamma_2 \right)^2 \right] \tag{31}$$

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The values x_3 and x_4 will be determined so as to ensure consistency with the assignment rule. Therefore

$$\widetilde{h}' \sim \log N \left(\gamma_2 \log x_3 + (\gamma_1 + x_4 \gamma_2) \left(\mu_{\widetilde{h}} + \mu_{\widetilde{a}} \right) ; (\gamma_1 + x_4 \gamma_2)^2 \left(\sigma_{\widetilde{h}}^2 + \sigma_{\widetilde{a}}^2 \right) \right),$$

and

$$a'h' \sim \log N \left(\mu_a + \gamma_1 \left(\mu_h + \mu_a \right) + \gamma_2 \mu_{\widetilde{h}}; (\gamma_1 + x_2 \gamma_2)^2 \left(\sigma_a^2 + \sigma_h^2 \right) + \sigma_a^2 \right).$$

The assignment rule one period ahead is then given by $\widetilde{h}' = x_1' \left(a'h' \right)^{x_2}$. Taking logarithms and using equation (30) yields

$$\log \widetilde{h}' = \log x_1 + (1 - x_2) \log \mathbf{g} + x_2 \left(\log a' + \log h' \right).$$

To preserve the assignment across generations, the restrictions

$$\begin{aligned}
\gamma_2 \log x_3 + (\gamma_1 + x_4 \gamma_2) \left(\mu_{\widetilde{h}} + \mu_{\widetilde{a}} \right) &= \log x_1 + (1 - x_2) \log \mathbf{g} \\
&\quad + x_2 \left(\mu_a + \gamma_1 \left(\mu_h + \mu_a \right) + \gamma_2 \mu_{\widetilde{h}} \right)
\end{aligned}$$

and

$$(\gamma_1 + x_4 \gamma_2)^2 \left(\sigma_{\widetilde{h}}^2 + \sigma_{\widetilde{a}}^2 \right) = x_2^2 \left((\gamma_1 + x_2 \gamma_2)^2 \left(\sigma_a^2 + \sigma_h^2 \right) + \sigma_a^2 \right)$$

need to be in place. In order to ensure that the growth rate is constant over time, μ_h and $\mu_{\widetilde{h}}$ must grow at the same rate. This implies the restriction

$$\gamma_2 \log x_3 + (\gamma_1 + x_4 \gamma_2) \left(\mu_{\widetilde{h}} + \mu_{\widetilde{a}} \right) - \mu_{\widetilde{h}} = \gamma_2 \log x_1 + (\gamma_1 + x_2 \gamma_2) \left(\mu_h + \mu_a \right) - \mu_h.$$

From the above equation and equation (30), it follows that

$$\log \mathbf{g}_{x_3} = (1 - x_4) \log \mathbf{g}.$$

Finally, what are the long-run values of x_2 and x_4? These curvature parameters will play a key role in the analysis of the balanced growth path. To obtain a restriction, consider the Euler equation (4) for teacher human capital in the worker's dynamic program and the Euler equation (??) for educational expenditures in the teacher's

dynamic programming problem. Substituting the conjectured assignment rule into Euler equations and eliminating the price functions, it transpires that x_2 must equal x_4. Using this fact, together with equations (27), (29), and (31), it follows that

$$\left(\gamma_1 + x_2\gamma_2\right)\sigma_{\tilde{a}} = x_2\sigma_a.$$

Therefore, the long-run value of x_2 is given by

$$x_2 = \frac{\gamma_1\sigma_{\tilde{a}}/\sigma_a}{1 - \gamma_2\sigma_{\tilde{a}}/\sigma_a}.$$

Observe that if $\sigma_{\tilde{a}} = \sigma_a$, $x_2 = \frac{\gamma_1}{1-\gamma_2} = 1$. Thus is rather intuitive. The long-run variances of worker and teacher human capital are completely pinned down by the variances in their intrinsic abilities. If these variances in intrinsic abilities are identical, then the long-run variances in worker and teacher human capital converge to the same constant. The assignment rule becomes linear and the model reduces to the standard A-K variety. This completes the description of the balanced growth path in the private economy. The analysis now turns towards establishing some properties of the public and private economies along a balanced growth path.

5 Public versus Private

This section contains the main results of this paper. It contrasts the properties of the balanced growth paths of the two regimes detailed above with respect to growth, inequality, mobility and equality of opportunity.

5.1 Economic Growth

The impact of inequality of growth has been the subject of several recent papers. For some interesting work that demonstrates that inequality is harmful for growth, see Alesina and Rodrik (1994) and Persson and Tabellini (1994). Imagine starting off the public and the private economies with the exact same distributions of worker and

teacher human capital, $h_0 \sim \log N \left(\mu_{h_0}; \sigma_{h_0}^2 \right)$ and $\widetilde{h}_0 \sim \log N \left(\mu_{\widetilde{h}_0}; \sigma_{\widetilde{h}_0}^2 \right)$. Further, let the distribution for worker human capital in the public economy and the distributions for worker and teacher human capital in the private economy along a balanced growth path be given by $h^{\mathrm{pub}} \sim \log N \left(\mu_{h^{\mathrm{pub}}}; \sigma_{h^{\mathrm{pub}}}^2 \right)$, $h^{\mathrm{priv}} \sim \log N \left(\mu_{h^{\mathrm{priv}}}; \sigma_{h^{\mathrm{priv}}}^2 \right)$ and $\widetilde{h} \sim \log N \left(\mu_{\widetilde{h}}; \sigma_{\widetilde{h}}^2 \right)$ respectively. The following proposition results.

Proposition 8 *Growth: The long-run growth rate of the private education regime exceeds that of the public education regime if and only if $\mu_{\widetilde{h}} - \mu_{h^{\mathrm{priv}}} > \log \underline{h} - \mu_{h^{\mathrm{pub}}}$, where $\mu_{\widetilde{h}}$ is the mean teacher (log) human capital in the private economy and $\underline{h}$ is the standards set for teachers in the public economy.*

Proof. Recall that in the public economy, human capital evolves according to

$$h_{t+1} = (a_t h_t)^{\gamma_1} (\underline{h}_t)^{\gamma_2} . \tag{32}$$

The short-run growth rate in the public economy is then given by

$$\log g_t^{\mathrm{pub}} = \log \mathbf{h}_{t+1} - \log \mathbf{h}_t, \tag{33}$$

where $\mathbf{h}_t$ and $\mathbf{h}_{t+1}$ stand for aggregate human capital in the current period and the following period respectively. Since human capital is log-normally distributed, aggregate human capital is given by

$$\log \mathbf{h}_t = \mu_{h_t} + \sigma_{h_t}^2/2. \tag{34}$$

Using equations (32) and (34), equation (33) may be written as

$$\left. \log g_t^{\mathrm{pub}} \right|_{\mathrm{SR}} = \gamma_2 \left[\log \underline{h}_t - \mu_{h_t} \right] + \gamma_1 \mu_a + \gamma_1^2 \sigma_a^2/2 - \underbrace{\sigma_{h_t}^2/2}_{\text{Heterogeneity reduces growth}} \left[1 - \gamma_1^2 \right] . \tag{35}$$

In order to facilitate interpretation, the above equation may be re-written as

$$\left. \log g_t^{\mathrm{pub}} \right|_{\mathrm{SR}} = \gamma_2 \left[\log \underline{h}_t - \log \mathbf{h}_t \right] + \gamma_1 \log \mathbf{a} - \gamma_1 \left(1 - \gamma_1 \right) \left(\sigma_{h_t}^2 + \sigma_a^2 \right)/2.$$

Along the transition to a balanced growth path, the short-run growth rate is stochastic since the variance of human capital changes over time. The long-run growth rate in

the public economy may be obtained by replacing $\sigma_{h_t}^2$ with its steady-state value $\sigma_h^2 = \gamma_1 \sigma_a^2 / (1 - \gamma_1^2)$. Hence

$$\log g^{\text{pub}}\big|_{\text{LR}} = \gamma_2 \left[\log \underline{h}_t - \log \mathbf{h}_t\right] + \gamma_1 \log \mathbf{a} - \left[\gamma_1 / (1 + \gamma_1)\right] \sigma_a^2 / 2$$

Turning to the private economy, recall that the evolution of human capital in the private economy is given by

$$h_{t+1} = x_{1t}^{\gamma_2} a_t^{\gamma_1 + x_{2t}\gamma_2} h_t^{\gamma_1 + x_{2t}\gamma_2},$$

where x_{1t} and x_{2t} are given by

$$\log x_{1t} = \mu_{\tilde{h}_t} - x_{2t}\left(\mu_{h_t} + \mu_a\right),$$

$$x_{2t} = \frac{\sigma_{\tilde{h}_t}}{\sqrt{\sigma_{h_t}^2 + \sigma_a^2}}.$$

Proceeding along exactly the same lines as the public economy, the short-run growth rate in the private economy is given by

$$\log g_t^{\text{priv}} = \gamma_2 \left(\mu_{\tilde{h}_t} - \mu_{h_t}\right) + \gamma_1 \mu_a + (\gamma_1 + x_{2t}\gamma_2)^2 \sigma_a^2 / 2 - \sigma_{h_t}^2 / 2 \left[1 - (\gamma_1 + x_{2t}\gamma_2)^2\right]. \quad (36)$$

Again, re-writing the above equation in terms of $\log \mathbf{h}_t$ and $\log \widetilde{\mathbf{h}}_t$ yields

$$\begin{aligned}
\log g_t^{\text{priv}}\big|_{\text{SR}} = {} & \gamma_2 \left(\log \widetilde{\mathbf{h}}_t - \log \mathbf{h}_t\right) + \gamma_1 \log \mathbf{a} - \left[(\gamma_1 + x_{2t}\gamma_2)(1 - (\gamma_1 + x_{2t}\gamma_2))\right] \left(\sigma_a^2 + \sigma_{h_t}^2\right) / 2 \\
& - \gamma_2 \left[\sigma_{\tilde{h}_t}^2 - x_{2t}(\gamma_1 + x_{2t}\gamma_2)^2\right].
\end{aligned}$$

Given that $x_{2t}^2 \left(\sigma_{h_t}^2 + \sigma_a^2\right) = \sigma_{\tilde{h}_t}^2$, the above equation simplifies to

$$\log g_t^{\text{priv}}\big|_{\text{SR}} = \gamma_2 \left(\log \widetilde{\mathbf{h}}_t - \log \mathbf{h}_t\right) + \gamma_1 \log \mathbf{a} - \left[\gamma_1 + \gamma_2 x_{2t}^2 - (\gamma_1 + x_{2t}\gamma_2)^2\right] \left(\sigma_a^2 + \sigma_{h_t}^2\right) / 2.$$

The effects of heterogeneity in the private economy are clearly complex, because both worker heterogeneity and teacher heterogeneity now enter the determination of growth. Moreover, they interact through matching, i.e. through x_{1t} and x_{2t}. Thus, one cannot simply vary $\sigma_{h_t}^2, \sigma_{\tilde{h}_t}^2, \sigma_a^2$ and $\sigma_{\tilde{a}}^2$ and compute their effects on $\log g_t^{\text{priv}}\big|_{\text{SR}}$

as if x_{2t} were constant. Finally, setting $\sigma_{h_t}^2 = (\gamma_1 + x_2\gamma_2)^2 \sigma_a^2 / \left(1 - (\gamma_1 + x_2\gamma_2)^2\right)$ in the equation above, the long-run growth rate is given by

$$\log g^{\text{priv}}\big|_{\text{LR}} = \gamma_2 \left(\log \widetilde{\mathbf{h}}_t - \log \mathbf{h}_t\right) + \gamma_1 \log \mathbf{a} - \left[\frac{\gamma_1 + \gamma_2 x_2^2 - (\gamma_1 + x_2\gamma_2)^2}{1 - (\gamma_1 + x_2\gamma_2)^2}\right] \sigma_a^2/2,$$

where $x_2 = \frac{\gamma_1 \sigma_{\tilde{a}}/\sigma_a}{1 - \gamma_2 \sigma_{\tilde{a}}/\sigma_a}$.

Growth Comparisons: Now, the log-ratio of growth rates between the private and the public economy in the short-run is given by

$$\log g_t^{\text{priv}} - \log g_t^{\text{pub}}\big|_{\text{SR}} = \gamma_2 \underbrace{\left\{\log \widetilde{h}_t - \log \underline{h}_t\right\}}_{\text{Losses from low standards}} + \underbrace{\left[(\gamma_1 + \gamma_2 x_{2t})^2 - \gamma_1^2 - \gamma_2 x_{2t}^2\right]}_{\text{Gains from positive matching}} \left(\sigma_a^2 + \sigma_{h_t}^2\right)/2$$

$$(37)$$

The second term is strictly positive since $x_{2t} > 0$. Hence a sufficient condition for the short-run growth rate to be higher in the private economy is $\mu_{\tilde{h}_0} > \log \underline{h}$. What about the long-run? Recall that the long-run variances in the public and the private economies are given by

$$\sigma_{h_{\text{LR}}}^2\big|_{\text{pub}} = \frac{\gamma_1^2}{1 - \gamma_1^2}\sigma_a^2,$$

and

$$\sigma_{h_{\text{LR}}}^2\big|_{\text{priv}} = \frac{(\gamma_1 + x_2\gamma_2)^2}{1 - (\gamma_1 + x_2\gamma_2)^2}\sigma_a^2.$$

Therefore, the long-run growth rates in the public and private economies may be written as

$$\log g^{\text{pub}}\big|_{\text{LR}} = \gamma_2 \left[\log \underline{h}_t - \mu_{h^{\text{pub}}}\right] + \gamma_1 \mu_a + \gamma_1^2 \sigma_a^2/2 - \left\{\frac{\gamma_1^2}{1 - \gamma_1^2}\sigma_a^2\right\}/2 \left[1 - \gamma_1^2\right],$$

and

$$\log g^{\text{priv}}\big|_{\text{LR}} = \gamma_2 \left(\mu_{\tilde{h}} - \mu_{h^{\text{priv}}}\right) + \gamma_1 \mu_a + (\gamma_1 + x_2\gamma_2)^2 \sigma_a^2/2$$
$$- \left\{\frac{(\gamma_1 + x_2\gamma_2)^2}{1 - (\gamma_1 + x_2\gamma_2)^2}\sigma_a^2\right\}/2 \left[1 - (\gamma_1 + x_2\gamma_2)^2\right],$$

where $\mu_{h^{\text{pub}}}$ stands for the mean worker log human capital in the public economy and $\mu_{h^{\text{priv}}}$ and $\mu_{\tilde{h}}$ represent the mean worker and teacher log human capital in the private

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economy along a balanced growth path. The log-ratio of growth rates in the long-run then becomes

$$\log g^{\text{priv}} - \log g^{\text{pub}}\big|_{\text{LR}} = \gamma_2\left[\left(\mu_{\tilde{h}} - \mu_{h^{\text{priv}}}\right) - (\log \underline{h} - \mu_{h^{\text{pub}}})\right] + \underbrace{\left[(\gamma_1 + x_2\gamma_2)^2 - \gamma_1^2\right]}_{\text{Gains from positive matching}} \sigma_a^2/2$$

$$- \underbrace{\left[(\gamma_1 + x_2\gamma_2)^2 - \gamma_1^2\right]}_{\text{Losses from greater heterogeneity}} \sigma_a^2/2. \qquad (38)$$

Observe that the gains from positive assignment exactly cancel out the losses from the higher degree of heterogeneity. Therefore, the log-ratio of long-run growth rates becomes

$$\log g^{\text{priv}} - \log g^{\text{pub}}\big|_{\text{LR}} = \gamma_2\left[\left(\mu_{\tilde{h}} - \mu_{h^{\text{priv}}}\right) - (\log \underline{h} - \mu_{h^{\text{pub}}})\right].$$

∎

The above proposition represents one of the key result of this paper and brings out some of the main forces at work. Some of the terms used above will now be explained in greater detail and their relevance clarified in a series of remarks.

Remark 2 *Observe from equations (35) and (36) that all else equal, the higher the initial variance in worker human capital, the lower the short-run growth rate. Heterogeneity reduces growth simply because each dynasty exhibits diminishing returns to human capital accumulation. It is easy to see from equation (35) that if $\gamma_1 = 1$, heterogeneity has no effect on growth.*

Remark 3 *The greater the diversity in educational opportunities, the higher the short-run growth rate. This is easily seen by noticing the term $\gamma_1 + x_{2t}\gamma_2$ multiplying the variance of human capital in the private economy in equation (36) as opposed to γ_1 in the public economy in equation (35).*

Remark 4 *The first term on the right hand side of equation (37) represents the losses from low standards in the public economy. Such losses arise if it is too expensive to ensure that each teacher in the public economy will supply more human capital than*

the mean teacher log-human capital in the private economy. The second term on the right hand side of equation (37) represents the gains from positive matching. That these are in fact gains follows by noting that $x_{2t} > 0$. Again recall that positive assignment maximizes aggregate human capital since individual learning ability and teacher human capital are complementary in determining aggregate human capital.

Remark 5 *If standards could, in fact, be maintained at a high enough level (high enough to overcome the losses from low standards and the gains from positive assignment), the public economy will grow at a faster rate than the private economy in the short-run and the long-run.*

Remark 6 *The fact that gains exactly equals losses is not an artifact of the chosen functional forms for learning ability or the human capital production. It is an immediate fallout of growth stationarity.*

The intuition behind Proposition 11 can be simply put as follows. There are simply two considerations in the short-run as reflected by equation (37). First, gains from positive assignment in the private system and second, losses from low teacher standards in the public system. These effects reinforce one another and the short-run growth rate is higher in the private economy.

In the long-run, there is a trade-off. Complementarity between individual inputs and school resources argues in favor of providing the best student with the best teacher. The private system, thus facilitates the 'right' kind of assignment of students to teachers and this leads to positive benefits. The efficiency of the assignment process eventually leads to greater heterogeneity in the private economy. Heterogeneity, however, reduces economic growth as evidenced by the negative sign multiplying the variance of human capital in the short-run growth equation (35). As noted in the remark, heterogeneity reduces economic growth primarily because parental human capital exhibits diminishing returns in determining a child's human capital. Since the private economy leads to a more heterogeneous workforce in the long-run, the

negative effect on growth is larger. This argues in favor of homogeneity. Equal opportunity, however, does not come free. Such a cost is reflected in the resources needed to be expended to ensure that all working teachers are able to attain the cut-off level $\underline{h}$ as seen in equation (24). Further, the rigid pay structure for teachers leads them to accumulate and supply less human capital than they would in the private economy. This trade-off is important and considering the appropriate costs and benefits associated with each of these systems is crucial. From equation (38), it is also clear that the benefits of positive assignment *exactly* outweigh the long-run costs to heterogeneity in the private economy and the private system exhibits faster long-run growth if the costs to maintaining standards are high.[22]

How high can standards for teachers be maintained along a balanced growth path? Recall that in order to sustain economic growth in the public economy, standards for teachers need to be raised at the aggregate growth rate of the economy. From equations (19) and (21), it transpires that the cost to be incurred every period in order to get teachers who possess a human capital of $\underline{h}$ up to a level $\mathbf{g}\underline{h}$ is given by

$$\log \widetilde{\mathbf{e}} = 2 \log\left(\underline{h}\right) - \mu_{h^{\mathrm{pub}}} + \frac{\gamma_1}{\gamma_2}\mu_a - \left(\frac{1}{\gamma_2}\right)\mu_{\widetilde{a}} + \left(\frac{1}{\gamma_2^2}\right)\sigma_{\widetilde{a}}^2/2. \qquad (39)$$

To get a feel for how high standards can be set in the public economy, consider the ratio of aggregate educational expenditures to aggregate output. Further, assume that the distribution functions for intrinsic abilities for workers and teachers are identical and that the mean ability level equals one. It transpires that

$$\log \widetilde{\mathbf{e}} - \log \mathbf{y} = 2 \log \underline{h} - 2\mu_{h^{\mathrm{pub}}} - \log \kappa + \left(\frac{1 + \gamma_2^2}{\gamma_2^2} - \frac{\gamma_1^2}{1 - \gamma_1^2}\right)\sigma_a^2/2,$$

---

[22]One can ask whether borrowing constraints can alter this result? An analysis of the assignment problem in the presence of borrowing constraints is complicated in the general equilibrium set-up presented. However, some intuition can be obtained from the results: while borrowing constraints reduce the gains from positive assignment, they also reduce the losses from heterogeneity since long-run inequality declines. The point is more general and applies with any form of a mis-match in the assignment process.

where $\kappa = \left(\frac{1+r}{\alpha}\right)^{-\frac{\alpha}{1+\alpha}}$. Here, r and α stand for the real interest rate and capital's share of output respectively. It is easy to deduce that $\kappa < 1$ so that $\log \kappa < 0$. Is it possible that $\log \underline{h} > \mu_{\tilde{h}} - \mu_{h^{\text{priv}}} + \mu_{h^{\text{pub}}}$ (in which case, from Proposition 11, the public system will experience faster long-run growth)? If this inequality holds, then the above equation implies that

$$\log \tilde{\mathbf{e}} - \log \mathbf{y} > 2\left[\mu_{\tilde{h}} - \mu_{h^{\text{priv}}}\right] \underbrace{- \log \kappa + \left(\frac{1+\gamma_2^2}{\gamma_2^2} - \frac{\gamma_1^2}{1-\gamma_1^2}\right)\sigma_a^2/2}_{> 0}.$$

Note that with the exception of the term inside the brackets on the right hand side of the above equation, all the others are strictly positive.. Then, it transpires that if the log mean worker human capital $(\mu_{h^{\text{priv}}})$ is not too much higher than the log mean teacher human capital $(\mu_{\tilde{h}})$, $\log \tilde{\mathbf{e}} > \log \mathbf{y}$! In other words, aggregate educational expenditures will exceed aggregate output. This suggests that it is unlikely that standards can be maintained at a high enough high rate for the public system to dominate the private system in terms of its long-run growth performance.

5.2   Inequality

The analysis now shifts to an examination of inequality. There have been several recent analyses of wage inequality and its causes. For some early work on the determinants of wage inequality for the last three decades, see Katz and Murphy (1992).

Proposition 9 *Wage Inequality: The private education economy possesses a higher degree of inequality than the public education economy.*

Proof. Recall from Proposition 5 that the variance of human capital in the public regime along a balanced growth path is given by $\frac{\gamma_1^2}{1-\gamma_1^2}\sigma_a^2$ and is time-invariant. Therefore, the ratio of the variance in income in the private economy to that in the public economy is given by

$$\frac{\sigma_{priv}^2}{\sigma_{pub}^2} = \left[\frac{(\gamma_1 + x_2\gamma_2)^2}{\gamma_1^2}\right]\left[\frac{1-\gamma_1^2}{1-(\gamma_1 + x_2\gamma_2)^2}\right] > 1. \tag{40}$$

∎

   The proposition above demonstrates that the sorting of individuals in the private economy will in fact lead to greater inequality. Observe from equation (40) that the ratio of wage inequality in the private system to that in the public system is entirely determined by the curvature of the assignment process, x_2 and the parameters governing the human capital production function. When $x_2 = 0$, every student in the private economy is educated by a teacher of the same quality, hence inequality is identical across the two regimes. Furthermore, if the parental effect in quite strong relative to the effect of a teacher in determining a child's human capital ($\gamma_1 > \gamma_2$), the ratio would not be too large. The increase in inequality will be determined by the curvature of the assignment process, x_2.

   What is the impact of sorting on long-run inequality? The issue of marital sorting seems to have been given some recent focus. Kremer (1997) criticizes recent work by arguing that sorting does not have much of an impact on long-run inequality. In particular, he shows that moving from an economy with no segregation to one with complete segregation increases the steady-state standard deviation of education by approximately 9%. However, Fernandez and Rogerson (1999) find that increased sorting significantly increases income inequality. Whether or not one agrees with Kremer's estimate, this raises the question: is it possible to construct a model economy that admits the possibility that along a balanced growth path, sorting does not have too severe an impact on inequality? It turns out that the model economy presented does. An example might help the clarify the impact of sorting on inequality in the current context.

Example 1 *Let $\gamma_1 + \gamma_2 < 1$, and hence focus on a steady-state. The the ratio of the variance in the private economy to that in the public economy is still given by equation (40). To make things concrete, let $\gamma_1 = 0.25$ and $\gamma_2 = 0.05$. Then, using equation (40), the ratio of standard deviations in the economy with perfect sorting (private), to that in the economy with no sorting at all (public) equals 1.28 if $x_2 = 1$,*

1.11 if $x_2 = 0.5$, and 1.05 if $x_2 = 0.25$. When $x_2 = 1$, the implications of sorting on inequality become quite severe: the standard deviation of human capital rises by 28%! As x_2 decreases, so does the impact of sorting on inequality.

It seems to be taken for granted that the private economy would be less preferable to a public economy on grounds on equity. From a normative standpoint, even if it were the case that the private system entails more inequality, it might be preferable on grounds of equity, if it permitted a substantially higher degree of intergenerational mobility. The joint determination of inequality within a generation and economic mobility across generations along a balanced growth path makes the analysis in this paper rather unique and helps to shed light on the rather neglected aspect of social mobility. The model presented here provides closed-form solutions for the cross-sectional distribution of growth rates. As will be seen shortly, this statistic contains all the relevant information needed to make statements about economic mobility. The following proposition contrasts the cross-sectional distribution of growth rates in the two economies.

Proposition 10 *The Cross-sectional Distribution of Growth Rates: The cross-sectional distribution of growth rates in the public education regime has lower mean and lower variance than that under the private regime.*

Proof. Recall from Lemma 4.2 and 4.5 that the cross-sectional distribution of growth rates under the two regimes are given by

$$g^{pub} \sim \log N \left(\log \mathbf{g}^{pub}; \frac{2\gamma_1^2}{1 + \gamma_1} \sigma_a^2 \right)$$

and

$$g^{priv} \sim \log N \left(\log \mathbf{g}^{priv}; \frac{2(\gamma_1 + x_2\gamma_2)^2}{1 + \gamma_1 + x_2\gamma_2} \sigma_a^2 \right).$$

Recall that proposition 11 (together with the discussion on teacher standards) establishes that $\mathbf{g}^{pub} < \mathbf{g}^{priv}$. The proof of the proposition follows from noting that

$\frac{2\gamma_1^2}{1+\gamma_1}$ increases in γ_1 since $\gamma_1 \in (0,1)$ so that $\frac{2(\gamma_1+x_2\gamma_2)^2}{1+\gamma_1+x_2\gamma_2} > \frac{2\gamma_1^2}{1+\gamma_1}$. ∎Before turning to a discussion of intergeneratonal mobility, it is useful to compare the public and private economies in terms of 'equality of opportunity'.

5.3   Equality of Opportunity

In order to make precise statements on the degree of equality of opportunities in each economy, consider the stochastic process transforming the current distribution of human capital into the distribution of human capital next period

$$\log h' = \alpha_1 \log h + \alpha_2 \log a + \text{constant}, \tag{41}$$

where α_1 is the degree of correlation in human capital and α_2 represents the (absolute) importance of ability. When $\alpha_1 = 0$, the child's human capital becomes entirely random and the above stochastic process may be thought of as characterizing a *meritocracy*. On the other hand, when $\alpha_2 = 0$, there is complete transmission of human capital from parents to children and this situation may be thought of as depicting an *autocracy*. The evolution of human capital in the public and private economies are special cases of the above process when $\alpha_1 = \alpha_2 = \gamma_1$ and $\alpha_1 = \alpha_2 = \gamma_1 + x_2\gamma_2$ respectively. Following Atkinson (1980) define *equality of opportunity* by

$$E \equiv \frac{Var\left[\alpha_2 \log a\right]}{Var\left[\log h'\right]}. \tag{42}$$

Observe that according to the above definition, complete opportunity equalization $E = 1$, will arise only when the assignment of students to teachers can *negate* the effect of parental human capital in determining a child's human capital. Such a situation will arise with a negatively assortative matching. Recall that a negative assignment constitutes the efficient allocation if individual ability and teacher human capital are substitutes in producing a child's human capital. Using equation (42), the following result obtains.

Lemma 11 *The degrees of equality of opportunity in the public and private economies are given by $E^{pub} = 1 - \gamma_1^2$, and $E^{priv} = 1 - (\gamma_1 + x_2\gamma_2)^2$.*

The above lemma indicates that the public economy entails a greater degree of opportunity equalization. Observe that, all else equal, the higher the persistence in human capital, the lower the degree of opportunity equalization. Note, however, that the private economy amplifies the effect of own ability as well. This trade-off is captured in an analysis of intergenerational mobility to which the analysis next turns.

5.4   Intergenerational Mobility

The private system amplifies the effect of own ability, and this effect enhances mobility in the sense of increased randomness. However, it also leads to a greater degree of persistence in human capital across generations. A comparison of intergenerational mobility involves these trade-offs. The trade-off may be most easily captured by observing that while the public system entails a greater degree of 'equality of opportunity', the private system entails a greater degree of 'inequality in outcomes' with respect to the random component (ability).

In a recent paper, Fields and Ok (1996), derive a measure for the absolute level of mobility from some axioms and suggest the measure $E\left|h' - h\right|$. Such a measure is intractable in the current context. Hence the analysis proceeds with a closely connected measure of *relative* mobility. The measure of mobility 'employed here is

$$M = E\left|\log h' - \log h\right|. \tag{43}$$

This measure captures the expected value of the absolute change in position. Note that the higher the value of M, the greater the mobility in the process $\log h \to \log h'$ (in the sense defined by Fields and Ok).[23] The following proposition characterizes the ranking of economic mobility.

Proposition 12 *Intergenerational Economic Mobility: The private education economy permits a greater degree of intergenerational mobility in earnings than the public*

---

[23]This does not, however, imply that there is more mobility in the process $h \to h'$. The analysis shall, nevertheless, focus on the measure M defined above due to its tractability.

education economy.

Proof. The measure of economic mobility M, involves computing the expectation of the absolute value of a normally distributed random variable. If $X \sim N\left(\mu; \sigma^2\right)$, then the distribution of $|X|$ is said to be of the folded normal variety. From the Handbook of the Normal Distribution (1982 p. 34), the expected value of a folded normal distribution is given by

$$M = E|X| = \sigma\sqrt{2/\pi}\exp\left(-\mu^2/2\sigma^2\right) - \mu\left[1 - 2\Phi\left(\mu/\sigma\right)\right], \qquad (44)$$

where $\Phi\left(\cdot\right)$ is the CDF of a standard normal distribution. Taking partial derivatives with respect to μ and σ yields

$$
\begin{aligned}
\frac{\partial E|X|}{\partial \mu} &= \sigma\sqrt{2/\pi}\exp\left(-\mu^2/2\sigma^2\right)\left(-2\mu/2\sigma^2\right) - \left[1 - 2\Phi\left(\mu/\sigma\right)\right] + 2\mu\Phi_1\left(\mu/\sigma\right)\left(1/\sigma\right) \\
&= \left(\mu/\sigma\right)\left[2\Phi_1\left(\mu/\sigma\right) - \sqrt{2/\pi}\exp\left(-\mu^2/2\sigma^2\right)\right] - \left[1 - 2\Phi\left(\mu/\sigma\right)\right] \\
&= \left(\mu/\sigma\right)\left[2\Phi_1\left(\mu/\sigma\right) - 2\Phi_1\left(\mu/\sigma\right)\right] - \left[1 - 2\Phi\left(\mu/\sigma\right)\right] \\
&= \left[2\Phi\left(\mu/\sigma\right) - 1\right] > 0 \text{ since } \mu/\sigma > 0,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial E|X|}{\partial \sigma} &= \sqrt{2/\pi}\exp\left(-\mu^2/2\sigma^2\right) + \sigma\left(-\mu^2/2\right)\left(-2/\sigma\right)\sqrt{2/\pi}\exp\left(-\mu^2/2\sigma^2\right) \\
&\quad + 2\mu\left(-\mu/\sigma^2\right)\Phi_1\left(\mu/\sigma\right) \\
&= 2\Phi_1\left(\mu/\sigma\right)\left[1 + \mu^2\left(1 - 1/\sigma^2\right)\right] > 0.
\end{aligned}
$$

Observe that mobility is higher, the greater the growth rate of the economy, as well as the greater the inequality in growth rates. The proof of the proposition is then immediate since the private economy possesses a higher mean and a standard deviation in growth rates (see Proposition 14). ∎

Example (Mobility without Growth): The measure of mobility introduced in equation (44) is rather complicated. It is easy to see, however, that when one abstracts from economic growth ($\mu = 0$) and focuses on a steady-state, which transpires for instance

if $\gamma_1 + \gamma_1 < 1$ and $x_2 \neq (1 - \gamma_1)/\gamma_2$, the measure of economic mobility simplifies to

$$M = \sigma\sqrt{2/\pi}. \tag{45}$$

In other words, mobility is linear in the standard deviation of the cross-sectional distribution of growth rates. This is rather intuitive. Economic mobility is a property of the transition function which gives rise to the equilibrium distribution, and the cross-sectional distribution of growth rates characterizes the precise manner in which such a transition occurs. The variance in growth rates associated with the *steady-state* of the stochastic process (41) is given by

$$\begin{aligned} \sigma^2 &= (1 - \alpha_1)^2 \sigma_h^2 + \alpha_2^2 \sigma_a^2 \\ &= \left(\frac{2}{1 + \alpha_1}\right)\alpha_2^2\sigma_a^2. \end{aligned} \tag{46}$$

Using equation (46), the measure of mobility (45) reduces to

$$M = 2\sqrt{\frac{2}{\pi(1 + \alpha_1)}}\alpha_2\sigma_a.$$

Observe that the mobility measure, rather intuitively, decreases with α_1 (in the sense of an increased *persistence* in status across generations) and increases with α_2 (in the sense of an increased importance of the *random component* in determining income).

Lemma 13 *Focusing on steady-states, the private economy exhibits a greater degree of intergenerational mobility in human capital than the public economy.*

Proof. The ratio of mobility in the private economy to that in the public economy is given by

$$\frac{M^{priv}}{M^{pub}} = \left(\frac{\gamma_1 + x_2\gamma_2}{\gamma_1}\right)\sqrt{\frac{1 + \gamma_1}{1 + \gamma_1 + x_2\gamma_2}} > 1.$$

■

A quote from Friedman (1962) aptly describes the trade-off between the two 'types' of inequality.

The confusion of these two kinds of inequality is particularly important precisely because competitive free enterprise capitalism tends to substitute the one for the other...capitalism undermines status and induces social mobility.

- Milton Friedman (1962)

In the current context, capitalism undermines status *relative* to its impact on ability. In other words, the effect of ability overrides the effect of parental human capital. In summary, the sorting of students to teachers according to their learning ability will have a positive effect on growth (if the costs associated with maintaining teacher's standards are high), an adverse effect on inequality, and enhances economic mobility. It is interesting to note that the degree to which this happens, will depend crucially, on the value of x_2. As the system draws more and more distinctions among teachers, x_2 rises, and so does growth, inequality and intergenerational mobility. Equality of opportunity, however, is lower and the analysis makes clear that the notion of merit pay, in the sense used in this paper, is incompatible with the very notion of equality of opportunity.

6 Conclusions

It is instructive to review the main contributions of this paper. First, privatization of education leads to the specialization of teachers in skills and this promotes a better matching of students and teachers. This seems important in that one of the often touted reform proposals involves the capability of the system to draw considerable distinctions among teachers. This paper demonstrates that privatization accomplishes just that.

Second, the growth rate of the private economy exceeds that of the public economy if teacher standards are costly to maintain. The main result is that while heterogeneity imposes a burden on long-run growth, this loss from heterogeneity is exactly offset

by the benefits from positive matching. Further, since there are costs involved in providing every student with a teacher of the exact same human capital, the private system dominates the public system in terms of both short-run and long-run growth. The result is robust to the inclusion of aggregate spillovers, imperfections in the assignment process, and concavity in the determination of aggregate human capital. Furthermore, borrowing constraints are unlikely to alter the result. While borrowing constraints reduce the gains from positive assignment, they also decrease inequality and hence the losses from heterogeneity. This suggests that diversity in educational opportunities fosters growth, even if there are frictions in the assignment process.

Third, inequality within a generation is higher in the private economy, and the extent to which the inequality is higher is shown to depend exclusively on the ability of the public system to draw distinctions among teachers. In particular, the model economy can generate differential effects of sorting on inequality. The analysis employs an index of equality of opportunity due to Atkinson and demonstrates that the idea of merit pay is incompatible with the very notion of equal opportunity. Fourth, the analysis reveals that economic mobility would be higher in the private economy. While the private system increases the persistence in economic status from one generation to the next, it also amplifies the absolute importance of a child's ability in determining his human capital. The second effect dominates and social mobility is higher in the private economy.

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