

Economic Development and the Organization of Production

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Abstract

We present a heterogeneous agent model with occupational choice and endogenous accumulation of skills to examine the organization of production both within and across countries. Quality and quantity of workers are imperfect substitutes. The span of control is endogenously determined by the quality of workers assigned to an entrepreneur. We calibrate the model to match certain features of the US economy. It yields a number of empirical implications for firm heterogeneity, occupational choice and the life cycle dynamics of firm size and earnings. Varying the aggregate efficiency of economies, we find that entrepreneur and worker human capital can substantially improve our understanding of various empirical regularities pertaining to the organization of production across countries.

Keywords: sorting, occupational choice, human capital, development

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1 Introduction

Economists have long been interested in the sources of per capita income differences across countries. The relative importance of aggregate factor inputs and technology differences has been subject of considerable scrutiny. A recent growing literature, based on the work of Lucas (1978) and Hopenhayn (1992), investigates whether the allocation of resources inputs across firms can matter for aggregate outcomes.¹ Poorer countries are characterized by larger microeconomic gaps as documented by Banerjee and Duflo (2005), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). While there exists a body of work that analyzes the allocation of physical capital across firms as well as the importance of financial markets,² much less is known about the allocation of human capital. A robust empirical finding is that larger firms have persistently higher average products of labor and capital. We take the view that a firm is persistently more productive and large partly because its workforce is composed of highly skilled employees that allow an entrepreneur to spread his skills over a large scale. How is production by individuals with heterogeneous skills organized across firms within a country? Are aggregate factors affecting the organization of production?

The lack of firm growth and the large number of less productive, small firms in developing countries are well documented.³ What is the role of human capital and skills investment for understanding these patterns? Human capital is not directly observable. Yet, several pieces of evidence suggest that it matters for firms' outcomes. In many surveys, the relative scarcity of skills is mentioned as a primary obstacle for the growth of firms. In the World Bank Enterprises Surveys, an "inadequately educated workforce" is considered to be one of the main obstacles to conduct business. Using these same data, Gennaioli et al. (2013) find that entrepreneurial schooling is fundamental for understanding productivity differences across firms and conjecture that entrepreneurial human capital may play a central role in determining firm productivity. Bloom et al. (2013) find that management quality is strongly correlated with firm-level productivity and that the education of both managers and non-managers is associated with better management scores.

Motivated by the empirical evidence, we build a model of acquisition and allocation of human capital across firms and we distinguish between worker human capital and entrepreneur human capital. In the seminal occupational choice model of Lucas (1978), the most talented

¹See the survey by Hopenhayn (2014).

²See for instance Quadrini (2000); Cagetti and De Nardi (2006); Buera et al. (2011); Midrigan and Xu (2014).

³See Tybout (2000); Hsieh and Klenow (2014); Hsieh and Olken (2014); Bento and Restuccia (2014).

individuals are managers/entrepreneurs who leverage their talent by hiring the workers. The aggregate implications of his model are identical to the standard Neoclassical growth model since it features aggregation. This paper has been extended in several directions. In contrast to this body of work, our paper relaxes two important assumptions. We take seriously the view that a successful theory of firm behavior across countries should be consistent with several important microeconomic facts on firm heterogeneity and firm dynamics.

First, we endogenize the distribution of talent in the economy by modeling individuals' schooling and on the job training decisions. As in Ben-Porath (1967), individuals accumulate human capital both in school and on the job. Unlike standard human capital models, each individual decides to be an entrepreneur or a worker. This affects their human capital accumulation decisions. Precisely, workers accumulate human capital to match with larger firms that pay higher wages. And the most skilled workers change occupations later in their lifecycle and become entrepreneurs. Entrepreneur's human capital allows them to increase firm's productivity and the quality of workers they employ. The distribution of individual talent plays an important role in explaining economic development across countries and firm outcomes within a country. The endogenous determination of human capital allows us to assign a central role to entrepreneurial talent. This is consistent with several recent empirical contributions cited above. One of the contributions of our paper is to have an endogenous determination of entrepreneurial human capital and to quantify its impact on economic development. This also allows us to relate the model to observations on earnings growth and firm growth.

Second, we relax the efficiency units assumption. Most applications that emphasize entrepreneurial ability do not consider the role of worker human capital in explaining economic development and the organization of production. In Lucas (1978), an individual gets the same wage rate w independent of his skills when he chooses to be a worker. Rosen (1982) and Jovanovic (1994) allow for worker heterogeneity but assume the number of workers and their quality are perfect substitutes: only the efficiency units of labor matter and a worker with human capital h earns wh . In other words, a highly talented entrepreneur is indifferent between hiring two workers of some quality and a worker who is twice as productive. This parsimonious formulation has the unattractive feature that it remains silent on sorting patterns across firms. These prove critical for understanding both how production is organized within a country and for understanding differences in living standards across countries. Some imperfect substitutability between quantity and quality of workers leads to positive assortative matching. Precisely, we use a production function based on the work

of Garicano (2000) and Garicano and Rossi-Hansberg (2006). Individuals use their human capital to solve problems. The existence of firms allows the more talented individuals of the firms, the entrepreneurs, to spend time solving the less common, more complicated problems and delegating the more basic and common problems to a team of workers. This particular production technology assigns a central role to entrepreneurial input and it has the sensible implications that the span of control of entrepreneurs is constrained by worker human capital. Indeed, entrepreneurial time is a fixed input in each firm and the ability of the entrepreneur to delegate is constrained by the talent of its workers. More talented workers can solve more problems and will only use the scarce entrepreneurial time for more complicated problems. These complementarities in production lead the quantity and the quality of its workers to be imperfect substitutes from entrepreneurs point of view.

Our rich framework can be parsimoniously parameterized and yet yields a number of empirical implications. We calibrate the model to match time-series and cross-sectional features of the U.S. earnings distribution and firm distribution. Furthermore, as is clear from our simulations, the model is consistent with various features of firm behavior at the microeconomic level. The model is able to reproduce simultaneously firm size dynamics and earnings dynamics. Earnings grow over time in the model because of the accumulation of human capital and because of the possibility of matching with better entrepreneurs. The size of firms grows over time because entrepreneurs invest in human capital and are able to attract better workers. Our model is also consistent with the fact that larger firms are more productive and pay higher wages. And, there is variation in firm productivity and labor productivity at an efficient allocation.⁴ This comes from the imperfect substitutability between quality and quantity of workers and the resulting sorting between heterogeneous entrepreneurs and heterogeneous workers. A high productivity firm will hire high productivity employees and due to the endogeneity of the span of control, a large number of them. A low productivity firm will hire a small quantity of low quality employees. Hence, there exists dispersion in labor productivity in equilibrium. Further, the wage rate depends on the quality of human capital so that measuring the workforce of the firm with the wage bill will not lead to a vanishing dispersion in productivity. This contrasts with an efficiency-unit technology where dispersion in labor productivity can only be the result of frictions in the allocation of resources.

To discipline our cross-country analysis, we ask how much variation in aggregate efficiency

⁴Firm productivity measures how much output the firm produces given its inputs. Labor productivity measures revenue per worker.

we need to account for output per capita differences. Varying the aggregate efficiency across countries, the model generates many cross countries patterns of firms. In particular, our model endogenously reproduces the fact that firms start small and remain small in poor countries while firms experience substantial employment growth in richer countries. As the aggregate efficiency of the economy declines, the incentives to invest in human capital by both workers and entrepreneurs are reduced leading to lower wage growth and lower firm growth. Firm size is constrained by worker quality and is lower in countries where the average level of human capital is low. Last, the dispersion in input productivity is much higher in poorer countries. In our model economy, complementarities between workers and their entrepreneur skills combined with an occupational choice decision lead to a higher dispersion in firm productivity and labor productivity in countries with lower aggregate level of efficiency. This transpires because the selection into entrepreneurship is more severe in richer countries where it takes a disproportionate amount of talent to be an entrepreneur. Many individuals who would be entrepreneurs in poor countries decide, optimally, to work for others in richer countries. This lack of selection naturally leads to a higher dispersion of productivity in poor countries. It also implies that poorer countries are characterized by a very large fraction of small firms and a high entrepreneurship rate.

In terms of policy implications, our model suggests that the main focus should be to improve the distribution of skills. Dispersion in labor productivity across firms within countries and a higher dispersion in poorer countries are both outcomes of a competitive model where there is no room for improvements in the allocation of human capital conditional on the distribution of skills. This view is similar to Jovanovic (2014) who studies the role played by a learning friction in the labor market when the friction reduces output and training but where the equilibrium nevertheless maximizes the rate of growth of output per worker subject to the learning friction.

Our paper is related to a large literature on human capital, sorting and economic development. A broad area of research has examined the importance of sorting starting from the work of Becker (1973) and Sattinger (1975). Notably, some of the ideas in our paper are related to the important work of Kremer (1993) who proposes an O-ring production function in which quantity cannot be substituted for quality. Two keys ingredients in our analysis are the deviation from efficiency units and occupational choice both of which are formalized in hierarchical models developed by Garicano (2000) and Garicano and Rossi-Hansberg (2006).⁵ We use this particular production technology mainly because of its tractability and

⁵This class of hierarchical models has proved useful in many other contexts such as Trade (Antràs et al.

relevance to understand the role played by human capital, sorting and occupational choice in explaining both firm level and cross-country outcomes. Among other things, we find that the ability of a talented individual to spread his knowledge is limited in poorer countries because of the scarcity of talent. This makes it harder to delegate some of the tasks necessary for production. Yet, some of our results would continue to hold in other environments that features complementarities and multiple-worker firms. Eeckhout and Kircher (2012) derive implications of the assignment of multiple workers to firms with a very general production technology. Grossman et al. (2014) extend the previous paper by introducing multiple sectors and study the distributional consequences of trade. While some of our results would also hold in these settings, the focus of our work differs. This leads us to consider occupational choice and demonstrate that it is important for understanding how the organization of production vary across countries. We also endogenize the distribution of skills across countries by explicitly modeling human capital acquisition. A related paper by Bhattacharya et al. (2012) also examines human capital accumulation by entrepreneurs but workers are homogeneous and do not accumulate human capital. Hence, there is no sorting and the efficiency units assumption holds.

Other explanations have been proposed for the lack of firm growth and the presence of a large number of unproductive firm in poorer countries such as selection mechanisms as in Jovanovic (1982) or the presence of inefficiencies in financial markets as in Cooley and Quadrini (2001); Cagetti and De Nardi (2006); Buera et al. (2011); Midrigan and Xu (2014). While financial development is of great importance for the modernization of an economy and firm entry, it is less obvious that it can explain the fact that larger firms are more productive since firm productivity is very persistent. This should allow more productive firms to grow out of these financing constraints (see Midrigan and Xu, 2014). We abstract from these other forces in order to stress the fact that skills heterogeneity and occupational choice improve our understanding of various empirical regularities reported in these papers.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 describes the model calibration and examines the implications of the model for earnings and firm heterogeneity within a country. Section 4 presents the cross-countries analysis. Section 5 concludes.

(2006); Caliendo and Rossi-Hansberg (2012)) and Growth (Chatterjee and Rossi-Hansberg (2012); Garicano and Rossi-Hansberg (2012)).

2 Model

We consider an economy populated by overlapping generations of people who accumulate human capital using time and intermediate inputs as in Ben-Porath (1967). Individuals are endowed with an initial stock of human capital and a learning ability which remains fixed throughout their finite lifetime. Every period they choose whether to be entrepreneurs or workers as in Lucas (1978). Each entrepreneur supervises several workers in a knowledge hierarchy as in the theory developed by Garicano (2000) and further analyzed by Garicano and Rossi-Hansberg (2006). This section presents the environment and shows that the production technology leads to complementarities in production. We then derive optimal decisions and we characterize the equilibrium.

2.1 Environment

2.1.1 Production

Production involves problem solving and occurs in firms. At any point in time, an agent production depends on his human capital h , his time spent producing n and his occupation. A firm consists of an entrepreneur characterized by (h_e, n_e) and l_s workers characterized by (h_w, n_w) . We take these variables as given here to focus on the description of the production process. They will be determined endogenously later on.

Workers spend their unit of time in production n_w to draw problems. Depending on their human capital they are able to solve a fraction $G(h_w)$ of the problems they drew where G is a cumulative distribution function. When a worker cannot solve a problem, he communicates the problem to his entrepreneur with a communication cost per problem in units of time $c \in [0, 1]$ incurred by his entrepreneur. Thus, each worker needs his entrepreneur attention for $(1 - G(h_w)) n_w$ problems that he was not able to solve himself.

The entrepreneur has n_e unit of time for production which constrains his ability to supervise workers. With l_s workers, the entrepreneur spends $c(1 - G(h_w)) n_w l_s$ units of time supervising them. Using his time constraint, it follows that

$$l_s = \frac{n_e}{c(1 - G(h_w)) n_w} \quad (1)$$

Equation 1 shows that firm size is an increasing function of workers human capital h_w . The ability of an individual to spread his human capital is constrained by the human capital of his workers. This is different from Lucas (1978) where the size of firms is determined

by entrepreneurial talent and an exogenous “span of control” parameter. Here, the span of control of entrepreneurs is endogenously determined by workers’ human capital. Firm size depends on the entrepreneur human capital h_e through the matching function which assigns different workers to different entrepreneurs. Since there will be positive sorting at the equilibrium, more skilled entrepreneurs have larger firms.

As in Lucas (1978), entrepreneur human capital determines output and productivity. The fraction of problems an entrepreneur of type h_e can solve is $G(h_e)$. Therefore, the total number of problems solved y by the entrepreneur and its workers is

$$y = G(h_e) \times l_s \quad (2)$$

To illustrate the difference between this production process and the standard human capital model where firm production is not modeled, consider, again, an entrepreneur with human capital h_e and l_s workers with human capital h_w . If these individuals were not working in teams, output would be

$$G(h_e) + G(h_w) \times l_s$$

With the existence of firms, a talented individual has the possibility to spread his human capital h_e over a larger scale than if he were working for himself by employing l_s workers. The entrepreneur has a central role in determining firm productivity: a worker is endowed with his entrepreneur productivity rather than his own. This can be seen by comparing the two expressions above. When workers produce on their own, l_s is multiplied by $G(h_w)$ while it is multiplied by $G(h_e)$ with the existence of firms. There are indeed complementarities in production: a worker benefits from working for a more productive entrepreneur since the fraction of problem solved are effectively determined by his entrepreneur skills. Similarly, an entrepreneur benefits from working with better worker since they are able to solve more problem by themselves and use less of his limited time. In this setting, the efficiency units assumption does not hold. An entrepreneur is not indifferent between one worker of some quality and two workers that are half as productive. More workers allows to draw more problems but lead to a higher time-cost. More talented worker allows to economize on the entrepreneur time-endowment so a higher wage rate for more productive workers will ensure a well defined demand for quality.

Finally, physical capital k is not differentiated by quality. Some workers l_u do not participate in the problem-solving activities and instead only supply their raw-labor. Physical capital and raw labor enter the production function in a conventional Cobb-Douglas form.

Firm-level production function is thus:

$$z (G(h_e)l_s)^{\alpha\theta} l_u^{(1-\alpha)\theta} k^{1-\theta}$$

where z is an economy-wide efficiency level, α and θ are share parameters in $[0, 1]$. The size of the firm l is the sum of the number of workers participating to problem solving activities l_s and the number of workers providing raw labor l_u :

$$l = l_s + l_u$$

Note that worker time producing n_w does not appear in firm output. This is because n_w and the number of workers l_s are perfect substitutes. This anticipates a feature of the equilibrium. Sorting between workers and entrepreneurs will be based on human capital levels and not on human capital accumulation decisions.

Some evidence in favor of this production technology can be found in firm-level regressions reported by Gennaioli et al. (2013). They find that the impact of average worker education on firm productivity is of roughly the same magnitude as the impact of the entrepreneur education. Given that we are comparing one individual in the firm to all its workers, it suggests a primordial role for entrepreneurial talent.

2.1.2 Human Capital Accumulation

Individuals are born with human capital h_1 distributed according to cdf $F_1(\cdot)$ truncated above at an arbitrary large value. Individuals are endowed with one unit of time each period. They spend a fraction n_t of their time producing and a fraction $1 - n_t$ accumulating human capital for the future. Producing human capital also uses market inputs x which represents schooling and on-the-job training expenses. There are J types of agents who differ in their ability to learn s^j . The production technology is

$$h_{t+1} = s^j ((1 - n_t)h_t)^{\gamma_1} x^{\gamma_2} + (1 - \delta) h_t, 1 < t \leq T - 1$$

δ is the depreciation rate of human capital. This production technology was first proposed by Ben-Porath (1967). We adopt the standard interpretation of the time-period where $n = 0$ as the schooling period while $n \in (0, 1)$ corresponds to on-the-job training. To keep the model tractable, we assume human capital is general (not specific) and labor markets are competitive. In such a setting, the cost of on-the-job investment will be borne by workers

(Becker, 1973).

Let the distribution of human capital be f_{tj} for individuals in period $t = 1, 2, \dots, T$ and with ability $j = 1, \dots, J$. f_{1j} is the initial exogenous distribution of human capital. In period $t + 1$, the density evolves according to human capital accumulations decisions.

2.1.3 Demographics

We assume that each individual has ρ children at age B . It implies that the number of people of age a at time t denoted $N(a, t)$

$$N(a, t) = \rho N(B, t - a)$$

and $N(T, t) = 0, t > T$. It is easy to check that the stationary distribution of the population ϕ satisfies

$$\phi(a) = \frac{\rho^{-\frac{a}{B}}}{\sum_{a'=0}^T \rho^{-\frac{a'}{B}}}$$

2.2 Optimal Decisions

2.2.1 Entrepreneur's Problem

The entrepreneur's value function in period t is $V_{it}^e(h_e)$ where e stands for entrepreneur, t indexes age and i indexes learning ability s^i . The entrepreneur chooses his time spent in production n_e , intermediate inputs x_e , workers' human capital h_w , raw labor l_u and physical capital k . He solves thus,

$$\begin{aligned} V_{it}^e(h_m) = \max_{n_e, x_e, h_e, l_u, k} & \left(z \left(\frac{G(h_e)n_e}{c(1 - G(h_w))} \right)^{\alpha\theta} l_u^{(1-\alpha)\theta} k^{1-\theta} - w(h_w) \frac{n_e}{c(1 - G(h_w))} \right. \\ & \left. - w_u l_u - p_k(r + \delta_k)k - p \cdot x_e \right. \\ & \left. + \beta W_{it+1} \left(s^i ((1 - n_e)h_e)^{\gamma_1} x_e^{\gamma_2} + (1 - \delta) h_e \right) \right) \end{aligned} \quad (3)$$

Time spent producing by workers n_w and the number of workers are perfect substitutes leading to a wage function $\tilde{w}(h_w, n_w) = w(h_w)n_w$ that is linear in n_w . w_u is the wage rate for worker providing raw-labor. This is a small open economy with a fixed interest rate r and without borrowing constraints. We normalize the price of the final output good to 1. p_k is the relative price of capital and δ_k the depreciation rate of physical capital.

The continuation value W is the maximum of the value of being a worker or being an entrepreneur in the next period.

$$W_{it}(h) = \max \{V_{it}^w(h), V_{it}^e(h)\}, 0 < t \leq T \quad (4)$$

Solving for the optimal choices of physical capital and raw labor, the entrepreneur problem simplifies to:

$$\begin{aligned} V_{it}^e(h_m) = & \max_{n_e, x_e, h_w} \left(A \left(\frac{G(h_e)n_e}{c(1-G(h_w))} \right) - \frac{w(h_w)n_e}{c(1-G(h_w))} - p \cdot x_e \right. \\ & \left. + \beta W_{it+1} (s^i ((1-n_e)h_e)^{\gamma_1} x_e^{\gamma_2} + (1-\delta)h_e) \right) \end{aligned} \quad (5)$$

where A is a constant.⁶ Optimal worker quality h_w satisfies:

$$w'(h_w) = g(h_w) \frac{AG(h_m) - w(h_w)}{1 - G(h_w)} \quad (6)$$

The left hand side of this equation is the marginal cost of hiring a marginally better worker. The right hand side is the marginal benefit which is a combination of being able to solve more problems and an increased ability to leverage individual talent by managing a larger team.

In Equation 6, time spend producing, either n_w or n_e , does not appear. As a consequence, matching between worker and entrepreneur depends only on the current level of human capital. Time spent producing does not affect the sorting patterns because from the point of view of the entrepreneur, time spent producing and number of workers are perfect substitutes. Second, each entrepreneur will hire one type of worker. This is only made possible by the continuity of the distribution of human capital in the population.

The optimality conditions with respect to x_e and n_e are

$$\begin{aligned} p & \leq \beta s \gamma_2 (1-n_e)^{\gamma_1} h_e^{\gamma_1} x_e^{\gamma_2-1} W'_{it+1} (s ((1-n_e)h_e)^{\gamma_1} x_e^{\gamma_2} + (1-\delta)h_e) \\ \frac{AG(h_e) - w(h_w)}{c(1-G(h_w))} & \leq \beta s \gamma_1 (1-n_e)^{\gamma_1-1} h_e^{\gamma_1} x_e^{\gamma_2} W'_{it+1} (s ((1-n_e)h_e)^{\gamma_1} x_e^{\gamma_2} + (1-\delta)h_e) \end{aligned}$$

where the first equation holds with equality if $x > 0$ and the second equation holds with

⁶Precisely, $A = z^{\frac{1}{\alpha\theta}} w_u^{\frac{\alpha-1}{\alpha}} (p_k(r+\delta_k))^{\frac{\theta-1}{\alpha\theta}} \left(C_1^{(1-\alpha)\theta} C_2^{1-\theta} - C_1 - C_2 \right)$ with $C_1 = ((1-\alpha)\theta)^{\theta} (1-\theta)^{1-\theta}$ and $C_2 = ((1-\alpha)\theta)^{\frac{\alpha-1}{\alpha}} (1-\theta)^{\frac{1-(1-\alpha)\theta}{\alpha\theta}}$.

equality if $0 < n_e < 1$. At an interior solution, taking the ratio of the two optimality conditions leads to:

$$x_e = \frac{\gamma_2(1 - n_e)}{\gamma_1} \frac{\frac{AG(h_e) - w(h_w)}{c(1 - G(h_w))}}{p}$$

The entrepreneur has an incentive to accumulate human capital since it allows him to solve more problems and to be matched with better workers which increases both firm size and productivity. The costs of accumulating human capital are forgone earnings while learning new skills.

We substituted the time constraint of the entrepreneur that determined firm size in the objective function so that the time spent by workers producing does not appear. To formalize things, the entrepreneur chooses to hire workers of human capital h_w . These workers may be of different ages t and ability s^j . Let l^{tj} be the number of worker of each type. Using the time constraint of the entrepreneur, it is clear that the equation

$$c(1 - G(h_w)) \sum_{j=1}^J \sum_{t=1}^T l^{tj} n_{wtj} = n_e$$

holds. There are an infinite number of linear combinations of l^{tj} that satisfy the constraint. We pick one such that the labor market clears (see below) and satisfies the constraint.

2.2.2 Worker's Problem

We now turn to the worker's problem which is a standard human capital decision problem with two additional features. First, the wage rate $w(h)$ is not necessarily linear in human capital and second, the continuation value W takes into account the fact that a worker may switch occupations and become an entrepreneur as he gets older. Formally, the value function of a worker of age t , ability s^i and with human capital h_w is

$$V_{it}^w(h_w) = \max_{n_w, x_w} n_w w(h_w) - p \cdot x_w + \beta W_{it+1} (s^i ((1 - n_w) h_w)^{\gamma_1} x_w^{\gamma_2} + (1 - \delta) h_w) \quad (7)$$

The optimality conditions with respect to x_w and n_w are

$$\begin{aligned} p &= \beta s \gamma_2 (1 - n_w)^{\gamma_1} h_w^{\gamma_1} x_w^{\gamma_2 - 1} W'_{it+1} (s ((1 - n_w) h_w)^{\gamma_1} x_w^{\gamma_2} + (1 - \delta) h_w) \\ w(h_w) &\leq \beta s \gamma_1 (1 - n_w)^{\gamma_1 - 1} h_w^{\gamma_1} x_w^{\gamma_2} W'_{it+1} (s ((1 - n_w) h_w)^{\gamma_1} x_w^{\gamma_2} + (1 - \delta) h_w) \end{aligned}$$

where the first equation holds with equality if $x > 0$ and the second equation holds with equality if $n_w \in (0, 1)$. At an interior solution, taking the ratio of the two FOCs leads to

$$x_w = \frac{\gamma_2(1 - n_w)}{\gamma_1} \frac{w(h_w)}{p}$$

A worker has incentives to accumulate human capital since it allows him to be matched with a better entrepreneur which will pay him a higher wage rate. His investment will depend on time and intermediate inputs in a proportion that depends on the human capital production technology parametrized by γ_1 and γ_2 .

2.2.3 Occupational Choice

An agent with human capital h chooses the occupation that gives the highest utility:

$$W_{it}(h; s) = \max \{V_{it}^w(h), V_{it}^e(h)\}$$

The technology to accumulate human capital is the same for workers and entrepreneurs. Workers and entrepreneurs cannot commit to long term contracts: they re-match every period. It follows that the occupational choice is independent of age and learning ability. In other words, it is purely static. Hence, every period the individual decides to be a entrepreneur or a worker according to

$$\max \left\{ \frac{AG(h) - w(h_w)}{c(1 - G(h_w))}, w(h) \right\}$$

where h_w is the optimal choice of worker human capital by the entrepreneur.

Most of our analysis resorts to numerical methods since relatively few theoretical results can be established as is usual with heterogeneous agent dynamic models. Yet, conditional on human capital accumulation decisions, Proposition 1 shows that the organization of production features positive sorting and the set of entrepreneurs and workers is connected with entrepreneurs more talented than workers. We relegate the proof to the Appendix since it is a simple generalization of similar results in Antràs et al. (2006) and Garicano and Rossi-Hansberg (2006).

Proposition 1. *If the working time-weighted distribution of human capital is absolutely continuous and compact-valued and if an assignment function m exists, there exists \tilde{c} such that if $c < \tilde{c}$,*

1. *equilibrium features positive sorting: $h_m = m(h_w)$ with $m' > 0$.*
2. *the set of entrepreneurs and the set of workers is connected such that below (above) an endogenous threshold h^* , an individual decides to be a worker (entrepreneur).*

Proof. See the Appendix. □

We showed that the set of entrepreneurs and workers is connected so that there is a marginal individual with human capital h^* who is indifferent between the two occupations,

$$\frac{AG(h^*) - w(h_w^*)}{c(1 - G(h_w^*))} = w(h^*) \quad (8)$$

where h_w^* denotes the optimal worker quality chosen by the marginal entrepreneur.⁷ Workers can either be problem-solvers or supply their raw labor. There is a threshold \tilde{h} above which a worker becomes a problem-solver and below which he supplies raw-labor and perceives a wage w_u such that:

$$w(\tilde{h}) = w_u$$

The function $m(h_w) = h_e$ depicts the allocation of workers to entrepreneurs. It is defined from $[\tilde{h}, h^*]$ to $[h^*, \bar{h}]$. The lowest human capital problem-solvers (type \tilde{h}) are assigned to the lowest human capital entrepreneurs h^* and the highest human capital workers h^* are assigned to the highest human capital entrepreneurs \bar{h} .⁸ These are summarized by the two boundary conditions:

$$\begin{aligned} m(\tilde{h}) &= h^* \\ m(h^*) &= \bar{h} \end{aligned}$$

Workers in the interval $[\underline{h}, \tilde{h}]$ supply their raw labor and our model is silent on their assignment to firms since they are all equivalent from the entrepreneur point of view.

⁷This sharp dichotomy between workers and entrepreneurs is a feature of many models of occupational choice. Yet, it is likely that in practice the lowest human capital entrepreneur does not have a higher human capital than the highest human capital worker. There are several ways to account for this feature of the data that would not alter the main message of this paper. For instance, individuals could differ along two dimensions: human capital h and entrepreneurial talent as measured by the communication cost c (which could vary across individuals). McCann et al. (2014) provide a theoretical model with such a feature. We do not pursue it here since it is not essential to our main point.

⁸The highest human capital entrepreneur can be formally defined as $\bar{h} = \max \{h_{tj} : f_{tj}(h) > 0, t = 1, \dots, T, j = 1, \dots, J\}$

2.3 Equilibrium

An equilibrium is characterized by the individual policy functions $\{n_{wtj}, n_{etj}, x_{wtj}, x_{etj}, l_s, l_u, k\}$ and the equilibrium objects $\{f_{tj}, m, w, h^*, \tilde{h}\}$. These functions and variables are determined by the following conditions:

1. m, l_u, k, x_{etj} and n_{etj} are determined by entrepreneurs' first-order conditions.
2. x_{wtj} and n_{wtj} is determined by workers' first-order conditions.
3. h^* and \tilde{h} are determined by two indifference conditions for marginal individuals.
4. w and l_s are determined by labor market equilibrium and entrepreneurs' time constraint.
5. f_{tj} is determined by the human capital production technology.

The labor market is competitive and the wage function w is such that demand equals supply in the labor market. Precisely, an entrepreneur is endowed with a fixed time input that he partially allocates towards solving problems. Firm size is then determined by the production technology that constrains labor demand by each entrepreneur. The matching function m determines the allocation of workers to entrepreneurs given the technological constraint and the time spent producing by each agent. The wage function sustains this allocation as an equilibrium outcome in the sense that w is such that the optimal choice of worker type, derived in Equation 6, is satisfied for each entrepreneur.

Formally, labor demand $l^{\tau itj}(h_w)$ of workers h_w , aged τ and ability s^i , by an entrepreneurs h_e , aged t and ability s^j satisfies entrepreneurs time constraint:

$$c(1 - G(h_w)) \sum_{i=1}^J \sum_{\tau=1}^T l^{\tau itj}(h_w) n_{w\tau i}(h_w) = n_{etj}(h_e)$$

For any $h_w \leq h^*$, equilibrium in the labor market for worker aged $\tau = 1, \dots, T$ with ability $s^i, i = 1, \dots, J$ is such that:

$$\int_{\tilde{h}}^{h_w} f_{\tau i}(h) dh = \sum_{j=1}^J \sum_{t=1}^T \int_{h^*}^{m(h_w)} l^{\tau itj}(m^{-1}(h)) f_{tj}(h) dh$$

The left-hand side of the equation is the supply of workers in period τ , with ability i and skill level below h_w . The right-hand side is the demand for these workers by entrepreneurs

in period t with ability j . Differentiation with respect to $h \leq h^*$ gives:

$$f_{\tau i}(h) = m'(h) \sum_{j=1}^J \sum_{t=1}^T l^{\tau i t j}(h) f_{t j}(m(h))$$

This gives $J \times T + 1$ equations. But there are $(J \times T)^2 + 1$ unknowns: $l^{\tau i t j}$ and m' for every $h \leq h^*$. The multiplicity of solutions comes from the perfect substitutability between producing time and number of workers. This is not an important issue since these different solutions lead to the same output, employment and wages. The only difference between these solutions is the age distribution of workers across firms of the same productivity level. Since our theory has little to say on these objects we make the simplest additional assumptions to obtain a unique solution. We assume that given h , entrepreneurs of different ages and skills have the same labor demand for workers of particular age up to a factor of proportionality equal to the time an entrepreneur spends producing $n_{etj}(h)$:

$$l^{\tau i t j}(h) = \tilde{l}^{\tau i}(h) \times n_{etj}(m(h))$$

We now have a system of $J \times T + 1$ equations with $J \times T + 1$ unknowns $\tilde{l}^{ti}, t = 1, \dots, T; i = 1 \dots, J$ and m' . The labor market equilibrium condition can be rewritten:

$$m'(h) = c(1 - G(h)) \frac{\sum_{i=1}^J \sum_{t=1}^T f_{tj}(h) n_{wti}(h)}{\sum_{j=1}^J \sum_{t=1}^T n_{etj}(m(h)) f_{tj}(m(h))} \quad (9)$$

The derivative of the matching function at a particular human capital level is equal to the ratio of the density of workers at this level divided by the density of entrepreneurs they are matched with divided by the size of the firm.

2.4 Parametrization

The initial distribution of human capital F_1 is lognormal with mean μ_h and variance σ_h^2 and is truncated at an arbitrarily large value. The distribution of learning ability s^j is a discrete approximation of a log-normal distribution with mean μ_s and variance σ_s^2 .⁹ The distribution of problems g is exponentially distributed with parameter λ : $g(h) = \lambda e^{-\lambda h}$. We model individual decisions from age 6 until retirement at age 65. The model period corresponds to

⁹We follow the procedure proposed by Kennan (2006).

Table 1: Data Moments

Moments	Data	Model
Gini coefficient of lifetime earnings	0.3	0.3
Average years of schooling	12.5	12.6
Schooling expenditures	4.2	4.1
Wage rate at age 55/ age 25	1.9	1.7
Average plant size	10.7	11.3
Entrepreneurship rate	10.1	9.9
Average plant size at age 40 / age 5	4.0	3.9
Plant-size - wage premium	4.0	4.3

Table 2: Model Parameters

c	λ	α	μ_s	σ_s	σ_h	γ_1	γ_2
0.84	0.05	0.72	-2.06	1.15	2.47	0.58	0.34

a year. We set the depreciation rate $\delta = 0$ since most of the decline in earnings towards the end of the life cycle is due to the decline in hours worked. The discount factor corresponds to an annual interest rate of 5%. The average level of human capital at age 6, μ_h is a scaling parameter and is normalized to a convenient value. We set the capital share θ to 0.33, a commonly used value.

The solution algorithm is as follows. For given parameters values, we use initial guesses for the policy functions to simulate the distribution of human capital in the economy $\{f_{tj}\}$. Given the distribution of human capital, we derive the matching function and the wage function to clear the labor market. We then update the policy functions that solve each agent decision problem. We repeat this procedure until we obtain convergence of the matching function. The Appendix describes the details of the numerical algorithm.

3 Model Calibration and Implications for Earnings and Firm Size Distribution

By nesting a model of occupational choice and endogenous skill accumulation in a framework in which the span of control is endogenous, we develop a rich framework that yields a number of empirical implications. This section examines the implications of the model for the cross sectional and time series patterns of individuals and firm outcomes. We calibrate the parameters $(\lambda, c, \alpha, \gamma, \gamma_x, \mu_s, \sigma_h, \sigma_s)$ to match certain features of the U.S. economy. Table 1 lists the moments that we match. We report the calibrated parameter values in Table 2.

Some of the moments we target are fairly standard. We target 12.5 years of schooling following Barro and Lee (2013). The Gini coefficient of lifetime earnings is set to 0.3 which is the typical value that has been calculated using the PSID. This provides information on individuals heterogeneity in our model, and in particular the parameter σ_s and σ_h . According to the UNESCO Institute for Statistics, schooling expenditures represents 4.2% of GDP. It is useful to pin down the share of intermediate inputs in the human capital production technology γ_2 .

The remainder of this Section discusses how other moments help pin-down the model parameters. Our model is parsimoniously calibrated and for this reason we chose a conservative strategy. Some forces outside the model are likely to affect the moments we used and we took some care reflecting these concerns in our calibration strategy. As a result, the calibration leads to a fairly high communication cost c : the main benefit of creating firms for talented individuals is to allow them to deal with exceptions and spread their human capital over a larger scale.

3.1 Firm Heterogeneity

We consider an economy where individuals skills are central determinants of firm size, productivity and wages. As in Lucas (1978), entrepreneurial talent determines firm productivity in the sense that an entrepreneur endows his workers with his own human capital. A firm's productivity is equal to $AG(h_e)$ where h_e is the entrepreneur human capital. Hence, at an efficient allocation, there exists dispersion in productivity across firms because some entrepreneurs are more talented than others. The existence of productivity dispersion across firms in equilibrium is a feature of several economic models. What is new to our framework is that labor productivity is not equalized across firms even though the allocation of human capital is efficient. We consider a production technology where the efficiency units assumption does not hold: labor quantity is an imperfect substitute for labor quality. Hence, the average productivity of labor is not equalized across firms. High productivity firms attract high productivity employees which keeps labor productivity high. Further because of the constraint on entrepreneur time, high productivity firms are large since the best entrepreneurs attract the best workers which allow them to delegate a larger set of tasks and only use their limited time for the more difficult problems. On the other hand, low quality entrepreneurs attract low quality workers leading to a low productivity of labor. Labor productivity dispersion is not due to idiosyncratic distortions and is instead due to the sorting between heterogeneous entrepreneurs and heterogeneous workers.

Table 3: Size of firms, education of workers and education of entrepreneurs

Size Category	Owners' education					
	Average	Drop-out	High school	Some College	College	Post-College
1-24	14.3	6.02	20.17	39.00	22.61	12.20
25-99	14.6	4.41	17.50	37.77	25.76	14.57
100+	15.1	2.75	14.04	33.42	26.35	23.44

Size Category	Workers' education					
	Average	Drop-out	High school	Some College	College	Post-College
1-24	13.2	15.17	30.65	36.52	13.68	3.97
25-99	13.5	12.78	31.08	35.97	15.45	4.72
100+	13.8	9.21	29.00	34.76	19.20	7.82

Source: Survey of Income and Program Participation (SIPP). To measure education, we group individuals into five broad education attainment categories based on the highest level of school or degree completed: high school drop-out, high school graduate, some college, college graduate and post-college. We remove individuals under 18 and over 65, in the agricultural sector or non-profit. Plant size is reported in one of three intervals: 1-24, 25-99, or more than 100.

Because better workers are assigned to better entrepreneurs, entrepreneurs delegate a larger sets of tasks to workers in more productive firms. In order to attract these workers, he needs to pay higher wages that sustain the efficient allocation of talent. Consequently, wages, output and labor productivity are all positively correlated with firm size.

Some empirical evidence in favor of these arguments exists in the literature. It is well known that larger firms are more productive and pay higher wages (see for instance Oi and Idson, 1999). The role of worker skills and entrepreneurs skills is documented in Gennaioli et al. (2013). Bloom et al. (2013) find that management practices can explain some of the productivity dispersion across firms. And they also find that good management practices are positively correlated with the fraction of workers with a college degree. Finally, there are large and persistent differences in productivity levels across firms within the same industry (see Syverson, 2011). In our model, these facts are not the result of inefficiencies and are the equilibrium outcomes of a frictionless economy where individuals skills matter.

To provide further evidence, we examine the education attainment of business owners and their workers in the Survey of Income and Program Participation (SIPP). We pool 1996, 2001, 2004 and 2008 panels. Owners are those that report that they own a business and work more hours in their business than their salaried job. We consider individuals aged between 18 and 65 who work in for-profit and non-agricultural companies. The SIPP data

report plant size in one of three intervals: 1-24, 25-99, or more than 100. For each plant size category, Table 3 reports average years of education and the proportion of businesses whose owners' education falls into five broad education attainment categories measured by the highest level of school or degree completed: high school drop-out, high school graduate, some college, college graduate and post-college. Similarly, for each size category we report the proportion of workers in each schooling category. Larger firms are owned by more educated individuals. More than 50% of firms with more than 100 employees are owned by workers with at least a college degree as opposed to 35% for firms with less than 25 employees. Similarly, larger firms employ more educated workers. For instance, firms with more than 100 employees have more than twice as many employees with a post-college degree relative to firms with less than 25 employees.

We also look at the impact of plant size on worker wage. This helps separate workers solving problems from workers supplying raw labor in our model. Because the efficiency units assumption holds only for the latter, a strong plant size-wage correlation is informative on the fraction of workers for which sorting matters. The wage measure is a self-reported hourly wage. Table 4 lists the coefficient on each plant size category dummy from the regression of log wage on plant size controlling for race, gender, age, industry and occupation. The results are presented with and without education dummies in, respectively, Column (1) and Column (2). Workers receive 17% more pay in establishments of 100+ workers than in plants with 1-24 workers and the corresponding coefficient is statistically significant. The plant-size wage gap percentage between plants with 25-99 workers and firms with 1-24 workers is also statistically significant and below 3%. The specification with education dummies only slightly attenuates the coefficient on plant size. This is as expected from our theory since education is an imperfect proxy for skills. It also suggests there are other motives that leads larger firms to pay higher wages. In our calibration, we target a premium of 4% which is in the lower range of the estimates reported in the literature (see the survey by Oi and Idson, 1999).

3.2 The Life-Cycle of Wages and Firms

The life-cycle of earnings and firm size have been documented in several papers but rarely have the two facts been connected. For instance, earnings of high school graduates increase by about 50% in the first 10 years of their working life in the PSID. On the firm side, it is well known that conditional on survival, young firms grow more rapidly than their more mature counterparts. Our model captures both facts simultaneously. Wages grow over time through

Table 4: Wage - Firm Size Premium

Variables	(1)	(2)
25-99	0.0272 (75.87)	0.0251 (72.93)
100+	0.1741 (101.11)	0.1615 (91.91)
Education Dummies	No	Yes
Observations	318680	318680
R^2	0.2272	0.3254

Source: Survey of Income and Program Participation (SIPP). Wage measure is self-reported hourly wage rate. OLS regression of log wage on dummies for each plant size bin controlling for age, race, gender, industry and occupations. The Table reports the number of observations and the R^2 . t-stats are shown in parentheses.

three channels: (1) human capital accumulation h_w , (2) time spent in production n_w and (3) matching with a better entrepreneur $m'(h_w) > 0$. The first two channels are standard in any human capital accumulation model dating back to at least Ben-Porath (1967). Most models of human capital accumulation do not attribute a role to the firm in explaining earnings growth of workers. Yet, matched employer-employee datasets (see Abowd et al. (1999) and more recently Card et al. (2013)) reveal the importance of firms in explaining individual earnings. This is exactly what our theory predicts: some of the earnings of an individual are due not only to his own human capital but also to the human capital of his entrepreneur. Similarly, the size of a firm grows through three channels: (1) the accumulation of human capital h_e , (2) time spent in production n_e and (3) the ability to attract better workers $m^{-1'}(h_e) > 0$.

We use these two features to discipline our model parameters. For individuals, the target value for the earnings growth is the ratio of wage rate at age 55 and wage rate at age 25 which is 1.9 in the PSID. This number is similar to the figure reported in studies that estimate lifecycle earnings profiles from the PSID. For firms, we use the numbers reported in Hsieh and Klenow (2014). They find that in the cross-section, the average plant over the age of forty is about eight times larger than the average plant under the age of 5. Following a cohort of new establishments in 1967, they find an even larger number: average plant size increases by a factor of 10 from birth to age 25. However, some growth in average employment of a cohort can be attributed to the exit of small establishments. Survivors grow by a factor of 4. Since our model is silent on entry and exit, we set the growth of firms to be equal to 4 which is a conservative number.

3.3 Occupational Choice

According to our model, entrepreneurs are on average older than workers and have a higher level of human capital. This is because human capital increases over time and consequently people cross the threshold after a certain age. In the calibrated model, the fraction of entrepreneurs increases from less than 5% at age 20 to around 15% at age 40. The high learning-ability types have a higher fraction of entrepreneurs. With 5 learning-ability types, the lowest group has fewer than 5% of entrepreneurs while the highest group has more than 20% of entrepreneurs. It formalizes an insight of Lucas (1978):

people tend to move from employee to managerial status later in their careers (as opposed to immediately upon entry to the workforce, as predicted by the theory above); those that make this transition tend to be among the most skilled employees. These facts suggest the existence of a kind of human capital which is productive both in managing and in working for others, and which is accumulated most rapidly as an employee.

These empirical implications can be found in the data. First, Table 3 showed that entrepreneurs are on average more educated than workers. Second, we look at the American Community Survey for 2008 to report the fraction of entrepreneurs by age. We use two different measures. The first one is the closest to the model and defines entrepreneurs in the data as being a business owner as opposed to being employed (and working for someone else). The fraction of business-owners in the data is 10.1% which we include as a calibration target. A broader interpretation of our model, sometimes adopted in the literature using hierarchical models,¹⁰ is to interpret the agents at the top of organization as managers. Using the occupation categories defined by Autor and Dorn (2013), we consider both the narrow category composed of “Chief executives” and the broader category “Executives, Administrative and Managerial Occupations”. In 2008, the proportion of CEOs in the sample is 0.7% while the proportion of individuals in the broad Managerial category is about 10.4%.

Figure 1 reports how the fraction of the population in one these categories varies by age on a log scale. Consistent with our model, the proportion of individuals choosing these particular occupation categories goes up with age. While about 0.1% of the population is categorized as CEO at age 20, this proportion rises to 1% at age 40 and remains constant until retirement. Similarly, the fraction of individuals classified as managers is about 2% at age 20, rises quickly to 8% at age 30 and reaches a plateau of 15% at age 40 until retirement.

¹⁰See for instance Caliendo et al. (2014).

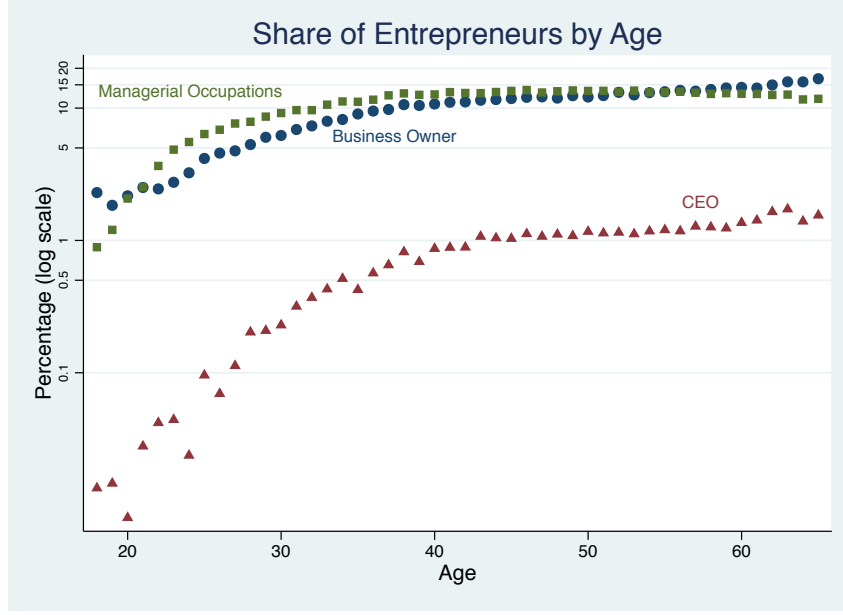


Figure 1: Occupational Choice by Age

Source: American Community Survey for 2008. Occupation measure refers to prior year's employment. The figure plots the percentage of the population in one of the three entrepreneurs measures by age for individuals between 18 and 65 years old. The y-axis is on log scale.

The fraction of business owners closely track the fraction of managers but show a steady rise with age.

A statistic closely related to occupational choice is the average plant size.¹¹ Hence, we hence include it as a calibration target. Using the U.S. Economics Census, we find that average plant size is 10.7 once we take into account self-entrepreneurs (to which we assign zero employees by construction). This helps pin down the communication cost c and the parameter of the problem distribution λ . We focus on the average plant size since it has a clear interpretation in our model. Our model could be extended in two directions to fit the entire firm size distribution. First, we could generalize the technology to more than two layers of production. Without additional sources of heterogeneity across firms, the equilibrium can only sustain two connected levels of hierarchy (see Garicano and Rossi-Hansberg (2006) for a formal proof). In such a setting, it is natural to think about a firm made of a manager and several employees. To have more than two connected levels of hierarchy it is necessary to introduce an additional source of heterogeneity. Caliendo and Rossi-Hansberg (2012) provide this extension where firms sell products of different qualities. We do not pursue it

¹¹They are approximately the inverse of one another.

here since the distribution of product qualities is likely to differ across countries and it would not change the qualitative predictions of the model. Second, we assume that firms have a finite horizon that corresponds to the working horizon of entrepreneurs. An extension of the model would consider how entrepreneurs could sell or transmit their human capital to other individuals when they retire.

With the proposed calibration, it turns out that entrepreneurs spend less time accumulating human capital than workers. There are three different forces at play. First, they have a higher opportunity cost of human capital accumulation. Second, to become entrepreneurs they must have previously accumulated a sufficiently high level of human capital. Finally, there is an opposing force: entrepreneurs have on average a higher learning ability and should accumulate more human capital all else equal. With the proposed calibration, the first two effects dominate.

4 Implications of the Model for Cross-Countries Differences

This section examines the implications of the model for cross-countries differences in the organization of production. To discipline our quantitative exercise, we match output per capita differences by varying the aggregate efficiency level z controlling for exogenous differences in demographic variables and the price of physical capital. This parameter z captures the various aggregate factors, such as institutions, geography, culture or luck, that impact output conditional on inputs.

We then examine our model ability to match firm size, firm growth, occupational choice, and firm-level productivity dispersion across countries. We find that individuals skills and human capital accumulation can substantially improve our understanding of these cross-countries regularities.

4.1 Cross-Countries Differences in Income

The first step to our cross-countries analysis is to vary the aggregate efficiency level z to match output per capita differences. We use the implied variations to evaluate the ability of our model to quantitatively explain differences in the organization of production across countries due to individuals skills and human capital accumulation decisions.

Table 5 reports GDP by deciles of output per worker from Penn World Table 8.0. There

Table 5: Cross-Countries Differences in GDP and aggregate efficiency.

Decile	GDP per capita	Lifespan	Fertility	p_k	z
US	1	77	2.07	1.00	1
90-100	0.87	80	1.65	1.00	0.93
80-90	0.74	79	1.87	0.97	0.81
70-80	0.51	76	1.45	1.14	0.68
60-70	0.35	74	1.91	1.23	0.60
50-60	0.25	70	1.87	1.35	0.52
40-50	0.19	71	2.41	1.10	0.42
30-40	0.12	66	2.69	1.47	0.37
20-30	0.08	62	3.58	1.44	0.28
20-10	0.04	54	4.44	1.34	0.22
0-10	0.02	53	4.79	1.22	0.15

Source: Penn World Table 8.0 (PWT 8.0), World Bank and CIA World Fact-book. GDP per capita is measured as average Real GDP over the years 2003 and 2007 at constant 2005 national prices divided by total workforce. Life expectancy comes from life expectancy at birth from the World Bank. The Fertility rate is measured at the total fertility rate adjusted for the infant mortality rate taken from the CIA World Factbook. The relative price of capital p_k is measured as the ratio of the “price level of capital formation” relative to the “price level of household consumption” from the PWT 8.0 and averaged over the years 2003-2007.

are large variations across countries in standards of living. We choose the value of z in each deciles to match output per worker allowing for exogenous differences in lifespan and fertility rate (which determines the age structure of population) and the price of capital. The last column of Table 5 reports the implied differences in aggregate efficiency normalizing z to one in the US.

Under our calibration strategy, human capital strongly amplifies aggregate efficiency differences across countries: a 7-fold difference in z explains a 50-fold difference in output per worker as is observed between the top 10 percent and bottom 10 percent of countries. The amplification lies in between the numbers reported by Erosa et al. (2010) and Manuelli and Seshadri (2014). We borrow from these two papers the idea that producing human capital requires some physical inputs which are more efficiently produced in richer countries.¹² We now turn our attention to how the human capital of workers and entrepreneurs vary across countries. Figure 2 reports the evolution of the problem solving-ability of different groups of individuals in the economy for different level of aggregate efficiency z .

Workers and entrepreneurs both have higher levels of human capital in richer countries.

¹²We have performed the exercise of this Section using a lower share of intermediate inputs in the production of human capital. As expected, it leads to a larger role for differences in z across countries but leaves the qualitative implications for the organization of production unaffected.

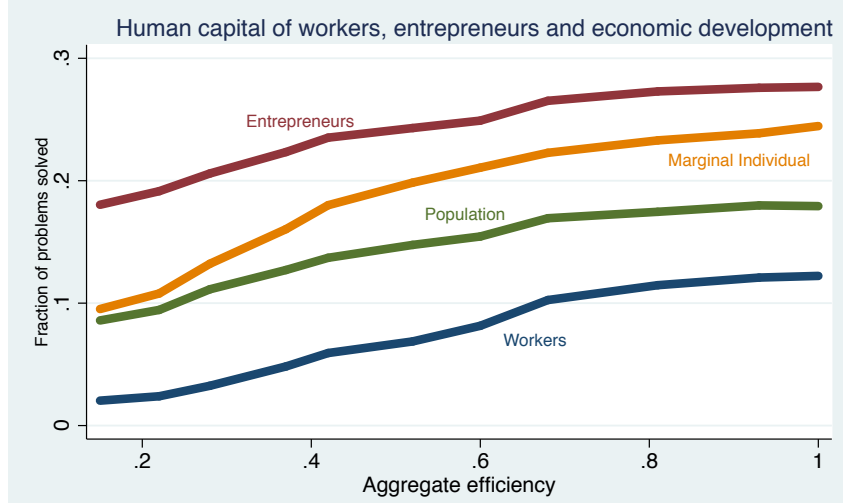


Figure 2: Human Capital of Workers, Entrepreneurs and Economic Development

The average problem-solving ability of entrepreneurs (workers) differs by a factor of 1.5 (6) between the top and the bottom deciles. These cross country variations in the distribution of human capital arise because of individual incentives to accumulate human capital that differ depending on the aggregate level of efficiency of the economy. Further, the average human capital of entrepreneurs is higher than the average human capital of workers in every country reflecting the fact that it is optimal for the most skilled individuals to become entrepreneurs and organize production carried out by less talented individuals.

The problem-solving ability of the marginal individual helps illustrate the main mechanism driving the negative relationship between the share of entrepreneurs and output per capita described in the next subsection. In the poorest country, the marginal entrepreneur has about the same talent as the average individual in the population. As aggregate efficiency increases, the threshold to become entrepreneur h^* increases, reflecting the fact that it requires more and more talent to become an entrepreneur as the economy gets richer. And the average talent of workers increases in the population permitting the creation of large firms. As the result, it requires a disproportionate amount of talent to be an entrepreneur in a richer economy where the gap between the marginal individual and the average individual is larger than in poorer countries.

Are the predictions of the model in line with estimates of human capital? While human capital is not directly observable, these decisions can be traced back to schooling and on the job-training statistics. In the dataset constructed by Barro and Lee (2013), average years of schooling are 12.5 in the US, 8.5 in the median country and 3.6 in the lowest decile

of income. Our model predicts, respectively, 12.6, 9.3 and 2.8. As for on-the-job training decisions, one measure is the steepness of earnings-profile. Lagakos et al. (2012) document that experience-earnings profiles are flatter in poor countries suggesting that lifecycle human capital accumulation is less intense in poorer countries. This is precisely what our model predicts.

Having disciplined our estimates of the aggregate efficiency z of different countries, we can now examine the impact on the organization of production. We first look at the implications for average plant size and occupational choice. Then, we look at firm growth and finally the dispersion of labor productivity.

4.2 Plant Size and Occupational Choice

Comparable firm size distributions across countries are notoriously difficult to find. Yet, some broad patterns have been reported in several studies.¹³ Two robust facts are the positive relationship between average plant size and GDP per capita as well as the negative relationship between the entrepreneurial rate and GDP per capita. The prevalence of smaller firms in poor countries has been measured using different metrics and we focus here on the average size of a plant since there is a clear mapping between worker quality and plant size in our model. Indeed, average plant size is lower in countries where the average level of human capital is low. To get a sense for whether the magnitude matches up with the evidence we use two different data-sets. First, we use the data-set constructed by Bento and Restuccia (2014) which uses census, survey and registry data from more than a hundred countries. We omit countries with a population less than half of one million and countries richer than the US. Second, we use the Global Entrepreneurship Monitor (GEM), collected by a not-for-profit company, Global Entrepreneurship Research Association. It conducted individual level interviews with representative sample of adult across a wide range of countries. Because it is at the individual-level, it captures all the firms, even the very small ones, both in the formal and the informal sectors. We refer the reader to Poschke (2014) which contains additional details on the sources and the construction of the survey. Each dataset has its own advantages. The former has a larger sample size for each country and the numbers comes from more reliable data sources. The latter has the advantage of being at the individual-level and is likely to cover small and informal firms better which are prevalent in poorer countries. Figure 3 reports average plant-size by GDP per capita in the data and in the model. The

¹³See Tybout (2000); Gollin (2008); Ramos and Santana (2013); Poschke (2014); Hsieh and Olken (2014); Bento and Restuccia (2014).

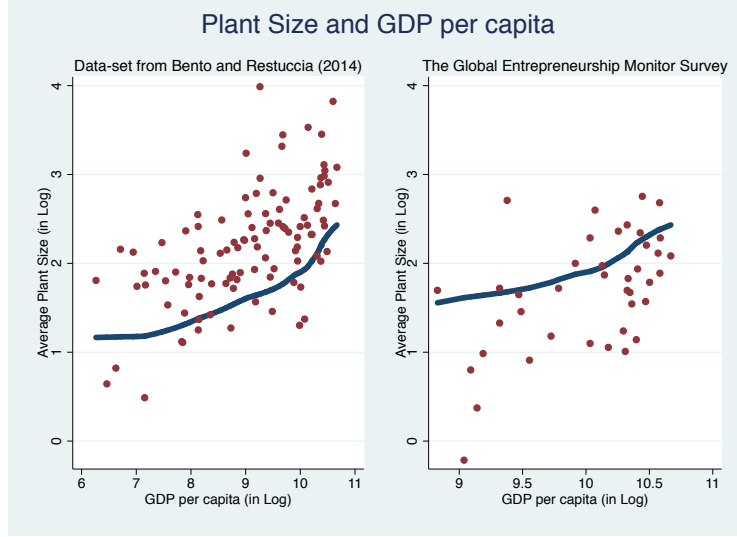


Figure 3: Average plant size across countries

Source: Left Panel: The Global Entrepreneurship Monitor (GEM) survey. Average plant size is pooled across the years 1999 to 2005. Solid line is from the model economy. GDP per Capita is from PWT 8.0. Right Panel: Data from Bento and Restuccia (2014) which combine census, survey and registry data for the manufacturing sector. Each dot represents a country in the database.

magnitudes implied by our model seem in line with evidence even though our exercise does not directly attempt to match average plant size across countries. Our model does a better job at reproducing the household-level data from GEM and it tends to underestimate average plant size in the data-set constructed by Bento and Restuccia (2014). This was expected since our model considers the decision of finite-lived agents and does not consider firms that have an horizon longer than their creator.

We now examine the implications of our calibrated model for occupational choice across countries. We use two definitions of entrepreneur. First, we use data on the share of entrepreneurs across different countries from the International Labor Organization (ILO). We calculate the proportion of employers and own-account owners in the population. We do not make a distinction between self-entrepreneurs since what is really critical for us is that as the economy grows a large fraction of the population decides to work for others and only the most talented individuals hire workers and endow them with their talent. Second, we use data from IPUMS who report individuals occupations into categories that are harmonized across countries. We report the share of individuals in the occupation “Legislators, senior officials and managers”. Figure 4 reports the results and performs the same calculations in the model. The fit is reasonably close to the data in the left panel. The model captures

part of the highly non-linear relationship between GDP and the fraction of business-owners in most of the distribution. It under-predicts the share of entrepreneurs in the lowest decile. This suggests that other forces, such as the imperfections in credit markets, are also in play especially in the poorest countries. In the right panel, model and data displays opposing patterns. There is a positive relationship between the fraction of “Legislators, senior officials and managers” and development. We believe part of the weakness in the model is due to the assumption of two layers of production. It seems very natural to think that the largest firms in rich countries consist of several layers of production where there is a first layer made of workers and several layers of managers.¹⁴

We emphasize that the aggregate efficiency level z has no direct effect on the number of entrepreneurs in the model. If the distribution of human capital was the same across countries, the share of entrepreneurs would be flat. It is the endogenous accumulation of human capital in rich countries that allows for the creation of large firms and a small number of very talented entrepreneurs. In poor countries, neither workers nor entrepreneurs accumulate much human capital. This causes firms to be small and a large share of individuals (even the one with average ability) to be pulled into entrepreneurship.

As discussed in Section 3.3, our model implies that the transition to entrepreneurial status happens later in individuals career, after he crosses the threshold level of human capital. It happens through endogenous investments in human capital. This process is less prevalent in poorer countries for two reasons. First, entrepreneurship is a less selective occupation: we show above that in the poorest countries the marginal entrepreneur has a level of human capital close to the population average. Second, there is less human capital accumulation and so the transition is likely to happen earlier in life. We test this prediction in the data by reporting the difference between the share of entrepreneurs in the age group 18 to 30 and the share of entrepreneur in the age group 30 to 65. This difference is increasing with GDP per capita as expected from our theoretical model for both business-owners (left panel) and managerial occupations (right panel).

Finally, while we rationalized these empirical regularities using variations in z , we could explain some of it by a higher communication cost c in poor countries. This can be interpreted as variations in the ability of entrepreneurs to leverage their talents. When we increase the communication cost c by 20%, average firm size decreases by 80% and the share of entrepreneurs increases by 40%. The intuition is simple: when c rises, the return to human

¹⁴See Caliendo and Rossi-Hansberg (2012) for a model featuring an endogenous number of layers and see Caliendo et al. (2014) for an empirical investigation using French data.

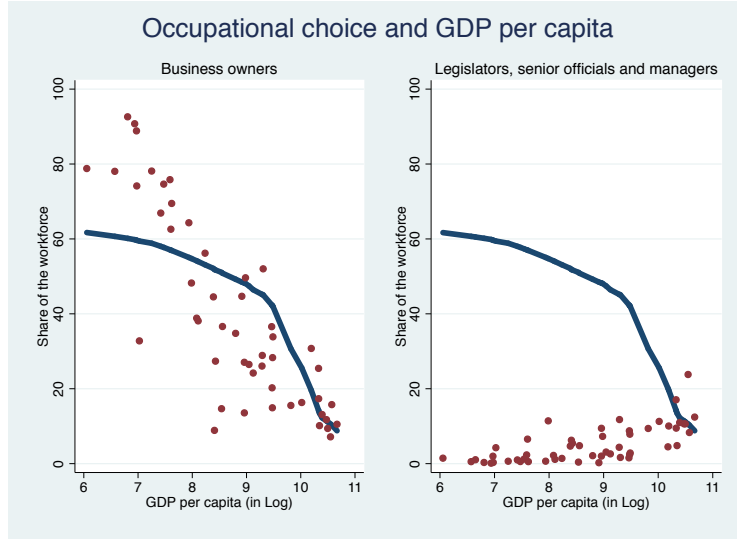


Figure 4: Fraction of entrepreneurs in the population across countries

Source: . Left Panel: the fraction of entrepreneurs is measured as the fraction of entrepreneurs and own account workers as a share of total workforce from International Labor Organization. Right Panel: the fraction of individuals in the occupation category “Legislators, senior officials and managers” in the total workforce from IPUMS. For each country, we took the latest year of survey available. Right Panel: GDP per capita is from PWT. 8.0. Solid line is from the model economy. Each dot represents a country.

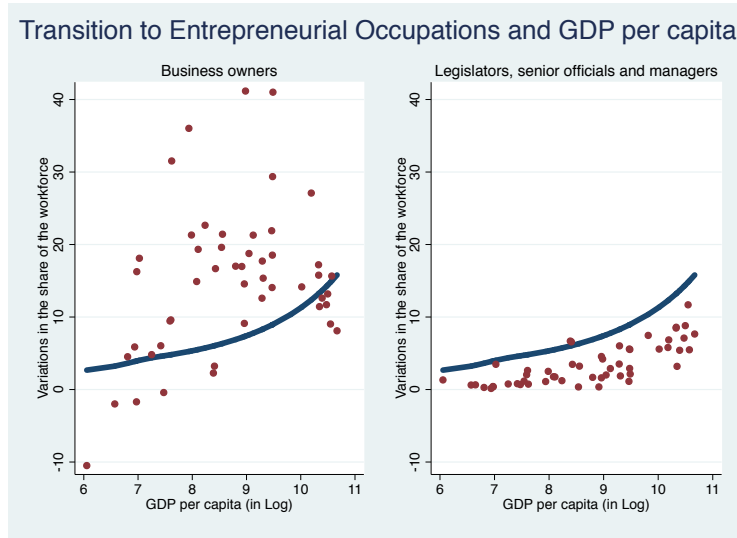


Figure 5: Fraction of entrepreneurs in the population across countries

Source: Left Panel: ILO. Right Panel: IPUMS. Difference between the share of entrepreneur in the age group 18 to 30 and the share of entrepreneur in the age group 30 to 65. GDP per capita is from PWT. 8.0. Solid line is from the model economy. Each dot represents a country.

capital accumulation is reduced. Entrepreneurs invest less in human capital and consequently firms do not grow as much. Hence, average firm size declines and there is larger fraction of entrepreneurs in the population.

4.3 Firm Growth

Hsieh and Klenow (2014) document that the average 40 year old plant employs almost eight times as many workers as the typical plant five years or younger in the U.S. In contrast, in poor countries, plants exhibit little growth in terms of either employment or output. In our model, there are two main channels that may prevent firms from growing in poorer countries because of the low level of human capital in the population. First, the human capital production technology uses both time and intermediate inputs. Intermediate inputs are less efficiently produced in countries with low aggregate efficiency level z . An entrepreneur has then less incentives to spend time away from production to improve his skills. It leads to lower firm growth through the same mechanism that leads workers to invest less in human capital, resulting in flatter age-earnings profiles. Second, an entrepreneur rewards to skills improvement are governed by the possibility to attract better workers and to increase firm size. The scarcity of talent in poorer countries limits an entrepreneur ability to increase his span of control.

There is various survey evidence where business-owners reports the lack of skills to be major impediment to the growth of firms in poor countries (see for instance Levy (1993)). We quantitatively examine this possibility in our model. We use data on output per capita, demographics and the price of capital in the US, Mexico and India and examine how far can our model go towards explaining differences in the life-cycle of plants across countries as documented by Hsieh and Klenow (2014). Figure 6 reports the model's predictions. Matching output per capita in Mexico and India requires a z , respectively, 51% and 17% lower than the US level. According to Hsieh and Klenow (2014), controlling for selection, firms grow by a factor of 4 in the US, 2 in Mexico and a little over 1 in India. Our theory predicts a factor of, 4 in the US, 2.6 in Mexico and 1.4 in India even though the life cycle of plants in Mexico and India is not used to discipline the model's parameters.

While the magnitudes are in line with the evidence, our model predict a flattening of firm growth as the entrepreneurs get older. This suggests that other forces are likely to prevent firm growth as a firm age. To match exactly the growth of firms across countries, we experiment by varying simultaneously z and the communication cost c . The latter parameter can be thought of capturing the level of trust or the probability of theft from employees.

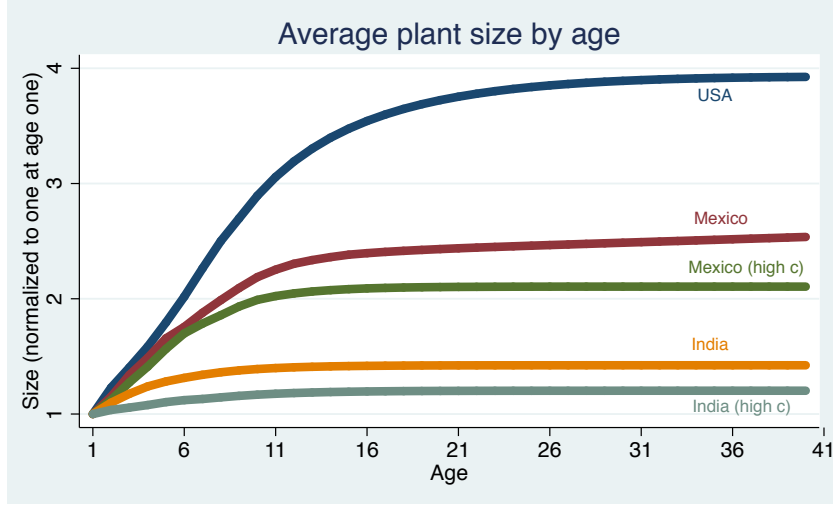


Figure 6: Life Cycle of Firms in the US, Mexico and India

Bloom et al. (2012) present a formal model of decentralization and trust along these lines and show its importance in explaining the organization of firms across countries. We view this exercise as a way of assessing the importance of other frictions. The results are also reported in Figure 6. We need to increase the communication cost by almost 30% in Mexico and by around 50% in India to match exactly the life-cycle of plants in these countries. As a result, the aggregate efficiency z goes up in both countries compared to our first experiment where c is constant across countries. Precisely, it goes up by 1% in Mexico and close to 5% in India.

4.4 Dispersion in Labor Productivity and Development

The model endogenously generates a higher dispersion in productivity across firms in poorer countries through two main forces: the selection into entrepreneurship and shifts in the distribution of skills. We first explain these mechanisms and then assess their quantitative importance. Figure 7 illustrates the two key forces at play.

The selection effect is illustrated on the left panel of Figure 7. It plots a distribution of problem solving abilities in the economy and shows the impact of decreasing the threshold level at which an individual becomes an entrepreneur, holding fixed the overall distribution. There is a larger fraction of individuals that are pulled into entrepreneurship in poorer countries where the average level of human capital is low. The area between the two marginal individuals measures the set of entrepreneurs in poor countries who would have been workers in richer countries. These individuals have a lower level of human capital than the en-

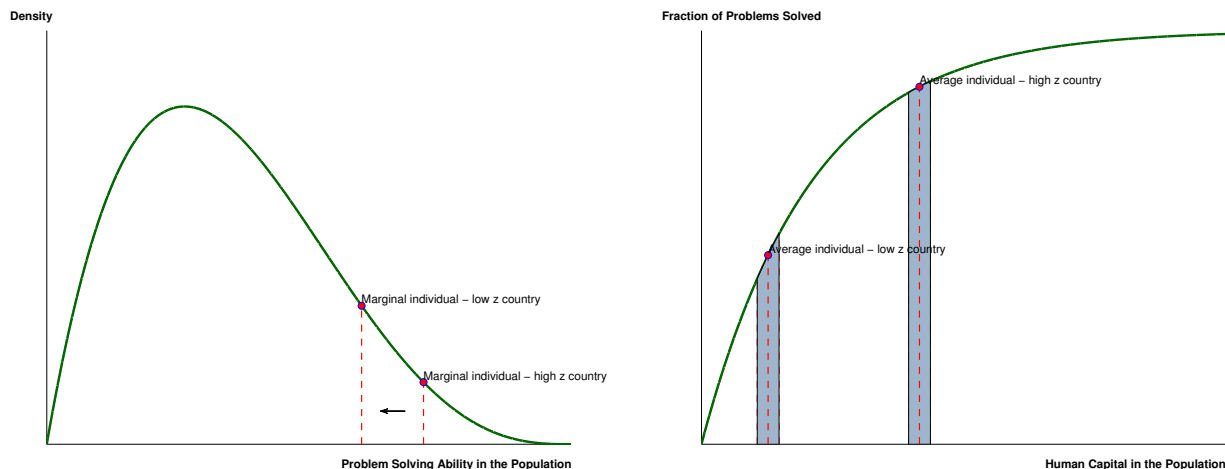


Figure 7: Firm Productivity Across Countries

trepreneurs, thereby contributing to the increased dispersion in firm productivity in poorer countries. This is precisely the long tail of low productivity firms that exists in India but not in the U.S as described by Bloom and Van Reenen (2007) and several follow-up papers. Essentially, our argument is that some measure of dispersion in productivity decreases when the degree of left truncation increases. While our quantitative results rely on parametric assumptions, the mechanism applies to a general family of distributions. Burdett (1996) shows that, for any continuous random variable X that is truncated below at \underline{x} , its variance V decreases with the degree of below truncation, $\frac{d}{d\underline{x}}V(X|X > \underline{x}) \leq 0$, if and only if the twice integrated survivor function is log-concave. This condition is satisfied by several well-known densities such as the Normal, Exponential and Uniform densities.

The second force at work is the shift in the skill distribution across countries as illustrated on the right panel of Figure 7. This is essentially a level effect. As aggregate efficiency increases, individuals have greater incentives to invest in human capital through either schooling or on-the-job training. The average level of human capital increases with economic development. Human capital is subject to diminishing returns and a small variation in the level of human capital leads to a greater variation in the problem solving ability of individuals in a poorer country than in a richer country. Hence, even if the same proportion of individuals decide to become entrepreneurs in both poor and rich countries, the dispersion in productivity will be higher in a poorer country due to diminishing returns. The right panel of Figure 7 represents the fraction of problems solved by a fixed interval of individuals depending on whether the average is high or low. As can be seen from this figure, there is more dispersion when the average is low than when it is high for a given dispersion in human

Table 6: Firm Productivity at 90th/10th Percentiles

Decile	z	Ratio relative to the US benchmark
US	1	1
90-100	0.93	1.03
80-90	0.81	1.04
70-80	0.68	1.07
60-70	0.60	1.13
50-60	0.52	1.16
40-50	0.42	1.21
30-40	0.37	1.33
20-30	0.28	1.58
20-10	0.22	1.83
0-10	0.15	1.91

capital. There is in principle another effect coming from the skill distribution: more human capital accumulation could lead to more inequality since it amplifies initial differences in human capital across entrepreneurs. From our simulation below, this last effect is not strong enough to reverse the selection effect and the level effect.

To illustrate the quantitative importance of the proposed mechanism, we report the ratio of productivity at 90th-percentile to productivity at 10th-percentile implied by the model at different levels of GDP per capita. The results are reported in Table 6. We take the US as a benchmark and we normalize the results to the level of dispersion in the US. The dispersion in productivity decreases with the level of development. Our model predicts that countries in the lowest decile have twice as much dispersion as the US. The effect is entirely due to the variation in aggregate efficiency level as there is no misallocation in the model. The effects are quite non-linear with noticeable increase in dispersion for countries with GDP below the median.

To relate our results, to Hsieh and Klenow (2009), we vary aggregate efficiency z , demographics and the price of capital relative to the US level to match the level of output per worker in China and India. We then calculate dispersion in productivity across firms and dispersion in labor productivity using both the number of employees and the wage bill to measure the number of employees (the equivalent of their revenue productivity measure). The rationale behind using the wage bill to measure the size of the firm is to measure the “efficiency” units of labor used by each firm. The numbers are reported in Table 7. We obtain, respectively, 30% and 90% more dispersion in productivity in China and India when employment is measured by the number of employees. If we measure employment by the

Table 7: 90/10 Ratio - Productivity and Labor Productivity in China, India and the US.

Employment Measure	Labor Productivity		Revenue Prod. (Hsieh/Klenow)
	Number of workers	Wage Bill	Wage Bill
USA	1.76	1.71	3.28
China	2.36	2.10	4.90
India	3.31	2.77	4.95
	normalized	normalized	normalized
China	1.34	1.23	1.49
India	1.88	1.62	1.50

wage bill, we get 20% more dispersion in China and 60% more dispersion on India. Measuring employment with the wage bill reduces labor productivity dispersion in poorer countries but does not eliminate it. Since we deviate from the efficiency units assumption, the wage rate is not a linear function of human capital and it is an imperfect proxy for human capital.

There are two important elements to our quantitative results and the work of Bloom and Van Reenen (2007) and Bloom et al. (2013) provide strong empirical support for these elements. First, management is critical in understanding firms productivities. Using a Census survey of over 30,000 plants across the US, they estimate that the increase in productivity associated with moving from the 10th to the 90th percentile of the management practices distribution can be as large as 60%. Further, Bloom et al. (2013) estimate that good management practices are correlated with the share of employees having a college degree and it rises with firm size. This is precisely what our theory predicts. Second, poor countries are characterized by a higher dispersion in management practice. Bloom and Van Reenen (2007) document that Chinese and Indian plants are characterized by a long tail of poorly managed firms which, again, is an implication of our model.

It is important to notice that we predict less dispersion than the numbers reported by Hsieh and Klenow (2009). We predict a 90/10 ratio of about half of the dispersion observed in the US. This suggests that there are several sources of misallocations documented in the literature (such as financial frictions or size-dependent policies) that would help matching the level of dispersion observed in the data. Studying their interaction with human capital is an important issue we leave for future research. Finally, several extensions of our framework could lead to more efficient dispersion. Entrepreneurial human capital is the only source of heterogeneity in our model. One could think of a more general model where high human capital entrepreneurs are matched with different products leading to an amplification of talent differences.

5 Conclusion

An active line of work suggests that the allocation of resources within countries is of first-order importance for understanding cross-countries income differences. This paper examines how productivity differences lead to different forms of organization of production through its impact on the allocation and accumulation of human capital.

We develop a model of human capital accumulation of workers and entrepreneurs with complementarities and sorting combining the work of Ben-Porath (1967), Lucas (1978), Garicano (2000) and Garicano and Rossi-Hansberg (2006). The model yields a number of empirical implications for earnings and firm heterogeneity within a country. Varying aggregate efficiency across countries to match GDP per capita differences, we find that human capital differences can explain a significant part of differences in firm size, firm growth, and occupational choice. We also show that in our economic environment, where there is only an aggregate variation in efficiency, richer countries are characterized by less firm productivity dispersion and less labor productivity dispersion.

Our model takes seriously the idea that entrepreneur and worker skills are important for understanding the organization of firms both within and across countries. We view this paper as a first step toward understanding the importance of sorting and entrepreneur and worker quality in the allocation of input factors, the organization of firms and economic development. In particular, these are affected by other factors such as credit markets and technology choices. We believe these are of first-order importance and their interaction with human capital accumulation decisions is an important topic worthy of further exploration.

To conclude, it is important to underscore that our paper does not suggest the absence of misallocation, a clear and real feature of the world. Rather we find that the allocation of talent is likely to differ across of countries with varying levels of efficiency which endogenously leads to varying dispersion in firm outcomes. The natural next step is to examine how idiosyncratic distortions lead to different effects in countries with varying level of efficiency. Indeed, the organization of production and endogenous acquisition of skills are likely to lead distortions to have different impact on countries that are in different stages of development. We leave these questions for future research.

References

- Abowd, J. M., Kramarz, F., and Margolis, D. N. (1999). High wage workers and high wage firms. *Econometrica*, 67(2):251–333.
- Antràs, P., Garicano, L., and Rossi-Hansberg, E. (2006). Offshoring in a knowledge economy. *The Quarterly Journal of Economics*, 121(1):31–77.
- Autor, D. and Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the U.S. labor market. *American Economic Review*, 103(5):1553–97.
- Banerjee, A. V. and Duflo, E. (2005). Growth theory through the lens of development economics. *Handbook of economic growth*, 1:473–552.
- Barro, R. J. and Lee, J. W. (2013). A new data set of educational attainment in the world, 1950–2010. *Journal of Development Economics*, 104:184–198.
- Becker, G. (1973). A theory of marriage: Part I. *The Journal of Political Economy*, 81:813–846.
- Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *The Journal of Political Economy*, 75(4):352–365.
- Bento, P. and Restuccia, D. (2014). Misallocation, establishment size, and productivity. Working Paper 517, University of Toronto.
- Bhattacharya, D., Guner, N., and Ventura, G. (2012). Distortions, endogenous managerial skills and productivity differences. *Review of Economic Dynamics*.
- Bloom, N., Sadun, R., and Van Reenen, J. (2012). The organization of firms across countries. *The Quarterly Journal of Economics*, 127(4):1663–1705.
- Bloom, N., Sadun, R., and Van Reenen, J. (2013). Management as a technology. mimeo, London School of Economics.
- Bloom, N. and Van Reenen, J. (2007). Measuring and explaining management practices across firms and countries. *The Quarterly Journal of Economics*, 122(4):1351–1408.
- Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and development: A tale of two sectors. *American Economic Review*, 101:1964–2002.

- Burdett, K. (1996). Truncated means and variances. *Economics Letters*, 52(3):263–267.
- Cagetti, M. and De Nardi, M. (2006). Entrepreneurship, frictions, and wealth. *Journal of Political Economy*, 114(5):835–870.
- Caliendo, L., Monte, F., and Rossi-Hansberg, E. (2014). The anatomy of french production hierarchies. *Journal of Political Economy*, forthcoming.
- Caliendo, L. and Rossi-Hansberg, E. (2012). The Impact of trade on organization and productivity. *The Quarterly Journal of Economics*, 127(3):1393–1467.
- Card, D., Heining, J., and Kline, P. (2013). Workplace heterogeneity and the rise of West German wage inequality. *The Quarterly Journal of Economics*, pages 967–1015.
- Chatterjee, S. and Rossi-Hansberg, E. (2012). Spinoffs and the market for ideas. *International Economic Review*, 53(1):53–93.
- Cooley, T. F. and Quadrini, V. (2001). Financial markets and firm dynamics. *American Economic Review*, pages 1286–1310.
- Eeckhout, J. and Kircher, P. (2012). Assortative matching with large firms: Span of control over more versus better workers. Mimeo, University of Edinburgh.
- Erosa, A., Koreshkova, T., and Restuccia, D. (2010). How important is human capital? a quantitative theory assessment of world income inequality. *The Review of Economic Studies*, 77(4):1421–1449.
- Garicano, L. (2000). Hierarchies and the organization of knowledge in production. *Journal of Political Economy*, 108(5):874–904.
- Garicano, L. and Rossi-Hansberg, E. (2006). Organization and inequality in a knowledge economy. *The Quarterly Journal of economics*, 121(4):1383–1435.
- Garicano, L. and Rossi-Hansberg, E. (2012). Organizing growth. *Journal of Economic Theory*, 147(2):623–656.
- Gennaioli, N., Porta, R., Lopez-de Silanes, F., and Shleifer, A. (2013). Human capital and regional development. *The Quarterly Journal of Economics*, pages 105–164.
- Gollin, D. (2008). Nobody’s business but my own: Self-employment and small enterprise in economic development. *Journal of Monetary Economics*, 55(2):219–233.

- Grossman, G. M., Helpman, E., and Kircher, P. (2014). Matching, sorting, and the distributional impacts of international trade. mimeo, University of Edinburgh.
- Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, pages 1127–1150.
- Hopenhayn, H. A. (2014). Firms, misallocation, and aggregate productivity: A review. *Annual Review of Economics*, 6(1):735–770.
- Hsieh, C.-T. and Klenow, P. (2009). Misallocation and manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403.
- Hsieh, C.-T. and Klenow, P. J. (2014). The life cycle of plants in india and mexico. *The Quarterly Journal of Economics*, 129(3):1035–1084.
- Hsieh, C.-T. and Olken, B. A. (2014). The missing "missing middle". *Journal of Economic Perspectives*, 28(3):89–108.
- Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica*, pages 649–670.
- Jovanovic, B. (1994). Firm formation with heterogeneous management and labor skills. *Small Business Economics*, 6(3):185–191.
- Jovanovic, B. (2014). Misallocation and growth. *American Economic Review*, 104(4):1149–71.
- Kennan, J. (2006). A note on discrete approximations of continuous distributions. mimeo, University of Wisconsin Madison.
- Kremer, M. (1993). The O-ring theory of economic development. *The Quarterly Journal of Economics*, 108(3):551–575.
- Lagakos, D., Moll, B., Porzio, T., and Qian, N. (2012). Experience matters: Human capital and development accounting. NBER Working Papers 18602, National Bureau of Economic Research.
- Levy, B. (1993). Obstacles to developing indigenous small and medium enterprises: an empirical assessment. *The World Bank Economic Review*, 7(1):65–83.
- Lucas, R. E. (1978). On the Size Distribution of Business Firms. *The Bell Journal of Economics*, 9(2):508.

- Manuelli, R. E. and Seshadri, A. (2014). Human capital and the wealth of nations. *American Economic Review*, 104(9):2736–62.
- McCann, R. J., Shi, X., Siow, A., and Wolthoff, R. P. (2014). Becker meets ricardo: Multi-sector matching with social and cognitive skills. mimeo, University of Toronto.
- Midrigan, V. and Xu, D. Y. (2014). Finance and misallocation: Evidence from plant-level data. *American Economic Review*, 104(2):422–58.
- Oi, W. Y. and Idson, T. L. (1999). Firm size and wages. *Handbook of labor economics*, 3:2165–2214.
- Poschke, M. (2014). The firm size distribution across countries and skill-biased change in entrepreneurial technology. IZA Discussion Paper 7991, IZA.
- Quadrini, V. (2000). Entrepreneurship, saving, and social mobility. *Review of Economic Dynamics*, 3(1):1–40.
- Ramos, R. and Santana, G. (2013). Distortions and the size distribution of plants: Evidence from cross-country data. mimeo, CEMFI.
- Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.
- Rosen, S. (1982). Authority, control, and the distribution of earnings. *The Bell Journal of Economics*, pages 311–323.
- Sattinger, M. (1975). Comparative advantage and the distributions of earnings and abilities. *Econometrica*, pages 455–468.
- Syverson, C. (2011). What determines productivity? *Journal of Economic Literature*, 49(2):326–65.
- Tybout, J. R. (2000). Manufacturing firms in developing countries: How well do they do, and why? *Journal of Economic literature*, pages 11–44.

Appendix

This Appendix is divided into two parts. We prove Proposition 1 and give the details of the numerical solution of the model.

5.1 Proof of Proposition 1

Defined the static rental rate as $R(h_m) = \frac{A G(h_m) - w(h_w)}{c(1 - G(h_w))}$ where h_w is optimally chosen. Using the envelope theorem:

$$R'(h_m) = \frac{A g(h_m)}{c(1 - G(h_w))}$$

leading the cross-partial derivative to be strictly positive:

$$\frac{dR'(h_m)}{dh_p} = g(h_w) \frac{A g(h_m)}{c(1 - G(h_w))^2} > 0$$

Since it is a maximum, the objective function is concave. Combined with $\frac{dR'(h_m)}{dh_p} > 0$, one can apply the implicit function theorem to find that:

$$\frac{\partial h_m}{\partial h_w} > 0$$

This proves positive sorting.

We now prove that the set of entrepreneurs and the set of workers is connected. By contradiction. Assume the set of workers W and the set of entrepreneurs E writes as

$$\begin{aligned} W &= [a_1, a_2] \cup [a_3, a_4] \\ E &= [a_2, a_3] \cup [a_4, a_5] \end{aligned}$$

with $m(a_1) = a_2$ and $m(a_2) = a_3$. Given a_1 and a_3 , we can solve for w_{13}, R_{13} and a_2 on (a_1, a_3) . Given a_3 and a_5 , we can solve for w_{35}, R_{35} and a_4 on (a_3, a_5) . For this to be an equilibrium, we need to make sure that agents in the interval (a_3, a_5) do not want to form teams with agents in the interval (a_1, a_3) .

Note that $R_{13}(a_3) = w_{35}(a_3)$. If $R_{13}(a_3) > w_{35}(a_3)$, agents (workers) with skills $a_3 + \epsilon$ would like to become entrepreneurs and agents at a_2 would be willing to worker for them at a wage marginally larger than $w_{13}(a_2)$.

If $R_{13}(a_3) < w_{35}(a_3)$, agents $a_3 - \epsilon$ would like to become workers and agent at a_4 would like to hire them at a wage marginally higher than $R_{13}(a_3)$. We prove that $\lim_{z \uparrow a_3} R'_{13}(z) > w'_{35}(z)$.

There are two cases to consider:

(1) $w_{13}(a_2) < AG(a_2)$. Since $R_{13}(a_3) = w_{35}(a_3)$,

$$w'_{35}(a_3) = g(a_3) \frac{AG(a_4)c(1 - G(a_2)) - AG(a_3) + w_{13}(a_2)}{(1 - G(a_3))c(1 - G(a_2))}$$

We want to show

$$\frac{Ag(a_3)}{c(1 - G(a_2))} > g(a_3) \frac{AG(a_4)c(1 - G(a_2)) - AG(a_3) + w_{13}(a_2)}{(1 - G(a_3))c(1 - G(a_2))}$$

which simplifies to

$$G(a_4) < \frac{1}{c} \frac{A - w_{13}(a_2)}{A - AG(a_2)}$$

This inequality is necessarily true since $G(a_4) < 1$, $c < 1$ and $w_{13}(a_2) < AG(a_2)$.

(2) $w_{13}(a_2) \geq AG(a_2)$. First we show that

$$w_{35}(a_3) = R_{13}(a_3) > AG(a_3)$$

Using the mean value theorem, for some $x \in [a_2, a_3]$

$$R_{13}(a_3) - AG(a_3) = (R_{13}(a_2) - AG(a_2)) + (R'_{13}(x) - Ag(x))(a_3 - a_2)$$

The first term in the RHS is positive since $R_{13}(a_2) = w_{13}(a_2)$ and by assumption $w_{13}(a_2) \geq AG(a_2)$. Regarding the second term in the RHS it is also positive since

$$R'_{13}(x) = \frac{Ag(x)}{c(1 - G(m^{-1}(x)))} > Ag(x)$$

and $c(1 - G(m^{-1}(x))) < 1$. We can then conclude $R_{13}(a_3) > AG(a_3)$.

Since $Ag(a_3) < R'_{35}(a_3)$, it is sufficient to show that $w'_{35}(a_3) < Ag(a_3)$ which simplifies to

$$\frac{AG(a_4) - w_{35}(a_3)}{A - AG(a_3)} < 1$$

This is true since we proved $w_{35}(a_3) > AG(a_3)$.

We now show that there is a contradiction. Precisely, a_4 would like to hire $a_3 - \epsilon$ at a

better wage than what he makes as an entrepreneur. Consider the rents that a_4 would get from hiring $a_3 - \epsilon$ at wage $R_{13}(a_3 - \epsilon)$:

$$\begin{aligned}\Pi(a_4, a_3 - \epsilon) &= \frac{A G(a_4) - R_{13}(a_3 - \epsilon)}{c(1 - G(a_3 - \epsilon))} \\ \lim_{\epsilon \rightarrow 0} \frac{\partial \Pi(a_4, a_3 - \epsilon)}{\partial \epsilon} &= \frac{R'_{13}(a_3) - w'_{35}(a_3)}{c(1 - G(a_3))} > 0\end{aligned}$$

Hence there are incentives for agents to form different teams.

Finally, we need to prove the existence of a $c^* > 0$ such that the allocation guaranteed to exist is such that $R'(h^*) > w'(z^*)$. Consider the incentives of an entrepreneur with ability \bar{h} to hire a worker with ability $h^* + \epsilon$. Her profits are given by

$$\begin{aligned}\Pi(\bar{h}, h^* + \epsilon) &= \frac{A G(\bar{h}) - R(h^* + \epsilon)}{c(1 - G(h^* + \epsilon))} \\ \lim_{\epsilon \rightarrow 0} \frac{\partial \Pi(\bar{h}, h^* + \epsilon)}{\partial \epsilon} &= \frac{w'(h^*) - R'(h^*)}{c(1 - G(h^*))}\end{aligned}$$

Substituting $w(h^*) = R(h^*) = \frac{A G(h^*) - w(\underline{h})}{c(1 - G(\underline{h}))}$ in the optimal choice of h_w for h^* :

$$w'(h^*) = g(h^*) \frac{c(1 - G(\underline{h})) A G(\bar{h}) - A G(h^*) + w(\underline{h})}{(1 - G(h^*)) c(1 - G(\underline{h}))}$$

We want to show that $w'(h^*) < R'(h^*)$ which is equivalent to

$$c < \frac{A - w(\underline{h})}{(1 - G(\underline{h})) A G(\bar{h})}$$

The left-hand side of the inequality provides an upper bound for c .

5.2 Numerical Solution

We use the following algorithm to solve the model.

1. Guess $n_{mtj}, n_{wtj}, j = 1, \dots, J, t = 1, \dots, T$ to obtain an estimate of f_{tj} .
2. Guess \tilde{h} , solve for m and h^* using:

$$m'(h) = c(1 - G(h)) \frac{\sum_{i=1}^J \sum_{\tau=1}^T f_{ti}(h) n_{w\tau i}(h)}{\sum_{j=1}^J \sum_{t=1}^T n_{mt}(m(h)) \times f_{tj}(m(h))}$$

and two boundary conditions:

$$\begin{aligned} m(\tilde{h}) &= h^* \\ m(h^*) &= \bar{h} \end{aligned}$$

3. solve for w and $w(\tilde{h})$ so that

$$w'(h_w) = g(h_w) \frac{A G(m(h_w)) - w(h_w)}{1 - G(h_w)}$$

and $w(h^*)$ satisfies the boundary condition:

$$\frac{A G(h^*) - w(\tilde{h})}{c(1 - G(\tilde{h}))} = w(h^*)$$

4. solve for the demand of raw labor l_u and the supply of raw labor. Adjust \tilde{h} and go back to Step 2 until reaching the equilibrium in the market for raw-labor.
5. Given m, w, \tilde{h}, h^* , solve the individuals problem by backward induction and discretizing the state space.
6. Check whether the policy functions and matching function have changed since the previous iteration. Go back to Step 2 until convergence.